MAS 3105 — June 15, 2006

Quiz 6 and Key — Prof. S. Hudson

1) Let S be the subspace of R^3 spanned by $\mathbf{x} = (2, 1, 1)^T$. Find a basis of $S^\perp$.

2) Find the distance from the point P(1,2,3) to the plane $x + y + 2z = 0$.

3) Choose ONE of these.

a) [based on MHW 4.1] Suppose that $F = \{\mathbf{w}_1, \ \mathbf{w}_2, \ \mathbf{w}_3\} = \{\mathbf{e}_3, \ \mathbf{e}_1 + \mathbf{e}_2, \ \mathbf{e}_1\}$ is a basis for R^3, and $L : R^3 \to R^3$. Suppose $L(\mathbf{w}_1) = 4\mathbf{w}_1$ and $L(\mathbf{w}_2) = 4\mathbf{w}_1 + 3\mathbf{w}_2$ and $L(\mathbf{w}_3) = 4\mathbf{w}_1 + 3\mathbf{w}_2 + 2\mathbf{w}_3$. Find the matrix representation of L with respect to F.

b) If A and B are similar, then rank $A =$ rank B.

c) State and prove (explain) the normal equations used in Least Squares problems.

Remarks and Answers: The average was about 48/60, very good for a Quiz 6. In the upper right corner, I averaged your best 5 quiz grades and gave you an approx semester letter grade. This does not yet include your HW or MHW grades. If you have not handed in MHW, or have very unusual HW/MHW scores, this estimate is probably not very accurate.

1) $\{ \ [-1/2 \ \ 1 \ \ 0]^T, \ [-1/2 \ \ 0 \ \ 1]^T \}$

2) The scalar projection of P onto the normal vector N $= 9/\sqrt{6}$.

3a) As in the HW/MHW, you can ignore the $\mathbf{e_j}$ part. The first column of the answer is $\mathbf{a_1} = [4 \ \ 0 \ \ 0]^T$, etc.

3b) From $A = S^{-1}BS$, get $SA = BS$. From HW, we know $N(SA) = N(A)$ so these two have the same nullity, the same width, thus the same rank. Now, show rank (BS)= rank (B). Since the S is on the right, it is not true that $N(BS) = N(B)$. So, you need a little trick (take the transpose first, which doesn't change the rank).