MAS 3105 April 5, 2007

Quiz 6 and Key Prof. S. Hudson

1) Write out explicitly the normal equations for this inconsistent system. But you don't have to solve them, to actually find the least squares solution.

$-x_1 + x_2 = 10$

$2x_1 + x_2 = 5$

$x_1 - 2x_2 = 20$

2) Let $V = C[0,1]$, with inner product $\langle f, g\rangle = \int_0^1 f(x)g(x)\, dx$. Use this to find $||x+1||$ .

3) Choose ONE of these.

a) [HW 5.2.13a,b] Assume $\mathbf{x} \in N(A^TA)$. Prove that $A\mathbf{x} \in N(A^T)$ and $A\mathbf{x} \in R(A)$ and $\mathbf{x} \in N(A)$.

b) State and prove the Fundamental Subspace Theorem (5.2.1).

**Remarks and Answers:** The average was about 40/60. The unofficial scale is A's 45-60, with 6 points per letter below that. I also computed your semester average, based on your best 5 out 6 quiz scores so far, and wrote a corresponding letter grade in the upper corner of your quiz. The scale for that was: A's 240-300, B's 210-240, etc (30 points per letter). I will include HW and MHW later, of course. If you have not handed-in MHW, for example, your actual grade is probably worse than what I wrote.

1) The normal equations are $A^TA\hat{x} = A^Tb$, so you substitute into that, and get

$$\begin{pmatrix} 6 & -1 \\ -1 & 6 \end{pmatrix} \hat{x} = \begin{pmatrix} 20 \\ -25 \end{pmatrix}$$

I intended for you to multiply out $A^TA$ and $A^Tb$, but gave full credit without that. I gave 2 points extra credit if you did so.

2) In $R^n$, we compute the norm of a vector from $||x|| = (x^Tx)^{1/2}$. In an inner product space, we use $\langle x, x\rangle$ instead. $(\int_0^1 (x+1)^2\, dx)^{1/2} = \sqrt{7/3}$.

3a) [This was HW, and we also did it in class]. The part about $Ax \in R(A)$ is from basic definitions (see pages 36-37). And $\mathbf{x} \in N(A^TA)$ means $A^TA\mathbf{x} = \mathbf{0}$ which means $A\mathbf{x} \in N(A^T)$. By the main thm (5.2.1), these show that $A\mathbf{x} \in S \cap S^{\perp} = \{\mathbf{0}\}$ (see Remark 1 on page 227).

3b) See the text. You should really include the paragraph *above* Theorem 5.2.1, too, though I wasn't very strict about that this time.