MAS 3105 Nov 21, 2008
Quiz 6 Prof. S. Hudson

1) Let $V = C[0, 2\pi]$ with inner product $\langle f, g\rangle = \frac{1}{\pi}\int_0^{2\pi} f(x)g(x)\,dx$.

a) Show that 1 is orthogonal to $\cos(5x)$ in V.

b) Compute $||\sin(3x)||$. [If you have forgotten some Calculus/Trig skills, you can replace $\sin(3x)$ by x^2, for partial credit].

2) Find the distance from the point P(1,2,3) to the plane $x + y + 2z = 0$.

3) Choose ONE of these.

a) [based on MHW 4.1] Suppose that $F = \{\mathbf{w}_1,\ \mathbf{w}_2,\ \mathbf{w}_3\} = \{\mathbf{e}_3,\ \mathbf{e}_1 + \mathbf{e}_2,\ \mathbf{e}_1\}$ is a basis for R^3, and $L : R^3 \to R^3$. Suppose $L(\mathbf{w}_1) = 4\mathbf{w}_1$ and $L(\mathbf{w}_2) = 4\mathbf{w}_1 + 3\mathbf{w}_2$ and $L(\mathbf{w}_3) = 4\mathbf{w}_1 + 3\mathbf{w}_2 + 2\mathbf{w}_3$. Find the matrix representation of L with respect to F.

b) Derive the formula for the projection matrix P for a Least Squares problem (it represents projection onto $R(A)$). Then, show that $P^2 = P$.

c) Show that if rank $(A) = n$ (= number of columns), then the normal equations have a unique solution $\hat{x}$. (You may use the textbook proof, or repeat HW 5.2.13, etc)

Remarks and Answers: The average was about 40/60. The scale for this quiz is: A-'s start at 48, B-'s at 41, C-'s at 35, etc). I will post a revised semester scale ASAP. I expect it will be about 1-2 points lower than the one posted on the Quiz 5 key.

1a) $\langle 1, \cos(5x)\rangle = \frac{1}{\pi}\int_0^{2\pi} \cos(5x)\,dx = 0$

1b) $\langle \sin(3x), \sin(3x)\rangle = \frac{1}{\pi}\int_0^{2\pi} \sin^2(3x)\,dx = 1$ (and $1^{1/2} = 1$). Recall that $\sin^2(3x) = (1 - \cos(6x))/2$.

2) The scalar projection of P onto N is $\frac{9}{\sqrt{6}}$.

3a)

$$\begin{pmatrix} 4 & 4 & 4 \\ 0 & 3 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$

3b) Get $P = A(A^TA)^{-1}A^T$ from the normal eqns. Then,

$$P^2 = A(A^TA)^{-1}\ [A^TA(A^TA)^{-1}]\ A^T = A(A^TA)^{-1}A^T = P$$

3c) See the text or lectures. Of course, the main idea is to show A^TA is nonsingular.