MAS 3105 AM April 3, 2008
Quiz 6 Prof. S. Hudson

1) Let S be the subspace of R^3 spanned by $\mathbf{x} = (2, 1, 1)^T$. Find a basis of $S^\perp$.

2) Find the least squares solution to the system (see Example 1, page 238).

3) Choose ONE of these.

a) State and prove the Fundamental Subspace Theorem (5.2.1).

b) Prove that Manhattan distance ($||x||_1$) defines a *norm* on R^2.

c) Show for every matrix A, that A^TA and A have the same rank.

Remarks and Answers: The average grade was about 49/60. I averaged your best 5 out of 6 quiz grades in the upper right corner (as on Quiz 5) and wrote your approximate semester grade there. The average for that was also about 49 or 50 out of 60. The unofficial scale for that, and for Quiz 6 is: A's 54-60, B's 49-53, C's 43-48, D's 38-42. I lowered this slightly in the C/D range after making the notes on your quiz.

1) Use GE and α, β as usual to get $S = \text{span}\ \{(-1/2,\ 0,\ 1)^T,\ (-1/2,\ 1,\ 0)^T\}$. It might be possible to do this in your head, but show your work (and/or explain your reasoning) for full credit.

2) I gave at least 10 points if you knew the normal equations and substituted into them successfully. Solving the system involves some fractions; the simplest method is probably to compute the inverse of A^TA using the adjoint method. $A^TA\hat{x} = A^Tb$ becomes

$$\begin{pmatrix} 9 & -7 \\ -7 & 11 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \quad \text{So,} \quad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{50} \begin{pmatrix} 11 & 7 \\ 7 & 9 \end{pmatrix} \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} 83/50 \\ 71/50 \end{pmatrix}$$

3) Most people chose a) or c); see the text. In a), you should explain why $R(A^T) \perp N(A)$ (see the paragraph above Theorem 5.2.1). I can see how you might think this wasn't part of the proof, so I only took off about 2 points for that. You are always welcome to ask me about such things before or during the quiz. Likewise, part c) was Problem 5.2.13, parts a thru c. Explain why $N(A^TA) = N(A)$.