MAS 3105 June 11, 2009
Quiz 6 and Key Prof. S. Hudson

1a) Let $P_1 = (1, 2, 2)$, $P_2 = (0, 1, 2)$ and $P_3 = (0, 1, 1)$. Find a nonzero normal vector $\mathbf{N}$, for the plane containing these three points in R^3. [You may be able to solve this without GE - if so, be sure to show enough work, or explain your reasoning].

1b) Find an equation for this plane.

2) Give an explicit example of a matrix $B \neq A$ which is similar to A:

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

3) Choose ONE and circle it. Answer on the back.

a) State and prove the Fundamental Subspace Theorem (5.2.1).

b) Prove (from the definition) that if A is similar to B and B is similar to C, then A is similar to C.

Bonus [5pts]: Give an example of an inconsistent system which has more than one Least Squares solution, and find at least two solutions.

Remarks and Answers: The average grade was approx 38 out of 60, a bit low, but not too bad. Problem 1a) was similar to a moderately-difficult HW problem 5.2.5; problem 2 should be pretty easy. The scale for this quiz is the same as for Q4.

I computed the sum of your best 5 quiz grades and wrote your semester grade in the corner, as before. The average for this sum was 237 out of 300. The two highest grades are approx 270/300. I used a scale similar to the one on the syllabus:

A's = 250-300, B's = 220-249, C' = 190-219, etc.

1a) Answer = $\mathbf{N} = [1, -1, 0]^T$ (or any nonzero scalar multiple of that).

Set $\mathbf{v_1} = \overrightarrow{P_2P_1} = [1, 1, 0]^T$ and $\mathbf{v_2} = \overrightarrow{P_3P_2} = [0, 0, 1]^T$, which lie in the plane. You can use others, as many as you want, but two are enough. Put these into a matrix A and compute $N(A^T)$, as in the Ch 5.2 HW. With the choice of $\mathbf{v_j}$ above, the system is already in RREF and easy to solve, but with other choices, you might need GE. Misc Remarks:

Unless this plane contains the origin (it doesn't), you cannot do the same thing with $\mathbf{v_1} = [1, 2, 2]^T$, etc, since these vectors will not be in the plane.

If you've had MAC 2313, you know the cross product can be used for this problem, but it cannot be used for similar problems in R^4 etc.

1b) $1 \cdot (x_1 - 1) - 1 \cdot (x_2 - 2) + 0 \cdot (x_3 - 2) = 0$ (which can be simplified, of course). Now, we can see that (0,0,0) does not fit.

2) Set $B = S^{-1}AS$ where S is almost any 2x2 matrix you choose (but don't choose $S = I$ or $S = A$, since you want to get $B \neq A$). So, there are many correct answers. The simplest choice that I saw was a elementary matrix with $S = S^{-1}$, eg:

$$B = S^{-1}AS = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

I gave partial credit, if your only mistake(s) were minor. But a bit less than usual, because your answer is easy to check. For example, det (B) = det $(A) = 1$. Soon, we will see that the two similar matrices must also have the same eigenvalues and the same *trace*, so $b_{11} + b_{22} = a_{11} + a_{22} = 2$.

3) See the text for a). Part b) was from the HW and was much easier. The most common error was to use the letter S too often. The word *similar* appears 3 times, so you need notation like S_1, S_2 and S_3.

Bonus: You don't want A to have full rank; you want its columns to be LD. For example, $x_1 + x_2 = 5$ and $x_1 + x_2 = 6$. Even without solving the normal equations, you might guess that this has many L.Sq. solutions, such as $\hat{x} = [5.5, 0]^T$ and $\hat{x} = [0, 5.5]^T$.

Nobody got full credit on this one; there were several answers which seemed close, but weren't quite legible.