MAS 3105 April 22, 2003
Final Exam Prof. S. Hudson

Name

Show all your work and reasoning for maximum credit. If you continue your work on another page, be sure to leave a note. Do not use a calculator, book, or any personal paper. You may ask about any ambiguous questions or for extra paper. If you use extra paper, hand it in with your exam.

1) Prove that the product of two orthogonal matrices is also an orthogonal matrix.

2) Find an orthogonal or unitary diagonalizing matrix for

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

3) Suppose that U and V are subspaces of W. Show that $U \cap V$ is also a subspace.

4) Let $L : R^2 \to R^2$ be a linear transformation. Assume

$$L((2,0)^T) = (2,6)^T \quad \text{and} \quad L((0,3)^T) = (6,12)^T$$

Find the matrix representation of L for the standard basis of R^2.

5) Answer True or False:

Every inconsistent linear system is overdetermined.
If det (A) is nonzero, then A is a product of elementary matrices.
Any two similar matrices must have the same eigenvalues.
If A is a 4x7 matrix then $Ax = 0$ has nontrivial solutions.
If $L : R^2 \to R^2$ is linear, and $\mathbf{x} \perp \mathbf{y}$ in R^2, then $L(\mathbf{x}) \perp L(\mathbf{y})$.
Every square matrix is similar to some upper triangular matrix T.
If A is Hermitian then its diagonal entries (the a_{ii}) are all real numbers.
For every square matrix A, det $(2A) = 2$ det (A).
Every normal matrix is diagonalizable.
If A is a 4x7 matrix then $A^T A = I$ is possible, but $AA^T = I$ is not.

6) Choose ONE:

A) Prove that $N(A) = R(A^T)^\perp$.
B) If A is nxn and has n orthonormal eigenvectors then A is normal.
C) An nxn matrix A is diagonalizable only if A has n linearly independent eigenvectors.

7) Let

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 12 \\ 6 \\ 18 \end{pmatrix}$$

a) Use the Gram-Schmidt process to find an orthonormal basis for $R(A)$.
b) Factor A into a product QR.
c) Use b) to solve the least squares problem $Ax = b$.

8) Given A below, find these determinants and label your answers clearly:

a) Find det (A^4)

b) Find det $(A^T A)$

c) Find det (A^{-1})

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{pmatrix}$$

Bonus (about 5 pts): Is it true that if $A^3 = O$ (a square zero matrix), then $A^2 = O$? Prove, or give a counterexample.