Correction to the Lecture Notes, Dec 4, 2006

Background: We had just diagonalized a rotation matrix A. It was not Hermitian, but it was *normal* (to be explained on Dec 6) so we were still able to find two eigenvectors which formed an orthonormal basis for C^2. So, A was diagonalizable by a unitary matrix U. We had gotten to:

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \sqrt{2}U = \begin{pmatrix} i & i \\ -1 & 1 \end{pmatrix} \quad D = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \quad \text{and} \quad A = UDU^H$$

Example: For practice with the idea of orthonormal basis, let's compute the coordinates of $\mathbf{x} = [1+i,\ 2+i]^T$ w.r.t. this basis (the columns of U). According to Thm 5.5.2, the first coordinate is

$$c_1 =< \mathbf{x}, \mathbf{u_1} >= [-i,\ -1] \begin{pmatrix} 1+i \\ 2+i \end{pmatrix} /\sqrt{2} = (-1-2i)/\sqrt{2}$$

Likewise, $c_2 =< \mathbf{x}, \mathbf{u_2} >= 3/\sqrt{2}$. We should check that $\mathbf{x} = c_1\mathbf{u_1} + c_2\mathbf{u_2}$ (see Thm 5.5.2):

$$c_1\mathbf{u_1} + c_2\mathbf{u_2} = [(-1-2i)\begin{pmatrix} i \\ -1 \end{pmatrix} + 3\begin{pmatrix} i \\ 1 \end{pmatrix}]/2 = \begin{pmatrix} 1+i \\ 2+i \end{pmatrix} = \mathbf{x}$$

Good!