MAA 3200, HW on Cauchy Sequences and R

Try to do these yourself. If you get stuck, you may find some of these answered in a book; maybe in the Morash book on reserve. As always, you can ask Alejandro or me for help (by email this time).

1) Prove that $x_n = (n+1)/n$ is Cauchy, using the definition of Cauchy.

2) Suppose that $\{x_n\}$ and $\{y_n\}$ are both Cauchy. Show that $\{x_n + y_n\}$ is also Cauchy.

3) Prove the lub Axiom for the real numbers, based on our definition of the reals (using Cauchy sequences). You can follow the outline given in class on Nov 23 (or find your own plan) and you can use any of the field axioms or order axioms you need.

4) Suppose $x_n$ is a bounded increasing sequence. Show that $\lim x_n = \text{lub } \{x_n\}$.

5) Given a bounded sequence $\{x_n\}$, let $a_m = \text{glb } \{x_m, x_{m+1}, \ldots\}$ (as in the completeness proof). Show that $\lim a_m$ exists (you may use a previous exercise). Note: this limit is called the *liminf* of $x_n$.

6) Let $S = \{(x, y) \in R^2 : 0 \leq x \leq y\}$. Show that $S$ is closed in $R^2$. This is similar to Exam 3, Problem 2. I suggest drawing a picture to help plan the proof.