

I will also assign some of the exercises from the book separately, but there are not many there about metrics, open sets, etc:

1) Define $\rho : R^2 \times R^2 \rightarrow R$ by $\rho((x_1, x_2), (y_1, y_2)) = |x_1 - y_1| + |x_2 - y_2|$. Each part of this problem below will use this metric instead of the usual one.

1a) Show that ρ is a metric (from the definition).

1b) Draw a picture of $B_2(0) = \{x \in M = R^2 : \rho(x, 0) < 2\}$. This is standard notation for a ball of radius 2 centered at 0 in a metric space [but maybe it is not in the Shilov book?]. Also, I am using “0” as an abbreviation for the origin in R^2 , so $0 = (0, 0)$.

1c) Prove that $P(1, 0)$ is an interior point of $B_2(0)$ using the definition of interior point (find a suitable radius and prove it works).

2) Give an example of a collection of open sets in R^2 such that the intersection of the collection is the closed unit ball. You can use the standard metric, or the one from Problem 1, whichever you prefer.

3a) Let $f(x, y) = xy$ on R^2 . Show that the set $U = \{(x, y) \in R^2 : f(x, y) > 0\}$ is open. You can use the standard metric in this one. Use the defn of open.

3b) Show that U can be partitioned into 2 components, A and B ; this means A and B are both open sets, and are disjoint and $A \cup B = U$.

4) Suppose that $x_n \rightarrow x \in R^2$, as defined in Section 3.31, using the standard metric in R^2 . Show that $x_n \rightarrow x$ is also true if we use the metric in Problem 1.