More HW on Shilov, Ch 3; Fall 2010

Read over Ch 3. At the very minimum, understand the vocabulary used in the section names [limit points, complete, compact, etc] and most of the proofs covered in class. If necessary, go lightly over homeomorphisms, and theorems not mentioned in class. Here are some problems:

1) Let $M = (0, 1) \subset R$ (so, M is a metric space). Show that $S = [1/2, 1)$ is closed in M.

2) Give examples of closed sets $S_n \subseteq R$, such that $\cup_{n=1}^{\infty} S_n = (0, 1)$. This not closed, of course.

3) Show that $Q \times Q$ is dense in R^2. Does this imply that R^2 has a countable dense subset ? [If so, this is a good example to remember].

4) Let $M = B_1(0) \subset R^2$ be the open unit ball. Show that this M is not complete by giving an example of a Cauchy sequence that does not converge in M.

5) [a longer one] Show that R^2 with one of the less-usual metrics [the one from Ch.3.16, or the one from the previous HW, whichever seems easier to you] is complete. I'm expecting you to mimic the proof of Theorem (d) on page 82, but you might find a way to *use* this theorem instead, as a shortcut. Or, maybe even use theorem 3.72.a.