

MAA 3200, Some practice problems for the final exam, mostly on limits

I will probably work through some of these on Thursday, Dec 2. Try to do the rest yourself before the exam. Prepare for all kinds of basic limits; one-sided limits, limits at infinity, limits of a sequence or a function, the basic limit theorems, etc.

1) If $x_n \rightarrow 2$ then $x_n^2 \rightarrow 4$.

2a) If $\lim_{x \rightarrow a} f(x) = L_1$ and $\lim_{x \rightarrow a} f(x) = L_2$ then $L_1 = L_2$ (limits are unique).

b) Show that $\lim_{x \rightarrow 2} 3x + 1 \neq 5$. One way is to use 2a) but you should also be able to prove this directly from the definition [negated] of limit. The proof is fairly similar to that of 2a).

c) Show that $\lim_{x \rightarrow 0} x/|x|$ does not exist (without using the famous Calculus theorem about one-sided limits). Assume it exists and get a contradiction. This is also similar to 2a) and 2b)

3) Prove some basic theorems about limits such as:

3a) If $x_n \rightarrow X$ and $y_n \rightarrow Y$ then $x_n y_n \rightarrow XY$.

3b) Squeeze Thm: If $\forall x \in R^1, f(x) \leq g(x) \leq h(x)$ and $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$, then $\lim_{x \rightarrow a} g(x) = L$ too.

3c) If $\{x_n\}$ converges, it is Cauchy.

3d) The product of two Cauchy sequences in a metric space M is Cauchy. This is similar to 3a, but do not use convergence in your proof, since M might not be complete.

4) $\lim_{x \rightarrow \infty} \sin(x)/x = 0$. You could use a modified Squeeze Theorem for this, but should also be able to prove this from the defn (and from the well-known formula $|\sin(x)| \leq 1$).

5) $[0, 1]^2$ is the closed unit square in R^2 . Show that it is compact (use the Shilov definition). Since we did so little with this in class, you can regard this problem as optional; it might make a good bonus question.

6) $\lim_{x \rightarrow 3^+} \frac{(3-x)x}{|3-x|} = -3$

7) Show that the sequence $(1/2n, (n+2)/n)$ converges in the metric space $M = R^2$. Part of this problem is to know (or guess) the definition of convergence in a metric space other than R^1 .

8) Compute $\liminf (-1)^n + 1/n$.

If you have trouble with any of these before Thursday afternoon, you can ask me in my office. After that, you may try email, but do it asap (certainly before Sunday night).