Limit HW for MAA 3200
Fall 2003

A) [HW2] Prove that $\lim_{x\to 2} 3x + 1 = 7$ using the definition of limit, and the ϵ - notation. Try to write your own proof based on Ch 3 proof strategies. If necessary, see a Calculus text for help.

B) [HW2] Disprove: $\lim_{x\to 2} 3x + 1 = 8$, using the definition and negations, but don't use exercise A. See Ch.3.2 for help; you will prove a $\exists\epsilon > 0$ (etc) sentence. You might start that by setting $\epsilon = 0.5$.

C) [HW3] Prove that if a limit exists, it is unique. That is, if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} f(x) = M$ then $L = M$, using the definition.

Hint: If you don't know the "triangle inequality", look that up first. Strategy 1: Assume that $L \neq M$ to get a contradiction; you can then specify ϵ based on the size of $|L - M|$. This is similar to problem B.

Strategy 2: Imitate the proof of remark 2.4 of Wade.

Notice that one could prove B from A and C (but don't prove B that way).

D) [HW3] Prove that $\lim_{x\to 3} x^2 = 9$. You can find examples like this in a Calculus book, or see example 3.3 of Wade. You may probably set $\delta = min(1, \epsilon/100)$ or something similar.

Optional for now: E) Prove that $\lim_{x\to 3}[f(x) + g(x)] = \lim_{x\to 3} f(x) + \lim_{x\to 3} g(x)$, assuming the last two limits exist. Since there are 3 limits, you may need to use a δ_1, a δ_2 a δ_3 in your proof.