MAA 3200 — November 14, 2000
Exam III — Prof. S. Hudson

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**Name**

Show all your work and explain your reasoning. Don't use your own paper, a calculator or book. You may ask for extra paper or for clarifications. Each problem is 20 points.

1) Short Answer:

Give an example of a number system with additive inverses that is not a field.

Given that $s_1 = 1$ and $s_2 = 1$ and $s_{n+1} = s_n + s_{n-1}$ (for $n \geq 2$), find $s_8$.

What is the *trichotomy* property in an ordered field?

Give an example of a sequence that is not bounded, with $\lim s_n/n = 0$.

2) Answer True or False; do not justify.

In every field, $0 < 1$.

$Z_5$ is a field.

$Q$ is a complete ordered field.

$\forall x, y \in N$, $s_x(\sigma(y)) = \sigma(s_x(y))$.

$s_2 : N \to N$ is onto.

Let $U = Q$. If $\{s_n\}$ converges, then it is Cauchy.

The set of all polynomial functions with rational coefficients is countable.

3,4) Choose TWO to prove:

A) Prove that multiplication is commutative in $Z$. Use the definitions - you may assume anything you need about $N$.

B) Addition in $N$ is associative. Use induction.

C) $<$ is transitive in $N$ (use the definition of $<$ and basic properties of $+$ and/or $\cdot$).

D) Suppose that $\{s_n\}$ is a Cauchy sequence with a subsequence that converges to $L$. Show that $\{s_n\}$ also converges to $L$.

5) Choose ONE to prove:

A) Suppose that $\{s_n\}$ is a sequence of real numbers and that $s_n \leq M$ for all $n \in N$, and that $\lim s_n = L$. Prove that $L \leq M$.

B) Show that if $A$ and $B$ are denumerable, then so is $A \times B$.

C) Show that $\lim(s_n + t_n) = \lim s_n + \lim t_n$, (assuming the two on the right exist).

BONUS: Prove