Some remarks on HW 3, MAA 3200, Fall 2020

This is a proof of HW **problem 6.4.4** about Martian currency. The math version of the statement is:

For all $n \geq 12$, $\exists\ a,\ b \in N$ such that $3a + 7b = n$.

Since this is in Section 6.4, you should try to use Strong Induction [and I think any other strategy will fail, or will be harder]. Before starting to write, you should experiment. For example, if $n = 12$ we can set $a = 4$ and $b = 0$. If we add 1 to a, we can handle $n = 15$. Likewise, 18, 21 etc are clearly OK. The case $n = 13$ shouldn't be too hard [but it is not directly related to 12, which reinforces the idea of using Strong induction, rather than regular induction], and when we solve that, we also have 16, 19, etc. So, it seems we should handle 13, 14 and 15 as special cases [like a basis step] and then write up our '+3 idea' as a SI proof.

Proof: Note that $12 = 3 \cdot 4$, and $13 = 3 \cdot 2 + 7$ and $14 = 2 \cdot 7$. These equations prove the theorem in the cases $n = 12, 13, 14$. For $n \geq 15$, we will use Strong Induction on n.

Let $n \geq 15$. Assume the theorem is true for $15 \leq k \leq n - 1$ [We also know it is true for $12 \leq k \leq 14$]. We will prove it for n. Let $k = n - 3$, so $12 \leq k \leq n - 1$. So the theorem is true for k, and $k = 3a_k + 7b_k$, with $a_k,\ b_k \in N$. Setting $a = a_k + 1$ and $b = b_k$, we get $n = 3a + 7b$, with $a,\ b \in N$. Done.

Note: Some books use a different form of Strong Induction, which requires a basis step. For example, you can find problems like this one in Rosen's book *Discrete Math* [which we use at FIU]. You'll see proofs like the one above, where the cases n = 12, 13 and 14 are handled in the basis step. There are so many different versions of induction that it seems impractical to list them all. With some practice, you can usually invent valid versions yourself, to fit the new problems you may face. Hopefully, your reader will recognize that your new method works. Perhaps all this sounds rather loose, but I've never seen it cause serious trouble.

Note 2: Some of the proofs I graded used reasoning like this: If we want to go from n to $n + 1$, we can replace two 3's by a 7. The problem is, we don't know n uses two 3's [for ex: n=14 wouldn't]. It should be possible to make this idea work, by using cases, but it seems harder this way.

Quick Remark on **problem 5.3.12**: this one says $\exists B'$ [etc]. So, your proof should include a line [near the start] of the form "Define $B' =$" [etc]. See page 377, basic proof strategy # 7. After some thought, I'd probably set $B' = \text{Ran}(f)$ and then think about how to prove $f^{-1} : B' \to A$ (see the definition of function).

Quick Remark on **problem 7.1.3** [not graded]: Parts a and b shouldn't be too hard, by finding specific functions f that work, such as $2x$ and $\tan(x)$. For c, you can show that $(0,1) \sim (-\pi/2, \pi/2)$ and then use part b and transitivity [or use a composite function, which is the same thing]. Part d seems relatively hard, without some of the theorems that come later, in 7.2 or 7.3. That's mainly because $(0,1]$ contains an endpoint. Here is one idea for a proof:

Set $A_Q = (0,1) \cap Q$ and $B_Q = (0,1) \backslash Q$. Set $C_Q = (0,1] \cap Q$ and $D_Q = (0,1) \backslash Q = B_Q$. Show $A_Q \sim C_Q$ (show both are denumerable) and put it all together with thm 7.1.2.

Another [similar] idea is to find a specific $f : (0,1] \to (0,1)$. This may take some trial and error, but try this; Let $f(x) = (n+1)^{-1}$ if $x = n^{-1}$ for some $n \in N$, and let $f(x) = x$ otherwise. Does this example remind you of the Hilbert Hotel story ?