HW on Number Systems; Fall 2010

1) Short Answer Questions:

a) Which famous proof method is used the most here, especially for properties of N ?
b) Why are the proofs about Z and Q usually shorter than those for N ?
c) How does Q differ from Z ? [Which operations, partial orders, etc, are defined in Q but not Z]
d) Which of our four number systems are fields ? (maybe more than one)
e) What is a binary operation on a set A ?

2) Before defining $+$ on N, we defined (for each fixed $m \in N$) a function $s_m : N \to N$ by:

a) $s_m(0) = m$

b) $s_m(\sigma(n)) = \sigma(s_m(n))$

Then, we defined $m+n$ as $s_m(n)$. Now, we can prove the commutative property, $m + n = n + m$, by induction on n. For your HW, just prove the basis step; set $n = 0$), and ETS $\forall m \in N,\ 0 + m = m + 0$.

Hints: apply the definition of $+$, to get a new ETS: $\forall m \in N,\ s_0(m) = s_m(0)$. Part a) further simplifies one side, so ETS: $\forall m \in N,\ s_0(m) = m$. Now, use induction on m, and use (a) and (b) as needed. [As of Oct 11, I plan to prove that s_m exists, tomorrow in class. If so, you may use formulas from my proof in your proof, if you state where they come from].

3) Prove trichotomy for $<$ on Z, using trichotomy for $<$ on N. Of course, you will need to use the definition of $<$ on Z and may have to think about what $=$ means for Z. Your proof will probably use cases.

4) Find these elements of N:

a) $\sigma(4)$
b) $s_3(2)$
c) $s_3(\sigma(2))$ and $\sigma(s_3(2))$ - are they equal?

5) Use the definition of $<$ in N to show that $3 < 10$. This isn't supposed to be hard; you can assume that 1, 3, 4, 7, 9 and 10 are in N, and you can use simple formulas like $1 + 3 = 4$ without proof.