

Everything (regrading requests, late HW, etc) is due on 12/3/00.

Our final is Wed. 12/10/03, 12:30pm to 3:15pm in the usual room. It will cover everything, but with emphasis on recent topics (see HW 7 and 8 and 9, including the both special HW's on the reals). Regarding older material, I'd especially recommend reviewing:

- 1) Exam II, page 2 (proofs with cases).
- 2) Exam III, problems 1 and 2 [and prepare for similar questions on Chs 2.4-3.3].
- 3) The definitions of commonly-used terms such as *onto*, *transitive*, *partial order*, *Cauchy sequence*,  $f(A)$ , *well-defined*, *associative*, *denumerable*, *subsequence*, *lub*, *limsup* etc.
- 4) Proofs similar to the HW in Velleman, Chs 4, 5 and 7.
- 5) Be able to define  $\mathbb{Z}$ ,  $\mathbb{Q}$  and  $\mathbb{R}$  using equivalence classes.

**Homework 9, not to turn in:** These relate to the 11/24 lecture notes (see also the related web page). Any of these could be on the final, but if you can do 2 out 3, that should be enough.

- 1) Prove step II.C of the trichotomy proof for  $R$ .
- 2) Prove the associative property of addition in  $R$ , that  $(a + b) + c = a + (b + c)$  directly from the definitions (Set  $a = [\{a_n\}]$  etc as in the proof of the commutative property done in class).
- 3) Refer to the proof of completeness, Claim 1. Use the WOP to prove (carefully) that if  $S_n$  is nonempty, then it has a least element  $q_n \in Q$ . Do not assume that all elements of  $S_n$  are positive.

**Textbook Proofs to know:** (suggestion: learn *at least* 4 of the first 6)

- 1)  $\mathbb{R}$  is uncountable.
- 2)  $\mathbb{R}$  is complete (give the outline, and state the 2 claims).
- 3) The Nested Interval Thm.
- 4) Thm 3.6 (about  $f(x_n)$ ).
- 5) The E.V.Thm 3.26

6) The I.V.Thm 3.29

7) [less likely ones] a) Every sequence has a monotone subsequence,  
b) The B.W.Thm., c) various pieces of the trichotomy principle for  $\mathbb{R}$ , d)  
Every Cauchy sequence in  $\mathbb{R}$  converges.

**Sample problems** I think reviewing old definitions, examples, and HW should be the best way to study. Also “play around” with the main theorems, so you are ready for True-False, give-an-example-of, etc. But after you do all that, you could also try these for more practice.

1) Assume  $a_n \rightarrow A$  and  $b_n \rightarrow B$ . Show

a)  $a_n + b_n \rightarrow A + B$

b)  $a_n b_n \rightarrow AB$

c) If  $a_n \leq b_n$  then  $A \leq B$

2) If  $a_n \rightarrow 0$  and  $b_n$  is bounded then  $a_n b_n \rightarrow 0$ .

3) Show that every bounded decreasing sequence converges.

4) If  $x = [\{q_n\}]$  then  $q_n \rightarrow x$ .

5) If  $a_n \rightarrow A$  then  $|a_n| \rightarrow |A|$ . Discuss whether the converse is true.

6)  $\limsup (-s_n) = -\liminf (s_n)$ .

7)  $\text{lub}(S \cup T) = \max(\text{lub } S, \text{lub } T)$