

The Growth Effects of Intrinsically Worthless Education

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Abstract. A positive relationship between individual earnings and education has been repeatedly justified by microeconomic theory and is also strongly supported by its empirical evidence. Although theoretical growth models have predicted even deeper links between education and growth, empirical work at the aggregate level has not been able to provide strong evidence consistent with this theoretical prediction. Some studies have attributed the inconsistency between micro and macro evidence to the primary signaling function of education, which does little to increase individual's productivity, and in turn is regarded as having no significant effects on growth. This paper develops a growth model taking individuals' innate ability into account, and explores the possibility that even if education only serves as means to signal innate ability, it still can have positive effects on growth if it helps to distinguish the high-ability labor from the low-ability, so that the most able individuals can be allocated to the knowledge creation sector and consequently increase economic growth.

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1. Introduction

Since Mincer's (1958, 1974) and Becker's (1966, 1975) seminal studies of the distribution of income and investment in human capital, microeconomic theory has repeatedly justified a positive relationship between individual earnings and education, which also has been strongly supported by empirical microeconomic evidence. In general, the return to an additional year of education, as measured by the coefficient on schooling in Mincer regressions, is about six to eleven percent (Card 1999). The presumption is that a worker with more schooling gets paid more because education has taught him valuable skills that increase his productivity. If this is true at the microeconomic level, simple aggregation suggests that human capital levels should be correlated with income levels, and that we should also be able to see a strong link between growth in per capita income and the expansion of education.

Theoretical growth models have suggested even deeper links between education and growth. Lucas (1988) includes individual educational investment as a factor input in the knowledge production function, and predicts that changes in the level of human capital permanently alter the growth rate of income. In Romer (1990), human capital is an essential input in the research sector which creates new technology; in his model income growth is closely related to the stock of human capital. However, compared to the microeconomic evidence, at the macroeconomic level there is no strong empirical support for this positive relationship between educational levels and growth. Sianesi and Reenen (2002) provide a comprehensive review of the empirical literature, which suggests positive but very weak relationships between education and growth. On average, each percentage point increase in secondary school enrolment rates is associated with a 2.5 percent increase in per capita GDP growth rate; an extra year of male secondary schooling is associated with a 1.4 increase in per capita GDP growth rate and a percentage point increase in human capital stock is associated with 0.3 percent increase in GDP. Moreover, it is very likely that even these results are biased upwards because of problems such as reverse causality and omitted variable bias, while the homogeneous-slope and linearity assumptions make the macro evidence even more fragile. Indeed a number of studies have attributed the inconsistency between the macro and micro evidence to econometric problems in the macroeconomic studies.

Another approach, the signaling hypothesis, considers the possibility that education does little to make individuals more productive, but instead serves as means to signal innate ability (Spence, 1973, 1974).

A number of attempts have been made to test the significance of this hypothesis of educational signaling, but the results have been mixed.¹ Knight and Sabot (1991) study workers in Kenya and Tanzania using data on ability, schooling, skills and wages and shows that the effect of schooling on wages is not because of signaling, but rather because schooling raises skills. Glewwe (1991) also finds no evidence of signaling using data on skills ability and wages in Ghana. Harmon, Oosterbeek and Walker (2000) conclude that repeated attempts to measure the signaling component of the return to education have yielded consistently small effects. Based on datasets where direct measures of ability are available, the inclusion of ability lowers the return to schooling by less than one percentage point. In contrast, Alderman et al. (1996) find that cognitive achievement, and not schooling attainment raises wages. Wolpin (1974) is widely interpreted as providing evidence against the screening interpretation by the author, but his results actually provide mild support for the screening hypothesis. Moreover, with an alternative approach by comparing “screened” and “unscreened” occupations, he concludes that the observed difference is a vindication of the educational screening hypothesis.

Thus, the evidence for signaling at the microeconomic level remains mixed. Riley (1979) argues that because of data limitations and the inadequate development of theoretical models of educational signaling and screening, these empirical studies are inconclusive.² Testing the signaling hypothesis is difficult in large part because of challenges in measuring innate ability and cognitive skills. Measurement error will attenuate the estimated effect of signaling, leaving as a residual an unduly large estimated role for the productivity-enhancing role of education. An alternative estimation strategy was pursued by Dale and Krueger (2002), who studied 1985 earnings of individuals in the High School Class of 1972, by matching individuals who were accepted to the same institutions but chose to go to different schools. They found that earnings almost ten years after college graduations did not depend upon the selectivity of the school they attended, but that the selectivity of schools that *rejected* the individual had a significant impact on subsequent earnings. Their findings, they concluded, were consistent with some of the earliest work on innate ability and educational quality:

The C student from Princeton earns more than the A student from Podunk not mainly because he has the prestige of a Princeton degree, but merely because he is abler. The

¹ For more details, see Taubman and Wales (1973), Layard and Psacharopoulos (1974), Wolpin (1974, 1977).

² Also see Kruger and Lindahl (2001).

golden touch is possessed not by the Ivy League College, but by its students.

Hunt (1963: p. 56)

Based on the signaling hypothesis, several studies have tried to take an alternative way to tackle the issue of the inconsistency between micro and macro evidence. Pritchett (1996) suggests that if education has primarily a signaling function, then the inconsistency between micro and macro evidence has a ready explanation. Since the signal is used by the employer as the determinant of wages and allocation of jobs to workers with different innate abilities, at micro level, the return to education reflects the innate ability that education signals rather than the productivity it enhances, therefore the increase in individual earnings doesn't necessarily imply the increase in productivity. Then from the macro point of view, more educated workers may not cause a consistent increase of growth rate.³

The purpose of this paper is to develop models to assess Pritchett's intuition. We analyze a two-sector endogenous growth model in the style of Romer (1990), in which workers' ability affects their productivity in the knowledge creation sector but not in the final goods sector. The analysis is conducted in two different settings: a simple social planner's model and a somewhat more complex decentralized model. Incorporating signaling into the social planner's problem, the model shows that – in contrast to Pritchett's argument – changes in the fraction of the population receiving education have significant but non-monotonic effects on the growth rate of income. When education levels are initially low, the growth rate rises with increasing education, before falling again with further increases in education. At the limits, when there is no education or when everyone receives the same level of education, the growth rate is identical. The intuition is straightforward: at the two limits education can play no signaling role and therefore cannot help the planner to allocate the most able workers to tasks where they are most productive. In the decentralized model, in contrast, changes in the fraction of the population receiving education have no effects on the growth rate of income. The decentralized model produces a result markedly different from the planner's problem largely because movements of educated and uneducated workers between the knowledge creation sector and the final good production sector two driven by equilibrium wage adjustments, neutralizes the effect of changes

³ An additional issue is that the argument emphasizes only the private return of the educational signal while ignoring its social return. Both Stiglitz (1975) and Wolpin (1977) suggest firms' output level may be increased if the workforce is homogeneous. The use of education as a screening device facilitates the construction of a homogeneous workforce and therefore reduces inefficiency.

in the relative supply of educated workers. As a result, we conclude with the somewhat ambiguous result that the growth effects of education, when education serves only as a signal, depend both on the details of the production technology and the degree of intervention by the social planner.

The rest of the paper is laid out as follows. Section 2 solves a simple social planner's problem to generalize the idea about how the expansion of education affects the average ability of workers in the knowledge creation sector by first driving it up then forcing it down, and how the growth rate changes in response. Section 3 analyzes the growth effect of educational signaling in the decentralized model. Section 4 concludes.

2. A Simple Planner's Problem

A simple illustration of the possible growth effects of education can be obtained from considering the allocation problem of a social planner. Assume output is given by the production function

$$Y = (A(1 - \gamma)L)^\beta K^{1-\beta}, \quad (1)$$

where Y is output, K is capital, A denotes knowledge, and $1-\gamma$ is the fraction of the labor force (which has unit mass) that is employed in the production of final goods. The knowledge production function satisfies

$$\dot{A} = A\theta_A\gamma L, \quad (2)$$

where θ_A is the average ability of labor employed by the knowledge creation sector, and γ is the fraction of the labor force employed there. Clearly, both the share and ability of labor engaged in the knowledge creation sector are key determinants of the rate of growth. Production is allocated between consumption and investment, according to

$$\dot{K} = Y - C. \quad (3)$$

The social planner's problem for this economy is to choose consumption and allocate labor efficiently between production and knowledge creation to maximize

$$\max_{c, \gamma} \int_0^{\infty} e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1-\sigma} dt. \quad (4)$$

This is a familiar model, and so the derivation is given in the appendix. The balanced growth, g , is given by the simple expression

$$g = \frac{\theta_A(\mu, \gamma) - \rho}{\sigma}, \quad (5)$$

which increases linearly with the ability of the workforce engaged in knowledge creation. This ability depends on two variables: the exogenous fraction of the population, μ , that is educated, and the endogenous fraction of the labor force engaged in knowledge creation. The latter is defined implicitly by

$$\gamma = \frac{\theta_A(\mu, \gamma) - \rho}{\sigma \theta_A(\mu, \gamma)}. \quad (6)$$

Equation (6) predicts that a social planner facing an increased ability among its knowledge creation labor force is inclined to increase the size of its knowledge creation sector. If θ_A were exogenous, (5) tells us that this unambiguously raises the growth rate.

However, θ_A is not exogenous and, moreover, the social planner cannot observe directly individual abilities. We construct a simple signaling framework as follows. The economy is populated by a continuum of infinitely-lived individuals with unit mass, and individual $i \in [0,1]$ has ability θ_i drawn from the distribution $F(\theta)$. There is an educational authority that admits a fraction, μ , of the population to a higher education program. The authority wishes to admit the most able individuals. To this end, the educational authority establishes an application procedure whereby each individual produces a signal y of his ability, where $E[\theta_i | y_i]$ is strictly increasing in y_i . Let $g(y)$ denote the unconditional distribution of signals. The educational authority establishes a minimum entry standard, y^* , satisfying $\mu = \int_{y^*}^{\infty} g(y) dy$.

While the educational authority necessarily observes each signal, the social planner does not. Instead, he observed only the binary signal, s , which takes the value one if the individual was admitted to higher education, and zero if not. Because ability matters only in the knowledge creation sector, the planner allocates only educated workers to knowledge creation when optimal employment in that sector is less than supply of educated workers. The planner employs all educated workers in knowledge creation, and supplements these workers with uneducated labor, when optimal employment in knowledge creation exceeds the supply of educated workers.

We are interested in seeing how an increase in the fraction, μ , of the population that is educated alters

θ_A . Because of the planner's information constraint, the effect an increase in μ has on growth is a little more complicated than is suggested by (5). It is, however, easy to see that for $\gamma < \mu$ the value of θ_A must coincide with the average ability of all people with education, i.e. $\theta_A = E[\theta|s=1]$, since the planner can populate the knowledge creation sector with random selections from only the educated group. In contrast, if $\gamma > \mu$, θ_A is a weighted average of the mean abilities of people from both the educated and uneducated groups, since in this case some labor without education would have to be allocated to the knowledge creation sector. Thus we have

$$\theta_A = E[\theta|s=1], \text{ if } \gamma \leq \mu, \quad (7)$$

and

$$\theta_A = \frac{\mu}{\gamma} E[\theta|s=1] + \frac{\gamma - \mu}{\gamma} E[\theta|s=0], \text{ if } \gamma \geq \mu. \quad (8)$$

It is easy to see that both $E[\theta|s=1]$ and $E[\theta|s=0]$ must be decreasing in μ . An increase in μ requires the educational authority to lower the entry standard, y^* . Doing so adds to the educated labor force individuals whose signals would have previously precluded admission to higher education, and it removes from the uneducated segment those that produced the highest signals consistent with not being admitted to higher education. Adding weight to the lower tail of the ability distribution of the educated, and taking weight away from the upper tail of the distribution of the uneducated lowers the expected ability of both groups.⁴ Equation (7) thus shows that when $\mu \geq \gamma$, any increase in μ will reduce θ_A , and from (5) this in turn induces a decline in the growth rate. Equation (8) reveals a more complicated situation. First, an increase in μ reduces $E[\theta|s=1]$ and $E[\theta|s=0]$, and this effect induces a decline in θ_A . Second, the weight μ/γ increases and, because $E[\theta|s=1] > E[\theta|s=0]$, this channel induces an increase in θ_A . Despite these offsetting effects, we are able to show in the appendix that the expression in (8) is strictly increasing in μ for all $\mu < \gamma$.

Figure 1 illustrates the relationship between μ and θ_A for any given γ . $E[\theta|s=1]$ is monotonically declining with μ , beginning at the upper limit of the distribution $F(q)$ (which we take to be infinite) for $\mu \rightarrow 0$, and declining to the population mean, $E[\theta]$, as $\mu \rightarrow 1$. Similarly, $E[\theta|s=0]$ is decreasing in μ , beginning at $E[\theta]$ when $\mu=0$, and falling to the lower limit of the distribution

⁴ Appendix B derives the expressions for how $E[\theta|s=1]$ and $E[\theta|s=0]$ responding to changes in μ .

(which in Figure 1 is taken to be zero) as $\mu \rightarrow 1$. The value of θ_A , is a weighted average of the two, indicated by the bold line. For $\mu < \gamma$, the weight on the educated population rises with μ sufficiently rapidly to offset the decline in expected values of both educated and uneducated workers, so θ_A rises. For $\mu > \gamma$, the weight on $E[\theta|s=1]$ is unity, so θ_A declines monotonically with any increases in μ beyond γ .

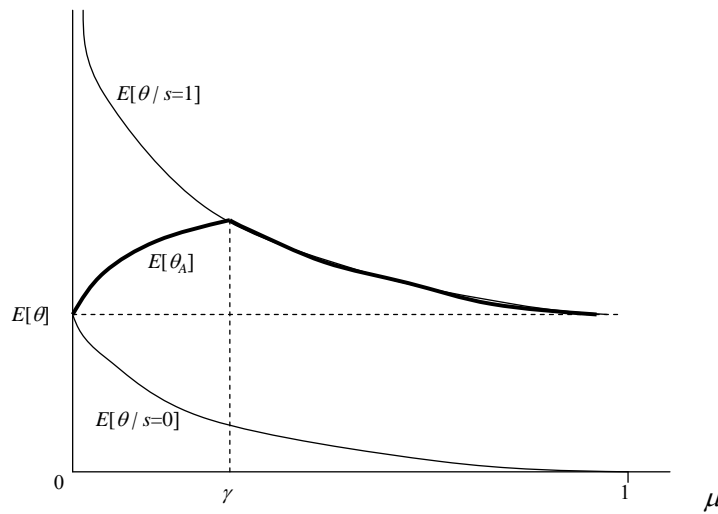


FIGURE 1. Expected ability and educational intensity

Only a minor additional step is needed to consider the effects of growth rates after incorporating the endogenous response of γ to changes in θ_A . When θ_A is rising, the optimal γ rises in response. This increase in γ dampens the direct effect of an increase in μ on θ_A , but does not overturn it. Thus, the balanced path growth rate of income, g , must rise. When θ_A is falling, the optimal γ falls and so does the growth rate of income. We therefore have the following result:

RESULT 1. *An increase in the fraction of the population that is educated raises [reduces] the growth rate of income when the educated population is smaller [larger] than the knowledge creation sector.*

3. A Decentralized Model

The model closely follows the growth model in Romer (1990), except for the specifications for the production functions in the final-goods and the knowledge creation sector. The economy consists of three sectors. First, the knowledge creation sector produces new knowledge, especially designs for new producer durables. In Romer (1990), new knowledge is produced with the use of the existing stock of knowledge A and human capital H_A . Our model departs from that by assuming that labor in the research sector is made up of people with and without college education, and the average abilities of both groups play an important role in the productivity of this sector. The production function is written as

$$\dot{A} = A(\theta_H H_A + \theta_L L_A) \quad (9)$$

where H_A is interpreted as college-educated labor employed in the knowledge creation sector. θ_H is the average ability of the college-educated group among the labor population. Similarly, L_A represents the labor employed in the knowledge creation who don't have college education and θ_L is the average ability of the labor group without college education.

Second, the intermediate-goods sector uses forgone consumption and designs x purchased from the knowledge creation sector to produce producer durables.

In Romer's model, the final-goods sector produces consumption or capital goods using labor, physical and human capital. Here, we replace the labor and human capital with workers with college education H_Y and workers without L_Y . The production function for the final goods takes the form of the constant elasticity of substitution (CES)

$$Y = (H_Y^\rho + L_Y^\rho)^{\frac{\alpha}{\rho}} \int_0^\infty x_i^{1-\alpha} di$$

For a fixed set of durable goods A , the production function can be written as

$$Y = (H_Y^\rho + L_Y^\rho)^{\frac{\alpha}{\rho}} A \bar{x}^{1-\alpha} \quad (10)$$

The CES production function still retains all the relevant properties of the production function in Romer's model such as the constant return to scale and diminishing returns to inputs. Two aspects of

the model are of particular interest: the steady-state growth rate and its change subject to educational expansion, measured by the percentage of workers among the labor population who have college education, μ . Since our analysis closely follows what has been done in Romer (1990), the main exposition is presented in the Appendix, and here we only focus on the final results.

On the balanced growth path, the stock of knowledge A and the output Y grow at constant exponential rates by definition. As discussed in the Appendix, the growth rate of output can be derived from the production function in the knowledge creation sector,

$$g = \frac{\dot{A}}{A} = \frac{(1-\alpha) \cdot [\theta_H \mu + \theta_L (1-\mu)] \cdot L - \rho}{1-\alpha + \sigma}$$

which can be simplified as

$$g = \frac{(1-\alpha) \cdot \bar{\theta} \cdot L - \rho}{1-\alpha + \sigma} \tag{11}$$

Equation (11) states that it is the average ability of the whole labor population $\bar{\theta}$ that actually matters for the steady-state growth rate. The variable μ is eliminated from the expression, implying that the expansion of college education has no effect on the growth rate along the balanced growth path. This result is different from the predictions generated in the previous social planner's model.

To see why the inconsistency happens, notice that in the previous model, education is used as an informational signal by the social planner to screen out the high ability labor and allocate them into the knowledge creation sector. In the case where the fraction of labor with college education, μ , is less than the size of the knowledge creation sector γ , as college education expands so that more high-ability labor can have college education, uneducated workers in the knowledge creation sector will be constantly replaced by those with college education. Consequently, the average ability of workers in this sector is expected to improve. As we have shown before, this leads to an increase in the size of the sector and the growth rate. In the decentralized model, however, to ensure an individual, who is either the educated type or the uneducated, has no incentive to move from one sector to the other, the wages for a certain type of labor in each sector must be equal. To meet this requirement, the ratio of the workers with college education to those without college education in the final-goods sector must satisfy the relation $\theta_L / \theta_H = (H_y / L_y)^{1-\rho}$, which indicates in our linear specification of R&D equation that the number of educated and uneducated labor devoted to knowledge creation would be restricted in a way that its change would offset the negative effect of the decline in the average ability

of educated workers θ_H and uneducated workers θ_L as education expands. In the end, it turns out that the weighted sum of the workers' ability in this sector would not be subjected to the signaling effect of education.

4. Conclusions

This paper models the interaction between educational signals and economic growth. We are interested in examining our primitive hypothesis that even if education doesn't raise a worker's productivity, it might be possible that economic growth can be induced only through the signaling effect of education. The results show that it might be true, depending on how we model it. Solving the social planner's problem explicitly yields the upward and downward trend of growth rate with respect to the increase of educational intensity, which implies that better signals of ability through education generates higher growth rates, vice versa. In the decentralized model, educational signals turn out to have no effect on the steady-state growth rate because the proportion of the two types of workers in each sector is confined in a way that the balanced growth rate depends only on the mean ability of the labor population.

The model can be extended in several ways for future research. First, the zero effect of educational signal in our decentralized model mostly arises from the specification of the two production functions. Other forms of production functions are worth investigating to see whether different results can be generated. Second, it is implicitly assumed throughout the paper that individuals with higher ability are more willing to apply to college because they are more likely to be admitted by producing better signals y . For our purposes, we ignored the possibility of counter-signaling by the highest-ability people, who may intentionally choose no college in order to distinguish themselves from the low-ability people, especially when education is overexpanded (c.f. Feltovich, Harbaugh and To (2002)). Therefore, incorporating individuals' signaling strategy into the model can be a further attempt. Another extension of the model is about introducing the ranking of educational institutions into the model. Suppose signaling of education matters for the growth rate, but the overexpansion of education weakens its effect. Ranking the educational institutions can be a way that we can think of to make the signal stronger. It would be interesting to model it and trace out its effect on economic growth.

Appendix

A. Derivation of the solution to the social planner's problem

The current value Hamiltonian is

$$H = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \lambda_1 \left((A(1-\gamma))^\beta K^{1-\beta} - c \right) + \lambda_2 (\theta_A \gamma A),$$

which yields the following necessary conditions:

$$c_t^{-\sigma} = \lambda_1, \tag{A.1}$$

$$\lambda_1 f_\gamma + \lambda_2 \theta_A L(t) A = 0, \tag{A.2}$$

and the two equations of motion:

$$\lambda_1 f_k = -\dot{\lambda}_1 + \rho \lambda_1, \tag{A.3}$$

$$\lambda_1 f_A + \lambda_2 \theta_A \gamma L(t) = -\dot{\lambda}_2 + \rho \lambda_2. \tag{A.4}$$

In (A.2) through (A.4), $f = (A(1-\gamma))^\beta K^{1-\beta} - c$.

For the balanced growth equilibrium, it must be the case that the growth rate satisfies $g = \dot{A}/A = \dot{c}/c = \dot{k}/k$, and that $\dot{\lambda}_1/\lambda_1 = \dot{\lambda}_2/\lambda_2$. Combining equations (A.2) and (A.4) allows us to simplify the equation of motion for λ_2 to

$$\frac{\dot{\lambda}_2}{\lambda_2} = \frac{1}{\gamma} g - \rho. \tag{A.5}$$

Since $\dot{\lambda}_1/\lambda_1 = -\sigma \dot{c}/c = -\sigma g$, differentiating both sides of equation (A.1) with respect to time and using equation (A.5) yields the balanced growth rate as a function of γ

$$g = \frac{\gamma \rho}{\gamma(1-\sigma) + 1 - \gamma}. \tag{A.6}$$

Finally, as $g = \theta_A \gamma$, equation (A.6) implies that along the balanced growth path the fraction of labor

devoted to the knowledge creation sector satisfies

$$\gamma = \frac{\theta_A - \rho}{\sigma\theta_A}.$$

Combining (A.6) and (A.7) yields equation (5) in the main text.

B. Proof of Result 1

By Bayes' rule, the planner's updated belief of an individual's ability θ , after observing his educational signal s , can be written as

$$f(\theta|s=1) = \frac{f(\theta) \cdot \int_{y^*}^{+\infty} f(y|\theta)dy}{\int_{y^*}^{+\infty} g(y)dy}, \quad (\text{B.1})$$

which yields the individual's expected ability

$$E(\theta|s=1) = \frac{\int_{-\infty}^{+\infty} \theta \cdot [f(\theta) \cdot \int_{y^*}^{+\infty} f(y|\theta)dy]d\theta}{\int_{y^*}^{+\infty} g(y)dy}. \quad (\text{B.2})$$

Expression (B.2) also reflects the average ability of all educated labor. Taking the derivative of (B.2) with respect to y^* yields, after some simplification,

$$\frac{\partial E(\theta|s=1)}{\partial \mu} = \frac{1}{\mu} [E(\theta|y^*) - E(\theta|s=1)] < 0, \quad (\text{B.3})$$

where use was made of the fact that $\mu = \int_{y^*}^{\infty} g(y)dy$ and hence that $dy^*/d\mu = -[g(y^*)]^{-1}$. For $\mu \geq \gamma$, $\theta_A \equiv E(\theta|s=1)$. Hence (5) and (B.3) establish that the growth rate is decreasing in μ when $\mu \geq \gamma$. Similarly, we obtain

$$\frac{\partial E(\theta|s=0)}{\partial \mu} = \frac{1}{(1-\mu)} [E(\theta|s=0) - E(\theta|y^*)] < 0. \quad (\text{B.4})$$

To establish that growth is increasing in μ for $\mu < \gamma$, differentiate (8) in the main text with respect to μ :

$$\begin{aligned} \frac{d\theta_A}{d\mu} = & \frac{1}{\gamma} \left(E[\theta|s=1] - E[\theta|s=0] \right) + \frac{1}{\gamma} \left(\mu \frac{\partial E[\theta|s=1]}{\partial \mu} + (\gamma - \mu) \frac{\partial E[\theta|s=0]}{\partial \mu} \right) \\ & - \frac{\mu}{\gamma^2} \left(E[\theta|s=1] - E[\theta|s=0] \right) \frac{d\gamma}{d\mu}. \end{aligned} \quad (\text{B.5})$$

From (6), we have

$$\frac{d\gamma}{d\mu} = \frac{\rho}{\sigma\theta_A^2} \frac{\partial \theta_A}{\partial \mu}. \quad (\text{B.6})$$

Substituting (B.3), (B.4) and (B.6) into (B.5) yields

$$\frac{d\theta_A}{d\mu} = \frac{1-\gamma}{\gamma(1-\mu)} \square \frac{\left(E[\theta|y=y^*] - E[\theta|s=0] \right)}{1 + \frac{\mu\rho}{\sigma\gamma^2\theta_A^2} \left(E[\theta|s=1] - E[\theta|s=0] \right)} > 0,$$

which completes the proof of Result 1.

C. The Decentralized Model

The model is built on the growth model in Romer (1990), except for the specifications for the production function in the final-goods sector and the R&D equation in the knowledge creation sector. Therefore, the profit maximization problem in each of the three sectors is solved in the same way as in Romer's model. First, because of the constant returns to scale in the production function, the final-goods sector is described by a representative price taking firm. The price of output Y is normalized to unity, and the price of the producer durables $x(i)$ is denoted by $p(i)$. Maximizing the firm's profit yields the demand function for $x(i)$,

$$p(i) = (1 - \alpha)(H_Y^\rho + L_Y^\rho)^{\frac{\alpha}{\rho}} x(i)^{-\alpha}. \quad (\text{C.1})$$

Second, the intermediate-goods sector takes the demand function for $x(i)$ as given and chooses an optimal price to maximize its profit, which is its revenue net of the interest cost on the ηx units of forgone output used to produce x durables,

$$\pi = \max_x p(x) \cdot x - r\eta x. \quad (\text{C.2})$$

Substituting (C.1) into (C.2) and taking the first order condition with respect to x gives the expression for the optimal price

$$\bar{p} = \frac{r\eta}{1-\alpha}, \quad (\text{C.3})$$

and

$$\bar{\pi} = \bar{p} \cdot \bar{x} - r\eta \bar{x} = \alpha \cdot \bar{p} \bar{x}, \quad (\text{C.4})$$

where $\bar{x} = (\bar{p}/(1-\alpha))^{-1/\alpha} (H_Y^\rho + L_Y^\rho)^{1/\rho}$.

Finally, the knowledge creation sector sets the price P_A for its design no higher than the present value of the monopoly profit that the intermediate sector can extract, that is

$$P_A = \frac{1}{r} \bar{\pi}. \quad (\text{C.5})$$

Substituting (C.3), (C.4) into (C.5), we get

$$P_A = \frac{\alpha(1-\alpha)}{r} (H_Y^\rho + L_Y^\rho)^\rho \bar{x}^{1-\alpha}. \quad (\text{C.6})$$

The R&D equation $\dot{A} = A \cdot (\theta_H H_A + \theta_L L_A)$ shows that all the income in the knowledge creation sector will go to the labor, which implies

$$w_{H_A} = P_A \cdot \theta_H \cdot A, \quad (\text{C.7})$$

$$w_{L_A} = P_A \cdot \theta_L \cdot A. \quad (\text{C.8})$$

In the final-goods sector, the optimal wages for both educated and uneducated workers are their marginal productivities, i.e.

$$w_{H_Y} = A \bar{x}^{1-\alpha} (H_Y^\rho + L_Y^\rho)^\rho \alpha \cdot H_Y^{\rho-1}, \quad (\text{C.9})$$

$$w_{L_Y} = A \bar{x}^{1-\alpha} (H_Y^\rho + L_Y^\rho)^\rho \alpha \cdot L_Y^{\rho-1}. \quad (\text{C.10})$$

In the equilibrium, workers at the same education level wouldn't have any incentive to move from one sector to the other. To guarantee this, we need to set $w_{HY} = w_{HA}$, and $w_{LY} = w_{LA}$, i.e.

$$P_A \cdot \theta_H \cdot A = A\bar{x}^{1-\alpha} (H_Y^\rho + L_Y^\rho)^\rho \alpha \cdot H_Y^{\rho-1}, \quad (\text{C.11})$$

$$P_A \cdot \theta_L \cdot A = A\bar{x}^{1-\alpha} (H_Y^\rho + L_Y^\rho)^\rho \alpha \cdot L_Y^{\rho-1}. \quad (\text{C.12})$$

Substituting P_A from equation (C.6) into (C.11) and (C.12) yields

$$(H_Y^\rho + L_Y^\rho)^{-1} H_Y^{\rho-1} = \frac{1-\alpha}{r} \theta_H, \quad (\text{C.13})$$

$$(H_Y^\rho + L_Y^\rho)^{-1} L_Y^{\rho-1} = \frac{1-\alpha}{r} \theta_L, \quad (\text{C.13a})$$

which gives a relation between θ_A and θ_L at the equilibrium,

$$\frac{\theta_L}{\theta_H} = \left(\frac{H_Y}{L_Y} \right)^{1-\rho}. \quad (\text{C.14})$$

The balanced growth rate g must satisfy

$$g = \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = \theta_Y H_A + \theta_L L_A.$$

Together with equation (C.13) and (C.14), the constraints $H_Y = \mu\bar{L} - H_A$ and $L_Y = (1-\mu)\bar{L} - L_A$ implies the growth rate $g = [\theta_H \gamma_c + \theta_L (1-\mu)] \cdot L - r / (1-\alpha)$, where L is the labor population. Since $g = \dot{c}/c = (r-\rho)/\sigma$, the growth rate g can be expressed in terms of the fundamentals of the model,

$$g = \frac{(1-\alpha) \cdot [\theta_H \gamma_c + \theta_L (1-\mu)] \cdot L - \rho}{1-\alpha + \sigma}. \quad (\text{C.15})$$

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