Numerical Problems

1. For a two-year bond, according to the expectations theory, the interest rate would be the average of the two one-year bonds, which is \((6\% + 4\%)/2 = 5\%\). Adding the risk premium of 0.5% gives an interest rate on the two-year bond of 5.5%.

   For the three-year bond, according to the expectations theory, the interest rate would be the average of the three one-year bonds, which is \((6\% + 4\% + 3\%)/3 = 4.33\%\). Adding the risk premium of 1.0% gives an interest rate on the three-year bond of 5.33%.

   The yield curve would show the interest rate on a one-year bond of 6%, the interest rate on a two-year bond of 5.55%, and the interest rate on a three-year bond of 5.33%, so it would be downward sloping, which is called “inverted” in the market.
2. (a) Real money demand is
\[ M^d/P = 500 + 0.2Y - 1000i \]
\[ = 500 + (0.2 \times 1000) - (1000 \times 0.10) \]
\[ = 600. \]
Nominal money demand is
\[ M^d = (M^d/P) \times P = 600 \times 100 = 60,000. \]
Velocity is
\[ V = PY/M^d = 100 \times 1000/60,000 = 1 2/3. \]
(b) Real money demand is unchanged, because neither \( Y \) nor \( i \) has changed.
Nominal money demand is
\[ M^d = (M^d/P) \times P = 600 \times 200 = 120,000. \]
Velocity is unchanged, because neither \( Y \) nor \( M^d/P \) has changed, and we can write the equation for velocity as
\[ V = PY/M^d = Y/(M^d/P). \]
(c) It is useful to use the last expression for velocity,
\[ V = Y(M^d/P) = Y/(500 + 0.2Y - 1000i). \]
1. Effect of increase in real income:
When \( i = 0.10 \),
\[ V = Y/[500 + 0.2Y - (1000 \times 0.10)] \]
\[ = Y/(400 + 0.2Y) \]
\[ = 1/[(400/Y) + 0.2]. \]
When \( Y \) increases, \( 400/Y \) decreases, so \( V \) increases. For example, if \( Y = 2000 \), then \( V = 2.5 \), which is an increase over \( V = 1 2/3 \) that we got when \( Y = 1000 \).
2. Effect of increase in the nominal interest rate:
When \( Y = 1000, V = 1000/[500 + (0.2 \times 1000) - 1000i] \]
\[ = 1000/(700 - 1000i) \]
\[ = 1/(0.7 - i). \]
When \( i \) increases, \( 0.7 - i \) decreases, so \( V \) increases. For example, if \( i = 0.20 \), then \( V = 2 \), which is an increase over \( V = 1 2/3 \) that we got when \( i = 0.10 \).
3. Effect of increase in the price level:
There is no effect on velocity, since we can write velocity as a function just of \( Y \) and \( i \). Nominal money demand changes proportionally with the price level, so that real money demand, and hence velocity, is unchanged.
3. (a) \( M^d = $100,000 - $50,000 - [$5000 \times (i - \bar{i}^m) \times 100] \). (Multiplying by 100 is necessary since \( i \) and \( \bar{i}^m \) are in decimals, not percent.) Simplifying this expression, we get
\[ M^d = $50,000 - $500,000(i - \bar{i}^m). \]
(b) \( B^d = $50,000 + $500,000(i - \bar{i}^m). \)
Adding these together we get \( M^d + B^d = $100,000 \), which is Mr. Midas’s initial wealth.
(c) This can be solved either by setting money supply equal to money demand, or by setting bond supply equal to bond demand.
\[ M^d = M^d \]
\[ \$50,000 - \$500,000(i - \bar{r}^m) = \$20,000 \]
\[ \$30,000 = \$500,000 i \quad \text{[Setting} \bar{r}^m = 0] \]
\[ i = 0.06 = 6\% \]

\[ B^d = B^s \]
\[ \$50,000 + \$500,000i = \$80,000 \]
\[ \$500,000i = \$30,000 \]
\[ i = 0.06 = 6\% \]

4. (a) From the equation \( MV = PY \), we get \( M/P = Y/V \). At equilibrium, \( M^d = M \), so \( M^d/P = Y/V = 10,000/5 = 2000 \). \( M^d = P \times (M^d/P) = 2 \times 2000 = 4000 \).

(b) From the equation \( MV = PY \), \( P = MV/Y \).

When \( M = 5000 \), \( P = (5000 \times 5)/10,000 = 2.5 \).

When \( M = 6000 \), \( P = (6000 \times 5)/10,000 = 3 \).

5. (a) \( \Delta P/P = -\eta \Delta Y/Y = -0.5 \times 6\% = -3\% \). The price level will be 3% lower.

(b) \( \Delta P/P = -\eta \Delta r/r = -(-0.1) \times 0.1 = 1\% \). The price level will be 1% higher.

(c) With changes in both income and the real interest rate, to get an unchanged price level would require \( \eta \Delta Y/Y + \eta \Delta r/r = 0 \), so \[ 0.5 \times (Y - 100)/100 - [0.1 \times 0.1] = 0 \], so \( Y = 102 \).

6. (a) \( \pi^e = \Delta M/M = 10\% \). \( i = r + \pi^e = 15\% \). \( M/P = L = 0.01 \times 150/0.15 = 10 \). \( P = 300/10 = 30 \).

(b) \( \pi^e = \Delta M/M = 5\% \). \( i = r + \pi^e = 10\% \). \( M/P = L = 0.01 \times 150/0.10 = 15 \). \( P = 300/15 = 20 \). The slowdown in money growth reduces expected inflation, increasing real money demand, thus lowering the price level.

7. (a) With a constant real interest rate and zero expected inflation, inflation is given by the equation \( \pi = \Delta M/M - \eta \Delta Y/Y \). To get inflation equal to zero, the central bank should set money growth so that \( \Delta M/M = \eta \Delta Y/Y = 2/3 \times .045 = .03 \) = 3%. Note that the interest elasticity isn’t relevant, since interest rates don’t change.

(b) Since \( V = PY/M, \Delta V/V = \Delta P/P + \Delta Y/Y - \Delta M/M \)
\[
= 0 + .045 - .03
\]
\[
= .015
\]
So velocity should rise 1.5% over the next year.