Numerical Methods MAD 5405 — Spring 2012
Home work

1. Due Jan 27.
   Write a program to generate the matrix A on Page 9.

2. Due Feb. 3.
   Page 24, Problem 5, 8.

3. Due Feb. 10

   (a) Page 24, Problem 14, 20

   (b) Run the following codes and try to explain: (1) Why was this experiment carried out ? (2) Why did the result came out as it did ?

```
    m = 1500;       %%% ( or 2000, 2500, 3000)
    Z = randn(m,m);
    A = Z'*Z;
    b = randn(m,1);
%%% (a)
    tic; x1 = A\b; toc;
%%% (b)
    A2 = A; A2(m,1) = A2(m,1)/2;
    tic; x2 = A2\b; toc;
%%% (c)
    I=eye(m,m); emin = min(eig(A));
    A3 = A-0.9*emin*I;
    tic; x3 = A3\b; toc;
%%% (d)
    A4 = A-1.1*emin*I;
    tic; x4 = A4\b; toc;
%%% (e)
    A5 = triu(A);
    tic; x5 = A5\b; toc;
%%% (f)
    A6 = A5; A6(m,1) = A5(1,m);
    tic; x6 = A6\b; toc;
```

4. Due Feb. 17
   Write programs that compute QR factorization $A = QR$ of an $m \times n$ matrix A with $m \geq n$ using CGS Algorithm (Classical Gram-Schmidt), MGS Algorithm (Modified Gram-Schmidt) and Householder transformation. Measure the accuracy of the results with residual $\|A - Q \cdot R\| / \|A\|$ and orthogonality $\|Q^T \cdot Q - I\|$. Run these programs with the following matrices:

$$A_1 = \begin{pmatrix} 1 & 1 & 1 \\ a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix} \qquad \text{for small value} \quad a$$

$$A_2 = \begin{pmatrix} 0.70000 & 0.70711 \\ 0.70001 & 0.70711 \end{pmatrix}$$

5. Due Feb. 24.
   Page 49, Problems 1, 3, 11, 16.

6. Due Mar. 2

   (a) Show that an $n \times n$ matrix A is irreducible if and only if its directed graph $G(A)$ is strongly connected.
   (b) Page 75, Problems 6, 7, 8, 9, 10.
   (c) Complete the proof of Theorem 4.9.
   (d) Write program that compute **reduced** QR factorization $A = QR$ of an $m \times n$ matrix A with $m \geq n$ using Householder transformation..
   And compute Qx from the output of the program.

7. Due Mar. 9
   (a) Show that block tri-diagonal matrix

$$\begin{bmatrix} D_1 & A_1 & & \\ B_1 & \ddots & \ddots & \\ & \ddots & \ddots & A_{n-1} \\ & & B_{n-1} & D_n \end{bmatrix}$$

   is consistently ordered when the D_i are diagonal.
   (b) Page 76, Problem 15, 16.

8. Due Mar. 23.
   (a) Page 76, Problem 4.19.
   (b) Consider the modal problem:

   $$-u''(x) = f(x), \qquad 0 < x < 1; \quad u(0) = u(1) = 0.$$

   Write programming to solve the corresponding linear system by Jacobi, Gauss-Seidel and SOR methods. Compare these iterative methods.

9. Due Mar. 30.

   Write programming to solve the above Modal program by Two-Grid method.

10. Due Apr. 6.

    Page 91, Problem 4, 8, 12.

11. Due Apr. 20.

    Write programming to compute all the zeros of a polynomial has only simple real zeros. (Page 112)