6 Control Charts for Variables

Learning Objectives

1. Understand the statistical basis of Shewhart control charts for variables
2. Know how to design variables control charts
3. Know how to set up and use $\bar{x}$ and $R$ control charts
4. Know how to estimate process capability from the control chart information
5. Know how to interpret patterns on $\bar{x}$ and $R$ control charts
6. Know how to set up and use $\bar{x}$ and $s$ or $s^2$ control charts
7. Know how to set up and use control charts for individual measurements
8. Understand the importance of the normality assumption for individuals control charts and know how to check this assumption
9. Understand the rational subgroup concept for variables control charts
10. Determine the average run length for variables control charts

6.1 Introduction

Many quality characteristics can be expressed in terms of a numerical measurement. A single measurable quality characteristic, such as dimension, weight, or volume is called a variable. Control charts for variables are used extensively. Control of the process average or mean quality level is usually done with the control chart for means, or the $\bar{x}$ chart. Process variability can be monitored with either a control chart for the standard deviation, called the $S$ chart or control chart for range, called the $R$ chart.
6.2 Control Charts for $\bar{x}$ and $R$

6.2.1 Statistical Basis of the Charts

Control Limits for $\bar{x}$ and $R$ charts

Suppose, a quality characteristic is normally distributed with mean $\mu$ and standard deviation $\sigma$, where both $\mu$ and $\sigma$ are known. If $x_1, x_2, \ldots, x_n$ is a sample of size $n$, then the sample mean,

$$\bar{x} : N\left(\mu, \frac{\sigma^2}{n}\right)$$

Then the probability is $(1-\alpha)$ that any sample mean will fall between

$$\mu + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \quad \text{and} \quad \mu - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Now, if $\mu$ and $\sigma$ are known then $\mu + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ and $\mu - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ could be used as upper and lower control limits on a control chart for sample mean. If we replace $Z_{\alpha/2}$ by 3, we get 3-$\sigma$ control limits. Since, $\mu$ and $\sigma$ are unknown most of the times, we have to estimate them from the observed samples.

Suppose that $m$ samples are available, each containing $n$ observations on the quality characteristic. Let $\bar{x}_1, \ldots, \bar{x}_m$ be the average of each sample. Also let $R_1, \ldots, R_m$ be the ranges of the $m$ samples. ($R = x_{\text{max}} - x_{\text{min}}$)

Control Limits for the $\bar{x}$ chart

$$UCL = \bar{x} + A_2 \bar{R}$$

$$\text{Center line}(CL) = \bar{x}$$

$$LCL = \bar{x} - A_2 \bar{R},$$

where $A_2$ is available from Appendix VI and

$$\bar{x} = \frac{\sum_{i=1}^{m} x_i}{m}, \quad \bar{R} = \frac{\sum_{i=1}^{m} R_i}{m}.$$
Control Limits for the $R$ Chart

$$UCL = D_4 \bar{R}$$
$$CL = \bar{R}$$
$$LCL = D_3 \bar{R},$$

where $D_3$ and $D_4$ are constants and available from Appendix VI (page 702) for various values of $n$. The $\bar{R}$ chart is good for small sample size.

6.2.2 Development and Use of $\bar{X}$ and $R$ Charts

Example 6.1, page 231 (Data in Table 6.1) There are 25 samples, that is $m = 25$. The sample size is 5, that is $n = 5$.

| Table 6.1 |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Sample Number   | Wafer 1         | Wafer 2         | Wafer 3         | Wafer 4         | Wafer 5         | $\bar{x}_i$  |
| 1               | 1.3235          | 1.4128          | 1.6744          | 1.4573          | 1.6914          | 1.5119         |
| 2               | 1.4314          | 1.3592          | 1.6075          | 1.4666          | 1.6109          | 1.4951         |
| 3               | 1.4284          | 1.4871          | 1.4932          | 1.4324          | 1.5674          | 1.4817         |
| 4               | 1.5028          | 1.6352          | 1.3841          | 1.2831          | 1.5507          | 1.4712         |
| 5               | 1.5604          | 1.2735          | 1.5265          | 1.4363          | 1.6441          | 1.4882         |
| 6               | 1.5955          | 1.5451          | 1.3574          | 1.3281          | 1.4198          | 1.4492         |
| 7               | 1.6274          | 1.5064          | 1.8366          | 1.4177          | 1.5144          | 1.5805         |
| 8               | 1.4190          | 1.4303          | 1.6637          | 1.6067          | 1.5519          | 1.5343         |
| 9               | 1.3884          | 1.7277          | 1.5355          | 1.5176          | 1.3688          | 1.5076         |
| 10              | 1.4039          | 1.6697          | 1.5089          | 1.4627          | 1.5220          | 1.5134         |
| 11              | 1.4158          | 1.7667          | 1.4278          | 1.5928          | 1.4181          | 1.5242         |
| 12              | 1.5821          | 1.3355          | 1.5777          | 1.3908          | 1.7559          | 1.5284         |
| 13              | 1.2856          | 1.4106          | 1.4447          | 1.6398          | 1.1928          | 1.3947         |
| 14              | 1.4951          | 1.4036          | 1.5893          | 1.6458          | 1.4969          | 1.5261         |
| 15              | 1.3589          | 1.2863          | 1.5996          | 1.2497          | 1.5471          | 1.4083         |
| 16              | 1.5747          | 1.5301          | 1.5171          | 1.1839          | 1.8662          | 1.5344         |
| 17              | 1.3680          | 1.7269          | 1.3957          | 1.5014          | 1.4449          | 1.4874         |
| 18              | 1.4163          | 1.3664          | 1.3057          | 1.6210          | 1.5573          | 1.4573         |
| 19              | 1.5796          | 1.4185          | 1.6541          | 1.5116          | 1.7247          | 1.5777         |
| 20              | 1.7106          | 1.4412          | 1.2361          | 1.3820          | 1.7601          | 1.5060         |
| 21              | 1.4371          | 1.5051          | 1.3485          | 1.5670          | 1.4880          | 1.4691         |
| 22              | 1.4738          | 1.5936          | 1.6583          | 1.4973          | 1.4720          | 1.5390         |
| 23              | 1.5917          | 1.4333          | 1.5551          | 1.5295          | 1.6866          | 1.5592         |
| 24              | 1.6399          | 1.5243          | 1.5705          | 1.5563          | 1.5530          | 1.5688         |
| 25              | 1.5797          | 1.3663          | 1.6240          | 1.3732          | 1.6887          | 1.5264         |

$\bar{x}_i = 37.6400$  $\bar{R}_i = 8.1302$

$\bar{x} = 1.5056$  $\bar{R} = 0.32521$
The \( R \) control limits are as follows:

\[
\begin{align*}
UCL &= 0.32521 \times 2.114 = 0.68749 \\
CL &= 0.32521 \\
LCL &= 0.32521 \times 0 = 0.
\end{align*}
\tag{3}
\]

The \( R \) chart has shown in Figure 6.2(b) as follows.

Since \( R \) chart indicates that process variability is in control, we may now construct \( \bar{x} \) chart. The \( \bar{x} \) control limits are as follows:

\[
\begin{align*}
UCL &= 1.5056 + 0.577 \times 0.32521 = 1.69325 \\
CL &= 1.5056 \\
LCL &= 1.5056 - 0.577 \times 0.32521 = 1.31795
\end{align*}
\tag{4}
\]

The \( \bar{x} \) chart is shown in Figure 6.2 (a) indicates that the process is in control. The control limits in (3) and (4) are called trial control limits for use in phase II, where monitoring of future production is of interest.

Since, both the \( \bar{x} \) and \( R \) charts exhibit control, we conclude that the process is in statistical control at the stated levels and adopt the trial control limits to control the future production.
Estimating Nonconformance rates and Process Capability

If the process measurements follow a normal distribution then the proportion of conforming items can be estimated by

\[ p = P(LSE < x < USL), \]

and the proportion of nonconforming can be estimated by

\[
p = 1 - P(LSL < x < USL) = 1 - P\left( \frac{LSL - \mu}{\sigma} < z < \frac{USL - \mu}{\sigma} \right)
\]

where \( \mu \) (process mean) and \( \sigma \) (process standard deviation) can be estimated by,

\[
\hat{\mu} = \bar{x}, \quad \text{and} \quad \hat{\sigma} = \frac{R}{d_2}
\]

respectively.

**Example:** Consider example 6.1, suppose that the specification limits are \( 1.50 \pm 0.50 \) mm. Find the proportion of nonconforming piston ring for this process.

The mean and process SD are respectively

\[
\hat{\mu} = \bar{x} = 1.5056, \quad \text{and} \quad \hat{\sigma} = \frac{R}{d_2} = \frac{0.32521}{2.326} = 0.1398
\]

The fraction of non-conforming wafers is

\[
p = 1 - P(1.00 < x < 2.00) = 1 - P\left( \frac{1.00 - 1.5056}{0.1398} < z < \frac{2.0 - 1.5056}{0.1398} \right) = 0.00035
\]

About 0.035 percent [350 parts per million (ppm)] of the wafers produced will be outside of the specifications.
Process Capability Ratio: (PCR)

Assume that the process measurements approximately follow a normal distribution, so that the process spread should encompass a range of about $6\sigma$. This range is called the actual process spread and the distance between USL and LSL is called the allowable process spread. Then the process capability index, denoted by $C_p$ can be defined by

$$PCR = C_p = \frac{USL - LSL}{6\sigma},$$

where $6\sigma$ is the spread of the process and came from the normality assumption. The estimated PCR is

$$\hat{PCR} = \hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}}$$

**Interpretation:**

i. If $C_p = 1.0 \Rightarrow$ marginally capable.

ii. $C_p > 1.33 \Rightarrow$ very good and commonly used by many companies.

iii. $C_p < 1 \Rightarrow$ Not capable.

**Note** Capability indexes are free of unit, which allow them to be used compare two entirely different process. For example inch, kg, or degree etc.

**Example:** Consider example 6.1 and assume that the specification limits for the process is $1.50 \pm 0.50 = [1.00, 2.00]$. The estimated process standard deviation is

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{0.32521}{2.326} = 0.1398.$$

Then the estimated PCR

$$\hat{C}_p = \frac{2.00 - 1.00}{6 \times 0.1398} = \frac{1.00}{0.8388} = 1.192.$$

Since the $PCR > 1$, we would conclude that process is capable. Another way, the quantity

$$p = \left(\frac{1}{C_p}\right) \times 100\% = \frac{1}{1.192} \times 100 = 83.89\%$$

The process uses up about 84% of the specification bands.
Figure 6.3, illustrated three cases of PCR.


Revision of control limits and central line: We always treat the initial set of control limits as trial limits, subject to subsequent revision. The effective use of control chart will require revision of the control limits, such as every week, every month or say, 25, 50 or 100 samples. Revised control limits may be used for at least 25 samples or subgroups. More on page 235.

Phase II Operation of the $\bar{x}$ and $R$ charts

Once a set of reliable limits are established we use the control chart for monitoring future production. This is called phase II control chart usage (Figure 6.4)
A run chart showing individuals observations in each sample, called a tolerance chart or tier diagram (Figure 6.5), may reveal patterns or unusual observations in the data.

![Figure 6.5](image1.png) Tier chart constructed using the Minitab box plot procedure for the flow width data.

**Control limits, Specification limits and Natural tolerance limits:** There is no mathematical or statistical relationship between control limits on $\bar{x}$ and $R$ charts and specification limits on the process. The control limits are driven by the natural variability of the process ie. by the natural tolerance limits of the process. See the relationship between Control limits, specification limits and natural tolerance limits in Figure 6.6.

![Figure 6.6](image2.png) Relationship of natural tolerance limits, control limits, and specification limits.
Rational subgroups:

- $\bar{x}$ charts monitor between-sample variability
- $R$ charts measure within-sample variability
- Standard deviation estimate of $\sigma$ used to construct control limits is calculated from within-sample variability
- It is not correct to estimate $\sigma$ using

$$s = \sqrt{\frac{\sum_{i=1}^{m} \sum_{j=1}^{n} (x_{ij} - \bar{x})^2}{m(n-1)}}$$

**Guide lines for design of the control limits:** To design the $\bar{x}$ and $R$ charts, one must specify the sample size, control limit width, and frequency of sampling to be used. It is not possible to give an exact solution to the problem of control chart design, unless the analyst has detailed information about both the statistical characteristics of the control chart tests and the economic factors that affect the problem. See more on page 238-239.

**Changing sample size on the $\bar{x}$ and $R$ charts:** We usually assume that the sample size $n$ is constant from sample to sample. However, there are many situations the sample size is not constant. See Example 6.2, page 241.

**Probability limits on the $\bar{x}$ and $R$ charts:** We generally express the control limits on $\bar{x}$ and $R$ charts as a multiple of the standard deviation of the statistic plotted on the charts. If the multiple chosen is $k$, then the limits referred to as $k$-sigma limits (usually $k=3$, called 3-sigma limits). However, it is also possible to define the control limits by specifying the type I error level for the test. Such control limits are called probability limits. See more on Page 242.

### 6.2.3 Chart based on the standard values

**Control Limits for the $\bar{x}$ chart**

- Upper Control Limit (UCL) = $\mu + A\sigma$
- Center line (CL) = $\mu$
- Lower Control Limit (LCL) = $\mu - A\sigma$.

where $A = \frac{3}{\sqrt{n}}$ is available from Appendix VI and
Control Limits for the $R$ Chart

$$\begin{align*}
UCL &= D_2 \sigma \\
CL &= d_2 \sigma \\
LCL &= D_1 \sigma,
\end{align*}$$

(6)

where $D_1 = d_2 - 3d_3$ and $D_2 = d_2 + 3d_3$ are constants and available from Appendix VI for various values of $n$.

6.2.4 Interpretation of $\bar{x}$ and $R$ charts

A control chart can indicate an out-of-control condition even though no single point plots outside the control limits, if the pattern of the plotted points exhibits nonrandom or systematic behavior. One should never attempt to interpret the $\bar{x}$ chart when the $R$ chart indicates an out-of-control condition. Here we briefly discuss some of the common patterns that appears on $\bar{x}$ and $R$ charts.

**Cyclic patterns:** Patterns may result from systematic environmental changes such as temperature, operator fatigue, regular rotation of operators or machines, or fluctuation in voltage or pressure etc. See Figure 6.8.

**Mixture pattern:** A mixture pattern can also occur when output product from several sources (such as parallel machines) are fed into a common stream which is then sampled for process monitoring purposes. See Figure 6.9.

**A shift in process level:** May result from the introduction of new workers, methods, raw materials, machines, change in the inspection method or standard etc. See Figure 6.10.

**A trend:** Causes, operator fatigue, presence of supervisor, temperature, tool wear, systematic causes of deterioration. See Figure 6.11.
Stratification: A tendency for the points to cluster artificially around the center line (see Figure 6.12).

6.2.5 The Effect of nonnormality on $\bar{x}$ and $R$ Charts

A fundamental assumption in the development of $\bar{x}$ and $R$ control limits is that the underlying distribution of the quality characteristics is normal. However, in practice there are many situations we may have data that do not follow normality assumption. More on page 246.
6.2.6 The Operating-Characteristic Function

The ability of $\bar{x}$ and $R$ charts to detect shifts in process quality is described by their operating-characteristic (OC) curves. If the mean shifts from in control value, say $\mu_0$, to another value say $\mu_i = \mu_0 + k\sigma$, then the probability of not detecting the shift on the first subsequent sample or the $\beta$ risk is

$$\beta = P\{LCL \leq \bar{x} \leq UCL | \mu = \mu_i = \mu_0 + k\sigma\}$$

(7)

Since $\bar{x} \sim N(\mu, \sigma^2/n)$, and the upper and lower control limits are $UCL = \mu_0 + L\sigma\sqrt{n}$ and $LCL = \mu_0 - L\sigma\sqrt{n}$, we may write the equation (7) as

$$\beta = \Phi(L-k\sqrt{n}) - \Phi(-L-k\sqrt{n})$$

(8)

To illustrate (8), we consider $L = 3$, $n = 5$, want to determine the probability of detecting a shift to $\mu_i = \mu_0 + 2\sigma$ on the first sample following the shift. Then $k = 2$ and

$$\beta = \Phi(3 - 2\sqrt{5}) - \Phi(-3 - 2\sqrt{5}) = 0.0708$$

(9)

The probability of not detecting a shift is 0.0708. Therefore, the probability that such a shift will be detected on the first subsequent sample is $1 - \beta = 1 - 0.708 = 0.9292$. To construct OC curve for $\bar{x}$ chart, plot the $\beta$ risk against the magnitude of $k$ for various values on $n$. See figure 6.13.
Detecting a shift on the \( r^{th} \) sample  The probability that the shift will be detected on the \( r^{th} \) sample is \( 1 - \beta \) times the probability of not detecting a shift on each of the initial \( r-1 \) samples. That is,
\[
(1 - \beta)\beta^{r-1}
\]

(which is a geometric distribution)  The expected number of samples taken before the shift is detected is the **average run length** or
\[
ARL = \frac{1}{1 - \beta}
\]

which is the mean of the Geometric distribution.

**Exercise 6.40 and 6.41, page 280.**

6.2.7 The Average Run Length for the \( \bar{x} \) Chart

The ARL can be expressed as
\[
ARL = \frac{1}{P(\text{one point plots out of control})}
\]

or
\[
ARL_0 = \frac{1}{\alpha}, \quad \text{for in-control ARL}
\]
\[
ARL_1 = \frac{1}{1 - \beta}, \quad \text{for out-of-control ARL}
\]

See Figure 6.15 for ARL curve.
Average Time to Signal (ATS)

\[ ATS = ARL \times h, \]

where \( h \) indicates the equal time interval between samples.

Expected number of individual units (I)

\[ I = n \times ARL, \]

where \( n \) is the sample size.

6.3 Control Charts for \( \bar{X} \) and \( S \) Charts

Generally, \( \bar{X} \) and \( R \) charts are widely used, however, \( \bar{X} \) and \( S \) charts are preferable over \( \bar{X} \) and \( R \) charts, when

(1) The sample size \( n \) is large, say \( n > 10 \).
(2) The sample size \( n \) is variable

6.3.1 Construction and operation of \( \bar{X} \) and \( S \) Charts

The control limits for \( S \) chart: when \( \sigma \) is known

\[
\begin{align*}
UCL &= B_{6}\sigma \\
CL &= c_{4}\sigma \\
LCL &= B_{c}\sigma,
\end{align*}
\]

\( (10) \)
where \( c_4, B_5 \) and \( B_6 \) are available from Appendix VI.

When \( \sigma \) is unknown then it must be estimated by analyzing past data. Suppose that \( m \) preliminary samples are available, each of size \( n \), and let \( S_i \) be the standard deviation of the \( i^{th} \) sample and we compute the following statistics

\[
\bar{S} = \frac{1}{m} \sum_{i=1}^{m} S_i, \quad \text{where} \quad S_i = \sqrt{\frac{1}{n-1} \sum_{j=1}^{n} (x_{ij} - \bar{x}_i)^2}
\]

The control limits for \( S \) chart: when \( \sigma \) is unknown

\[
UCL = B_4 \bar{S} \\
CL = \bar{S} \\
LCL = B_3 \bar{S},
\]

where \( B_3 = \frac{B_i}{c_4} \) and \( B_4 = \frac{B_6}{c_4} \) are available from Appendix VI.

The control limits for \( \bar{x} \) chart

\[
UCL = \bar{x} + A_2 \bar{S} \\
CL = \bar{x} \\
LCL = \bar{x} - A_2 \bar{S},
\]

where \( A_2 = \frac{3}{c_4 \sqrt{n}} \) is available from Appendix VI. Generally, the \( \bar{x} \) control limits based on \( \bar{S} \) will be different than limits based on \( \bar{R} \).

**Example 6.3, page 254** The control limits for \( \bar{x} \) chart are as follows:

\[
UCL = 74.001 + 1.427 \times 0.0094 = 74.014 \\
CL = 74.001 \\
LCL = 74.001 - 1.427 \times 0.0094 = 73.988
\]
The control limits for $S$ chart are as follows:

$$UCL = 2.089 \times 0.0094 = 0.0196$$
$$CL = 0.0094$$
$$LCL = 0 \times 0.0094 = 0$$  \hspace{1cm} (14)$$

The $\bar{x}$ chart has shown in Figure 6.17 (a) and the $S$ chart has shown in Figure 6.17 (b) and both figures indicated that the process is in control. Therefore, these limits could be adopted for phase II monitoring of the process.

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**Estimation of $\sigma$**

The statistic $\frac{\bar{S}}{c_4}$ is an unbiased estimator of $\sigma$. Therefore, estimator of the process standard deviation $\sigma$ is

$$\hat{\sigma} = \frac{\bar{S}}{c_4} = \frac{0.0094}{0.9400} = 0.01$$
6.3.2 The $\bar{x}$ and $S$ Control Charts with variable Sample size

Suppose that $m$ preliminary samples are available, and $n_i$ is the number of observations for the $i^{th}$ sample. Let $\bar{x}_i$ and $S_i$ be the sample mean and standard deviation of the $i^{th}$ sample respectively. Then the weighted average of the statistics are

$$\bar{X} = \frac{\sum_{i=1}^{m} n_i \bar{x}_i}{\sum_{i=1}^{m} n_i}$$

and

$$\bar{S} = \left[ \frac{\sum_{i=1}^{m} (n_i - 1) S_i^2}{\sum_{i=1}^{m} (n_i - 1)} \right]^{1/2}.$$

Then the control limits for $S$ chart are

$$UCL = B_4 \bar{S}$$
$$CL = \bar{S}$$
$$LCL = B_3 \bar{S},$$

where $B_3 = \frac{B_5}{c_4}$ and $B_4 = \frac{B_6}{c_4}$ are available from Appendix VI based on the sample size used in each individual group.

The control limits for $\bar{x}$ chart are

$$UCL = \bar{x} + A_3 \bar{S}$$
$$CL = \bar{x}$$
$$LCL = \bar{x} - A_3 \bar{S},$$

where $A_3 = \frac{3}{c_4 \sqrt{n}}$ is available from Appendix VI based on the sample size used in each individual group.
Example 6.4, page 256: The control limits for $\bar{x}$ chart are as follows:

$$UCL = 74.016$$
$$CL = 74.001$$
$$LCL = 73.986$$

The control limits for $S$ chart are as follows:

$$UCL = 0.022$$
$$CL = 0.0103$$
$$LCL = 0$$

The control charts based on the variable sample sizes in Table 6.4 are plotted in Figure 6.18.

Estimation of $\sigma$

First compute $\bar{S}$ for the most frequently occurring value of $n_i$ (in this case 5) as

$$\bar{S} = \frac{0.1715}{17} = 0.0101$$

which is close to the weighted $\bar{S} = 0.0098$ on page 246. Then the estimated $\sigma$ is

$$\hat{\sigma} = \frac{\bar{S}}{c_4} = \frac{0.0101}{0.9400} = 0.01,$$

where $c_4$ is used for $n = 5$ from Appendix VI.
6.3.3 The $s^2$ Control Chart

Most quality engineers use either the $R$ chart or the $s$ chart to monitor process variability, with $s$ preferable to $R$ for moderate to large sample sizes. Some practitioners recommend a control chart based directly on the sample variance $s^2$, the $s^2$ control chart. The parameters for the $s^2$ control chart are

\[
\begin{align*}
UCL &= \frac{s^2}{n-1} \chi^2_{\alpha/2,n-1} \\
\text{Center line} &= s^2 \\
LCL &= \frac{s^2}{n-1} \chi^2_{1-(\alpha/2),n-1}
\end{align*}
\]  

(6.32)

where $\chi^2_{\alpha/2}$ and $\chi^2_{1-(\alpha/2),n-1}$ denote the upper and lower $\alpha/2$ percentage points of the chi-square distribution with $n-1$ degrees of freedom, and $\bar{s}^2$ is an average sample variance obtained from the analysis of preliminary data. A standard value $\sigma^2$ could be used in equation (6.32) instead of $\bar{s}^2$ if one were available. Note that this control chart is defined with probability limits.


6.4 The Shewart Control Chart for Individual Measurements

There are many situations in which the sample size used for the process is $n=1$. See page 259 for such examples. For this chart, one need to compute moving range of two successive observations as

\[
MR_i = |x_i - x_{i-1}|
\]

Then the average moving range is

\[
\overline{MR} = \frac{\sum_{i=2}^{m} MR_i}{m-1}
\]
The control limits for individual measurements

\[
UCL = \bar{x} + 3 \frac{MR}{d_2} \\
CL = \bar{x} \\
LCL = \bar{x} - \frac{MR}{d_2},
\]

where \( \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \).

The control limits for moving range

\[
UCL = D_4 MR \\
CL = MR \\
LCL = D_3 MR
\]

Example 6.5, page 260

The control limits for individual measurements are

\[
UCL = 300.5 + 3 \times \frac{7.79}{1.128} = 321.22 \\
CL = 300.5 \\
LCL = 300.5 - 3 \times \frac{7.79}{1.128} = 279.78
\]

The control limits for moving range

\[
UCL = 3.267 \times 7.79 = 25.45 \\
CL = 7.79 \\
LCL = 0 \times 7.79 = 0
\]

Interpretation of the control charts (Figure 6.19), see page 261

Estimation of $\sigma$

$$\hat{\sigma}_1 = \frac{MR}{d_2}$$

$$\hat{\sigma}_2 = \frac{S}{c_4},$$

where $S = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$. Note: Here $n = 2$, because of moving average of sample size 2.

3

There are some real life examples (Example 6.7, page 268 to Example 6.11, page 270) have been discussed in this section.