Definitions

Game = Opportunity to gain utility through strategic interaction with at least one other "player."

Zero-sum Game = A game in which one player's loss is another player's gain (technically, this is a constant-sum game).

Strategy = A plan of action, covering all contingencies.

Outcome = The consequences of a plan of action, given what other players do.

Expected Utility = The value one gains (or loses) from the probable outcomes of a particular strategy.

Dominant Strategy = A strategy that is preferable to any other strategy (e.g., it brings better outcomes) regardless of what the other player does. To find the dominant strategy, consider the best strategy for each of the "opponent's" strategies. If the same strategy is always best, it is a dominant strategy.

Iterated Game = A game that is played more than once.

Pure strategy = Playing only one strategy. Single-play games have pure strategies.

Mixed strategy = Playing different strategies with a specific probability. Multiple-play (iterated) games may have either pure or mixed strategies.

Pareto Optimum = An outcome that is collectively preferable; there is no other outcome that would be better for both players.

Nash Equilibrium = An outcome for which neither player has any unilateral incentive to change its strategy. An outcome is a Nash equilibrium if each player would do worse by changing its strategy unilaterally.

Solution = A stable outcome, the result of rational choices in strategic interaction.

Minimax Theorem = Every finite, two-person, zero-sum game has an equilibrium solution (either pure, mixed, or both).

Game Solutions:

1. In a sequential game, search for a rollback equilibrium by starting at the final decision node(s) and working "back" to the beginning, eliminating all dominated pathways. The pathway that remains leads to the solution.

2. In games with simultaneous moves:
   a. Identify any dominant strategies. If at least one player has a dominant strategy, it will choose that strategy. The other player will then choose its "best" strategy, and the resulting outcome is the solution.
   b. Eliminate any inferior (dominated) strategies. If the game matrix is larger than 2 x 2, look for any strategies (rows or columns) that will never be chosen. Eliminate these strategies until only one outcome remains—this is the solution. If it is impossible to reduce the game to a single outcome, then search for Nash equilibria (see 2c).

(see over)
c. Search for Nash equilibria. If neither player has a dominant strategy (and if no inferior strategies can be eliminated), then ask—for each remaining cell in the matrix—whether either play would unilaterally change its strategy. If neither player would change its strategy unilaterally, that outcome is a Nash equilibrium.

d. Calculate a mixed strategy. In games with no Nash equilibria (and even in some games that do have Nash equilibria), a mixed strategy can be calculated if the game is played repeatedly (an iterated game). To find a mixed strategy equilibrium, choose the probability of playing each strategy that would make the other player indifferent between its own strategies.

Some Common Strategic Games:

Preferences are expressed ordinally as follows:

1 = best outcome
2 = second best outcome
3 = second worst outcome
4 = worst outcome

Prisoner’s Dilemma

<table>
<thead>
<tr>
<th></th>
<th>Silent</th>
<th>Confess</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silent</td>
<td>2,2</td>
<td>4,1</td>
</tr>
<tr>
<td>Confess</td>
<td>1,4</td>
<td>3,3</td>
</tr>
</tbody>
</table>

Pareto optimum = (Silent, Silent)
Nash equilibrium = (Confess, Confess)

Chicken

<table>
<thead>
<tr>
<th></th>
<th>Swerve</th>
<th>Don’t Swerve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swerve</td>
<td>2,2</td>
<td>3,1</td>
</tr>
<tr>
<td>Don’t Swerve</td>
<td>1,3</td>
<td>4,4</td>
</tr>
</tbody>
</table>

Pareto optimum = (Swerve, Swerve)
Nash equilibria = (Don’t Swerve, Swerve) or (Swerve, Don’t Swerve)

Stag Hunt

<table>
<thead>
<tr>
<th></th>
<th>Hunt Stag</th>
<th>Hunt Rabbit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hunt Stag</td>
<td>1,1</td>
<td>4,2</td>
</tr>
<tr>
<td>Hunt Rabbit</td>
<td>2,4</td>
<td>3,3</td>
</tr>
</tbody>
</table>

Pareto optimum = (Hunt Stag, Hunt Stag)
Nash equilibria = (Hunt Stag, Hunt Stag) or (Hunt Rabbit, Hunt Rabbit)