

INR 3303 — Practice Exercises 3  
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For the following two problems, calculate a mixed strategy equilibrium (i.e., the optimal mixed strategy for each player). Payoffs are cardinal (higher is better). Remember that the optimal mixed strategy is one that plays each strategy with a probability ( $p$ ) such that the opponent is indifferent between each of its strategies. To calculate this, set the expected utility equations for the opponents strategies equal to each other and solve for  $p$ .

1. Calculate the optimal mixed strategy for **both** Row and Column (**show all work, using the back of this page if necessary for your expected utility equations**):

		Column	
		<u>Left</u>	<u>Right</u>
Row	Up	1, 2	2, 1
	Down	2, 1	0, 4

For Row:  $2p + 1(1-p) = 1p + 4(1-p)$   
 $2p + 1 - p = p + 4 - 4p$   
 $p + 1 = 4 - 3p$   
 $4p = 3$   
 $p = 3/4$

Row should play Up 3/4 of the time  
 Row should play Down 1/4 of the time

For Column:  $1p + 2(1-p) = 2p + 0(1-p)$   
 $p + 2 - 2p = 2p$   
 $2 - p = 2p$   
 $2 = 3p$   
 $p = 2/3$

Column should play Left 2/3 of the time  
 Column should play Right 1/3 of the time

The mixed strategy equilibrium is (3/4 Up, 1/4 Down; 2/3 Left, 1/3 Right) for (Row; Column).

2. Suppose that two countries, Spinlandia and Octavia have periodically engaged in trade wars. Both wish to continue trading, but each wishes to stand firm occasionally ("war") in order to protect critical industries and to satisfy labor interests at home. Suppose that payoffs in a given industry, over time, are given (cardinally) in billions of dollars. Calculate the optimal mixed strategy for **both** Spinlandia and Octavia (**show all work, using the back of this page if necessary for your expected utility equations**):

		Octavia	
		<u>Trade</u>	<u>War</u>
Spin.	Trade	8, 8	0, 4
	War	4, 0	2, 2

For Spinlandia:  $8p + 0(1-p) = 4p + 2(1-p)$   
 $8p = 4p + 2 - 2p$   
 $8p = 2p + 2$   
 $6p = 2$   
 $p = 1/3$

Spinlandia should Trade 1/3 of the time  
 Spinlandia should War 2/3 of the time

For Octavia:  $8p + 0(1-p) = 4p + 2(1-p)$   
 $8p = 4p + 2 - 2p$   
 $8p = 2p + 2$   
 $6p = 2$   
 $p = 1/3$

Octavia should Trade 1/3 of the time  
 Octavia should War 2/3 of the time

The mixed strategy equilibrium is (1/3 Trade, 2/3 War; 1/3 Trade, 2/3 War) for (Spinlandia; Octavia).