

Abstract

Seawater intrusion is a continuing concern in south Florida and in many other coastal areas. Although there is extensive research on this topic, more tools are required to fully understand and predict the location and behavior of the freshwater/seawater boundary. The lattice Boltzmann method (LBM) is a radically different numerical tool because it is not based on discretization of a series of differential equations. It has been used to simulate fluid flow in various disciplines. Recent advances in LBM permit simulation of large-scale density-dependent ground water flow and heat/solute transport. These simulations can be accomplished while retaining the advantages of 'regular' lattice Boltzmann methods, such as solute/heat transport at high Reynolds numbers (characteristic of flow inside a conduit). We show how this method can be applied to density-dependent flows, including a Henry-like problem.

Introduction

Seawater intrusion is a classic density-dependent problem in hydrogeology. It must be fully understood in order to be able to predict and prevent groundwater deterioration in coastal areas. Various software programs have been developed and are being used to model coupled fluid flow and solute transport for density-dependent applications: SUTRA, SEAWAT, and HST3D are popular examples. All of the current programs are either finite difference or finite element methods. Density-dependent flow problems are exceptionally challenging for conventional numerical methods due to inherent non-linearity; definitive solutions are often elusive and a completely different modeling approach may be advantageous. LBM represents such a numerical tool because it is not based on discretization of a series of differential equations. Instead, its foundation lies in the kinetic theory of gasses as proposed by Boltzmann. A key advantage of the lattice Boltzmann method is that it has the ability to solve the Navier-Stokes equations in larger conduits and pores. Hence it allows for eddy diffusion brought on by inertial components of flow at higher Reynolds numbers, which may occur in some coastal aquifers. Simulation of these phenomena is not possible with traditional Darcy's law-based groundwater models. Some geologists and engineers have been able to successfully apply LBM to fluid flow and contaminant transport problems (Ginzburg and d'Humieres, 2003; Anwar et al., 2008). There are only a handful of scientists attempting to apply LBM to density-dependent flows in general (e.g., Shan, 1997; Dixit and Babu, 2006); even fewer have considered seawater intrusion (Servan Camas, 2007).

Lattice Boltzmann Method Basics

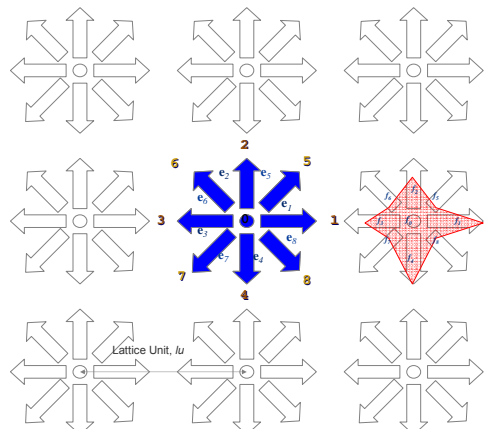


Figure 1: D2Q9 grid, e_{1-8} are unit velocity vectors, f_{1-8} are the directional density distribution function.

Method Continued

Most numerical modeling tools are based on discretization of differential equations. LBMs are based on discretization of the movement and interaction of hypothetical particles using a particle distribution function, (Sukop and Thorne, 2006). The moments of the particle distribution function provide the macroscopic fluid properties, e.g., density and velocity. In LBMs, time, space, and velocity directions are all discretized. Each lattice node, a discrete set of corresponding to a choice of directional densities, f_a , velocity directions e_a are stored. Directions in two dimensions are (0,0), ($\pm 1,0$), (0, ± 1), ($\pm 1, \pm 1$), and the lattice based on these nine directions is referred to as the D2Q9 grid (Figure 1).

At each time step, they are updated according to a collision step: $f_a^*(x, t) = f_a(x, t) - \frac{1}{\tau} (f_a^{eq}(x, t) - f_a(x, t))$ and a streaming step: $f_a(x + e_a, t + 1) = f_a^*(x, t)$ (Sukop and Thorne, 2006). The assumptions of unit time step $\Delta t = 1$ time step (ts) and unit node spacing $\Delta x = \Delta y = 1$ lattice unit (lu) have been incorporated. The key to the collision step is the equilibrium distribution function f_a^{eq} , which is defined per direction as: $f_a^{eq} = w_a \rho (1 + 3e_a \cdot u^{eq} + \frac{3}{2}(e_a \cdot u^{eq})^2 + \frac{3}{2}u^{eq} \cdot u^{eq})$ where $w_0 = 4/9$, $w_{1-4} = 1/9$, $w_{5-8} = 1/36$, $\rho = \sum f_a$ and $u^{eq} = \frac{1}{\rho} \sum f_a e_a$. The superscript on u is to distinguish it from the actual macroscopic velocity that is computed after the collision step.

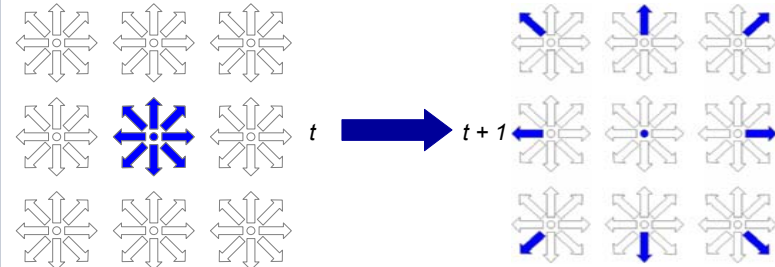


Figure 2: Depiction of streaming.

Additional details of LBMs including incorporation of buoyant forces can be found in Sukop and Thorne (2006).

Buoyancy

The first example is an initial value problem with half of the domain filled with saltwater.

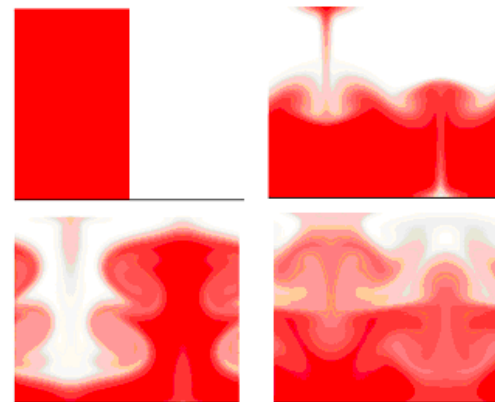


Figure 3: Initial condition upper left, approaching equilibrium bottom right. Solute (saltwater) shown in red, periodic domain except for bottom wall, $g = -0.0001 lu ts^{-1}$, $\delta\rho/\delta C = 0.4$, $\tau_0 = \tau_1 = 0.51 ts$.

The second example involves fluid flow through a narrow opening between two plates at different temperatures. We have compared our results with the analytical equation:

Buoyancy Continued

$$u = \frac{\Delta T g \beta \rho B^2}{12 \mu} \left[\left(\frac{y}{B} \right)^3 - \left(\frac{y}{B} \right) \right]$$

where: u = velocity, ΔT = temperature difference between plates, g = gravity, β = coefficient of volume expansion, ρ = fluid density, B = half width of opening, y = position in opening. Figure 3 is the result of the comparison.

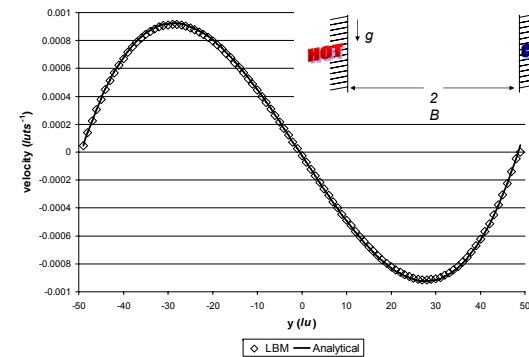


Figure 4: Comparison graph of analytical (solid line) and LBM (dashed line) solutions. Inset: Simulation domain, periodic top & bottom. $g = -0.000001 lu ts^{-1}$, $\delta\rho/\delta C = 0.5$, $\tau_0 = \tau_1 = 1 ts$, $\Delta C = 4^\circ$

The last example begins with a drop of salt water in the middle of the domain and continues with a constant salt source along the top boundary and a zero-concentration sink along the bottom. For this simulation the Rayleigh number is high enough to cause convection.

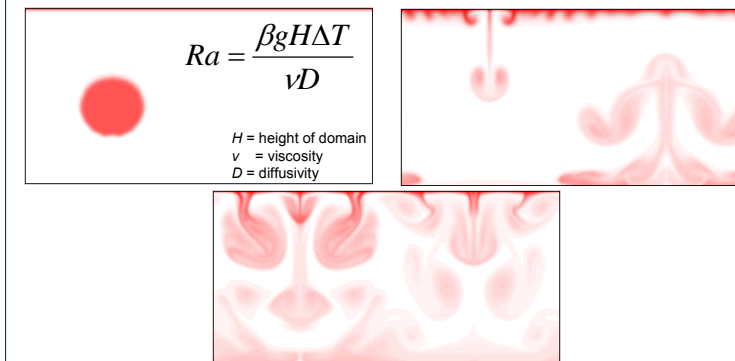


Figure 5: Convection caused by source at top and sink at bottom. Initial condition (saltwater on top, zero concentration boundary on bottom, periodic left & right, perturbation) top left to latest time frame bottom. $g = -0.0001 lu ts^{-1}$, $\delta\rho/\delta C = 0.2$, $\tau_0 = \tau_1 = .51 ts$, $\Delta C = .513 mu lu^{-2}$

Henry-Like Problem

The next step is to model the classic Henry problem. The Henry problem consists of a rectangular domain representing a confined aquifer with a freshwater inflow boundary condition on the right and a saltwater inflow boundary along the left. The top and bottom boundaries are closed (Henry, 1964). We were able to simulate a problem similar to the Henry problem. However we have yet to simulate the classic Henry problem with LBM using the physical parameters.

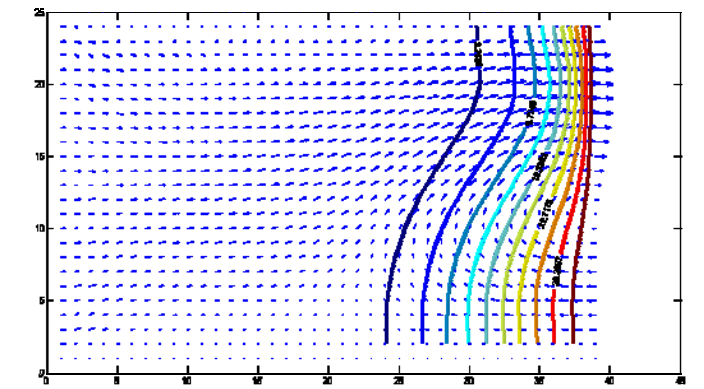


Figure 6: Henry-like problem showing the salt contours and fluid convection. Freshwater supply left, seawater supply right, closed top & bottom. $g = -0.00009165 lu ts^{-1}$, $\delta\rho/\delta C = 0.2$, $\tau_0 = 1 ts$, $\tau_1 = 0.503 ts$, $\Delta C = 0.2 mu lu^{-2}$

Conclusions

We used LBM to simulate three density-dependent flow problems. One available analytical solution was closely matched by LBM. Further work is needed to translate the physical Henry problem parameters into LBM inputs.

References

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