

If 0.1 mole of HOAc and 0.075 mole of NaOAc are combined in a solution with a total volume of 1 liter, what will the pH be at equilibrium?



Initial	0.1M	0	0.075M
Equilibrium	(0.1-x)	x	(0.075+x)

$$K_a = 1.8 \times 10^{-5} = \frac{[\text{A}^-][\text{H}^+]}{[\text{HA}]} = \frac{(0.075+x)x}{(0.1-x)}$$

we can solve the long way

Expand and solve as a quadratic (NO Assumptions)

$$1.8 \times 10^{-5} (0.1-x) = (0.075+x)x \rightarrow 1.8 \times 10^{-6} - 1.8 \times 10^{-5}x = 7.5 \times 10^{-2}x + x^2$$

rearranging

$$0 = -1.8 \times 10^{-6} + 1.8 \times 10^{-5}x + 7.5 \times 10^{-2}x + x^2$$

$$0 = -1.8 \times 10^{-6} + 7.5018 \times 10^{-2}x + x^2$$

$$x = \frac{-b \pm (b^2 - 4ac)^{1/2}}{2a}$$

$$= \frac{-7.5018 \times 10^{-2} \pm ((-7.5018 \times 10^{-2})^2 - 4(-1.8 \times 10^{-6}))^{1/2}}{2}$$

$$= \frac{-7.5018 \times 10^{-2} \pm ((5.6277 \times 10^{-3}) + 7.2 \times 10^{-6})^{1/2}}{2}$$

$$= \frac{-7.5018 \times 10^{-2} \pm (5.6349 \times 10^{-3})^{1/2}}{2}$$

$$= \frac{-7.5018 \times 10^{-2} \pm (5.6349 \times 10^{-3})^{1/2}}{2} = \frac{-7.5018 \times 10^{-2} \pm (7.50659 \times 10^{-2})}{2}$$

$$= \frac{4.797 \times 10^{-5}}{2} = 2.398 \times 10^{-5} \quad \text{OR} \quad \frac{-1.5008 \times 10^{-1}}{2} = -7.504 \times 10^{-2}$$

The only reasonable answer is $+2.398 \times 10^{-5}$

Note that -7.504×10^{-2} requires more reaction from AC than possible and gives a negative $[\text{H}^+]$ concentration (even when remembering that the initial $[\text{H}^+] = 10^{-7}$)

OR We can solve a much shorter way:

$$K_a = 1.8 \times 10^{-5} = \frac{[A^-][H^+]}{[HA]} = \frac{(0.075+x)x}{(0.1-x)}$$

IF $x \ll 0.075$ and 0.1 then the problem simplifies

$$K_a = 1.8 \times 10^{-5} = \frac{(0.075)x}{(0.1)} = 0.75x$$

$$\frac{1.8 \times 10^{-5}}{0.75} = x$$

$$x = 2.4 \times 10^{-5}$$

I like this solution best!