# PHY 5346 – PROBLEM SET 6

1) In a particular region of space the electric and magnetic fields are given by

$$\mathbf{E}(\mathbf{r},t) = -q/(4\pi\varepsilon_o r^2)\ \theta(vt-r)\ \mathbf{r\text{-}hat} \qquad \mathbf{B}(\mathbf{r},t) = 0$$

where $\theta(vt-r)$ is the step function defined to be 1 for positive argument and 0 for negative argument. Show that these fields satisfy Maxwell's equations and in the process, determine the volume charge and current densities, $\rho(\mathbf{r},t)$ and $\mathbf{J}(\mathbf{r},t)$, that give rise to these fields. Note that the charge density has two contributions, not one. Describe the physical situation represented by this charge density.

2) A coaxial cable consists of a very long cylindrical wire of radius a surrounded by a very long concentric cylindrical shell of inside radius b and outside radius c ($a<b<c$). The wire and shell carry currents of equal magnitude I in opposite directions along their lengths. Each current is uniformly distributed over the corresponding cross section.

a) Use Ampere's law to determine the magnetic field at arbitrary points in the three regions: (i) $\rho<a$, (ii) $a<\rho<b$, and (iii) $b<\rho<c$.

b) The wire and shell are both made of material with electrical conductivity $\sigma$. Find the electric fields within the wire and shell and then determine the Poynting vectors at arbitrary points within the wire and the shell.

c) Use the results of part b to calculate the power per unit axial length that flows *radially* into the wire and the shell.

d) Calculate the power dissipated per unit axial length of the system due to its electrical resistance and compare with the result of part c.

3) **Jackson, problem 6.11** – in part a, consider a small cylindrical pillbox intersecting the plane and use eq. (6.122). Note that the right hand side of this equation is just the $\alpha$ component of the force exerted by the EM radiation.

4) **Jackson, problem 7.1, *part b only*** -- First deduce the values of the amplitude and phase shift parameters from the Stokes parameters. Then write expressions for the electric fields in the two bases. To construct the requested figures, note that the lengths of the semi-major and semi-minor axes of the ellipse traced by the electric field vector are equal to the maximum and minimum of the electric field magnitude. Moreover, the tangent of the angle that the semi-major axis makes with the horizontal is just the ratio of $E_y$ to $E_x$ at the maximum field magnitude. Note that since the two components of **E** are not in phase, the square of the field amplitude is not necessarily equal to the square of the amplitudes of the components.

5) A plane electromagnetic wave of angular frequency $\omega$ traveling through a non-conducting and non-magnetic material with index of refraction $n_1$ encounters a flat boundary between the original material and a second non-conducting and non-magnetic material with index of refraction $n_2$. The electric field of the incident wave makes an angle $\phi_I$ with the scattering plane, so that that wave is a superposition of s and p-polarized waves, i.e.,

$$\mathbf{E_I} = E_I\,(\cos\phi_I\ \textbf{p-hat} + \sin\phi_I\ \textbf{s-hat}),$$

where **p-hat** and **s-hat** are unit vectors parallel and perpendicular to the scattering plane. The electric field of the transmitted wave makes an angle $\phi_T$ with the scattering plane, so that

$$\mathbf{E_T} = E_T\,(\cos\phi_T\ \textbf{p-hat} + \sin\phi_T\ \textbf{s-hat}).$$

a) Determine $\tan\phi_T$. Express your result in terms of $\tan\phi_I$ and the parameters $\alpha$ and $\beta$ defined by the expressions

$$\alpha = \cos\theta_T/\cos\theta_I \qquad \text{and} \qquad \beta = n_2/n_1$$

where $\theta_I$ and $\theta_T$ are the angles of incidence and refraction. Note that each component of the transmitted electric field depends only on the corresponding component of the incident electric field.

b) Find $E_T$ in terms of $E_I$, $\cos\phi_I$, $\sin\phi_I$, and the parameters $\alpha$ and $\beta$ defined above.

c) For the special case $\beta=\alpha$, determine the transmission coefficient T. Show that your result reduces to the correct results for pure p-polarization and for pure s-polarization. Note that because the transmission coefficient involves the *squares* of the field amplitudes, the transmission coefficient for mixed polarization is *not* equal to the sum of the pure s and p-polarization coefficients.

6) **Jackson, problem 7.4** --- keep in mind that the wave vector in a conductor is complex, so that application of the boundary conditions will result in reflected and transmitted waves that are not in phase with the incident wave.