PHY 5347 – PROBLEM SET 6

1) A particle of charge q and mass m moves with constant angular velocity ω in a circle of radius a in the xy-plane. Choose coordinates with the origin at the center of the circle and the angular velocity vector along the positive z-axis. Assume that the particle crosses the x-axis at time t=0 and that it moves relativistically.

a) Derive expressions for the position of the particle, its dimensionless velocity, and its acceleration as functions of time.

b) Starting from the Lienard-Wiechert expressions for the fields produced by a relativistic charged particle and using the results of part a, show that the electric field produced by the particle at the center of the circle has the form

$$\mathbf{E} = [A \cos(\omega t_r) + B \sin(\omega t_r)]\ \textbf{x-hat} + [A \sin(\omega t_r) - B \cos(\omega t_r)]\ \textbf{y-hat},$$

where t_r is the retarded time, and determine the coefficients A and B in terms of the given quantities.

c) Using the result of part b and the relation between **E** and **B**, show that at the center of the circle, the corresponding magnetic field is just that produced by a circular loop of current with $I = \omega q/2\pi$.

2) **Jackson, problem 14.4 –** note that for times that are not too long, it is usually an adequate approximation to replace **R-hat** with **r-hat**. In the second part, choose the position vector to lie along the x-axis at time t=0. Finally, note that both the angular distribution and the total power should be averaged over time so that the final results are not time-dependent.

3) **Jackson, problem 14-5 –** note in part a that for a central force, F = -dV/dr. The integral required in the second part can be performed by parts after shifting to the variable $u = zZe^2/r$.

4) **Jackson, problem 14.8 –** here you need to combine the relativistic expression for the acceleration of a charged particle in a uniform electric field with the relativistic Lienard equation for the radiated power. It is useful to transform the time integral into an integral over the trajectory assuming that for a large enough impact parameter b, the trajectory is approximately a straight line. The distance of the particle from the fixed charge at the point of closest approach is then just b. One can also assume in the integral that the velocity of the moving particle is approximately constant. Finally, note that if t=0 at the point of closest approach, then the limits of the original time integration are from $-\infty$ to $+\infty$.

5) Starting at time t=0, a particle with charge q and mass m moves along the z-axis like a damped harmonic oscillator. For times t<0, the velocity of the particle is zero; for t>0, its dimensionless velocity is given by

$$\beta(t) = \beta_o \cos(\omega_o t)\, e^{-\alpha t}$$

where α, β_o, and ω_o are all constants.

a) Derive an expression for the spectral distribution function $d^2W/d\omega d\Omega$ in the radiation zone in the nonrelativistic approximation ($t_r >> |x|/c$). Note that your result should be expressed in terms of the frequency ω and the angle θ between the z-axis and the position vector to the observation point.

b) Show that close to the resonance frequency ω_o and for weak damping ($\alpha << \omega_o$), the result of part a reduces to

$$c\, d^2W/d\omega d\Omega = (q\beta_o \sin\theta/4\pi)^2\, \omega^2/[(\omega-\omega_o)^2 + \alpha^2]$$