# Probability Questions

## Basic Probability

**Key Words:** Find the probability… “1” randomly selected…

**Formula(s):**

\[
P(A) = \frac{\text{# of times "A" happened}}{\text{Total # of observations}} \quad \text{OR} \quad P(A) = \frac{\text{# of ways "A" can happened}}{\text{Total # of possible outcomes}}
\]

## Addition Rule

**Key Words:** Find the probability… “1” randomly selected… “or”…

**Formula:**

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]

\[
P(A \cup B) = \frac{\text{# of subjects in group A}}{\text{Total}} + \frac{\text{# of subjects in group B}}{\text{Total}} - \frac{\text{# of subjects belonging to both A & B}}{\text{Total}}
\]

## Conditional Probability

**Key Words:** Find the probability… “1” randomly selected… “given that”…

**Formula:**

\[
P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{\text{Number of Subjects in Both Groups/Total}}{\text{Number of Subjects in Group B/Total}}
\]

*short cut: Only look at the numbers in the row or column relating to the “given that” condition, the total of that row or column is your denominator, then just get your numerator.

## Multiplication Rule

**Key Words:** Find the probability… (more than 1) randomly selected

- For example: 4 randomly selected ___: ___: ___: ___

**Formula:**

\[
P(A \cap B) = P(A) \cdot P(B) ; \text{ (Independent, with replacement)}
\]

**OR**

\[
P(A \cap B) = P(A) \cdot P(B \mid A) ; \text{ (Dependent, without replacement)}
\]

## Probability of At Least One

**Key Words:** Sometimes it will say, “Find the probability that at least one…”

**Formula:**

\[
P(\text{At Least 1}) = 1 - P(\text{None})
\]

## Binomial Probability

**Key Words:** example: Find the probability that 2 subjects out of 6 …

**Formula:**

\[
P(x) = \binom{n}{x} p^x (1 - p)^{n-x},
\]

\[p = \text{prob. of a success, } n = \text{number of trials, and } x = \text{number of successes}\]

*Note \[\binom{n}{x} = \frac{n!}{x!(n-x)!}\]
Non-Probability Questions

**Counting**

- **Key Words:** “In how many ways …”, or “How many …”
- **Formula(s):**

  Fundamental Counting Rule:
  \# of choices for 1st selection \times \# choices for 2nd selection \times \ldots \times \# choices for last selection

  Factorial Rule: \( n! \)

  Permutations: \( _nP_r = \frac{n!}{(n-r)!} \) (order matters)

  Combinations: \( _rC_n = \frac{n!}{r!(n-r)!} \) (order does not matter)

**Mean, Variance, and Standard Deviation for a Binomial Probability Distribution**

- **Key Words:** Find the mean (or Variance or Standard Deviation) …
- **Formulas:**

  (mean) \( \mu = n \cdot p \), (variance) \( \sigma^2 = n \cdot p \cdot (1 - p) \),
  
  (Standard deviation) \( \sigma = \sqrt{n \cdot p \cdot (1 - p)} \)

**Expected Value (mean of a probability distribution)**

- **Key Words:** Find the Expected Value
- **Formulas:** \( E(X) = \mu = \sum x \cdot P(x) \) (see table below)

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<tr>
<th>( x )</th>
<th>( P(x) )</th>
<th>( x \cdot P(x) )</th>
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<td>( \sum x \cdot P(x) )</td>
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**Variance and Standard Deviation of a Probability Distribution**

- **Key Words:** “Find the Variance for…” OR “Find the Standard Deviation for…”
- **Formulas:** \( \sigma^2 = \text{Var}(X) = \sum x^2 \cdot P(x) - \mu^2 \), St. dev \( \sigma = \sqrt{\sum x^2 \cdot P(x) - \mu^2} \)

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