Public and Private Production in a Two-Sector Economy

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Abstract

We develop a two-sector “non-scale” production model in which there are two types of firms, conventional profit-maximizing private firms, and what we call “public firms”, whose objective is to produce a specified quantity of government investment goods – determined by government policy – at minimum cost. Furthermore, the production functions of the two sectors need not in general coincide. Using this two-sector production set-up we characterize the equilibrium dynamics, and analyze a variety of fiscal disturbances. Because of the complexity of the model our analysis is carried out using simulations of a calibrated economy. We find that the effects of tax policies are remarkably robust with respect to the relative capital intensities of the two productive sectors. In contrast, the effects of government investment are much more sensitive to this aspect. One conclusion of this is that one can continue to employ the one-sector Ramsey model to analyze tax policy in the presence of public capital without seriously jeopardizing the analysis. But one has to be more careful in analyzing the impact of public investment itself.
1. Introduction

During recent years an increasing volume of research has focused on the role of government capital as a critical productive input. Without exception, this has been carried out using a one-sector model of production, in which public capital enters the aggregate production function together with private capital and labor; see e.g. Arrow and Kurz (1970), Baxter and King (1993), Futagami, Murata, and Shibata (1993), Fisher and Turnovsky (1998), Turnovsky (2003). This implicitly assumes that all production occurs in the private sector. The government then enters the private market and makes its purchases, in competition with private agents, using the resources it generates from tax revenues and from borrowing.

While this is a reasonable description and provides a good working model, in many instances governments may be better viewed as effectively conducting their own productive operations. Consider the following stylized description. A government passes legislation to invest in an airport facility, say. It sets out precise specifications of the project, which it then puts out for bids to private contractors who will hire labor and employ private capital to carry out the project. But instead of being free to hire productive factors to maximize his profit -- and thereby determine his output level (the size of the airport) endogenously -- the contractor is constrained (to win the contract) to construct the project, as specified by the government, most efficiently. Moreover, there is no reason to assume that the technology employed in the public project need be the same as that in the private sector. Indeed, one can plausibly argue that government investment projects that involve the nation’s infrastructure may well be more capital intensive (in private capital) than is the average technology employed in the private sector.

In this paper we develop a two sector model in which private output is produced in one sector by profit-maximizing firms. The production function in that sector depends upon the stocks of both private and public capital, as well as upon endogenously supplied labor. Public capital introduces a positive externality in production, so that the complete production function is one of overall increasing returns to scale in these three productive factors. Government capital is produced in a second sector by what we shall term “public firms”, hiring labor and private capital, with public capital also providing an externality. In hiring their productive factors these firms compete with the
private sector and therefore need to pay competitive factor returns. The objective of the public firm is to provide the new public capital, as determined by government policy, at minimum cost. Thus in contrast to the usual model in which the government uses its resources to finance its direct purchases of new investment, the government uses its resources to purchase the services of the productive factors that it employs. These resources are obtained by taxing capital income, labor income, and consumption, or by imposing non-distortionary lump-sum taxation.

To our knowledge, there are few other papers that disaggregate the production side of the economy in this way. Schmitz (2001) argues that in many economies, particularly Egypt, and to a lesser degree India and Turkey, the vast majority of investment goods are produced by the government, showing that this led to a major reduction in labor productivity. He also extends the model to allow for both public and private production of the economy’s investment good.¹

The model we employ is a “non-scale” growth model of the type introduced by Jones (1995a, 1995b), Segerstrom (1998), and others. This can be viewed as an extension of the neoclassical model, generalized to allow for non-constant returns to scale.² Like the neoclassical model, this has the property that the long-run equilibrium growth rate is determined by the interaction between the population growth rate and technological production parameters and is independent of government policy parameters. Also like the neoclassical growth model it has the advantage of being consistent with balanced growth under quite general production structures.

The fact that the long-run growth rate is independent of fiscal instruments does not mean that fiscal policy is irrelevant for long-run economic performance. On the contrary, fiscal policy has important effects on the levels of key economic variables, such as the per capita stock of capital and output. Moreover, the non-scale model typically yields slow asymptotic speeds of convergence, consistent with the empirical evidence of 2-3% per annum; see Eicher and Turnovsky (1999).³ This

¹ Arrow and Kurz (1970, Chapter 4) discuss a two sector model, referring to the private and public sector, but they do not disaggregate the production process in the way being proposed here.
² Interest in this model has grown as a result of increasing concerns about the endogenous growth model, which has centered around three objections: (i) Its frequent association with “scale effects”, meaning that the equilibrium growth rate is tied to the size of the economy, (ii) the weak empirical support for some of its policy implications, (iii) the knife-edge conditions on both preferences and technology that are required to generate ongoing growth. These objections, as well as some critiques of the non-scale model, are reviewed in more detail by Turnovsky (2000).
³ This benchmark was established in early work by Barro (1991), Mankiw, Romer, and Weil (1992). Subsequent studies suggest that the convergence rates are more variable and sensitive to time periods and the set of countries than originally suggested and a wider range of estimates have been obtained. For example, Islam (1995) estimates the rate of
implies that policy changes can affect growth rates for sustained periods of time so that their impacts during the transition from one equilibrium to another will eventually accumulate to potentially large influences on steady-state levels. Thus, although the economy grows at the same rate across steady states, the corresponding bases upon which the growth rates compound may be substantially different. These considerations suggest that attention should be directed to analyzing the impact of fiscal policy on the transitional dynamics and this has been the focus of a number of previous studies; see e.g. Auerbach and Kotlikoff (1987), Baxter and King (1993), King and Rebelo (1993), Devereux and Love (1994), Turnovsky and Chatterjee (2002), Turnovsky (2003).

Our objective is to analyze the effects of government investment and changes in the alternative tax rates in this two-sector productive economy. We set out the dynamic equilibrium of this economy and show how the stable adjustment is characterized by a two dimensional locus in terms of the two stationary variables, referred to as “scale-adjusted” per capita stocks of private and public capital. Because of its complexity it is necessary to analyze the equilibrium numerically, and to do this we calibrate the model to a benchmark economy and assess the numerical effects of various policy shocks relative to this benchmark. Different fiscal shocks have different impacts on government revenues, and to preserve comparability the shocks must be normalized in terms of their impacts on the government deficit. This can be done either with respect to their short-run effects or their intertemporal effects. We focus on the latter. Both the transitional adjustments and the eventual long-run equilibrium responses are considered. Particular attention is devoted to the welfare of the representative agent, both the time profile of instantaneous utility and intertemporal welfare, as represented by the present value of the accumulated benefits.

The major focus of the paper is to highlight the intertemporal dimensions of fiscal policy and the tradeoffs these involve for economic performance, especially growth and welfare, paying convergence to be 4.7% for non-oil countries and 9.7% for OECD economies. Evans (1997) obtains estimates of the convergence rate of around 6% per annum.

4 The fact that the transitional paths are two-dimensional introduces important flexibility to the dynamics. Convergence speeds vary over time and across variables, allowing different variables to follow different transitional paths; see Eicher and Turnovsky (1999). This characteristic is relevant to the empirical evidence of Bernard and Jones (1996), who find that while growth rates of output among OECD countries converge, the growth rates of manufacturing technologies exhibit markedly different time profiles. This contrasts with rigid dynamics imposed by the standard one-sector neoclassical growth model, or the familiar two-sector endogenous growth model.
particular attention to the role of the more general production structure. Overall, we find that the effects of pure tax policies are rather insensitive to the differences in the production structures between the private and public sectors. Thus, one general conclusion to be drawn is that the conventional one-sector model is a perfectly adequate vehicle for analyzing the aggregate effects of pure tax policies.

In contrast, the effects of government investment, irrespective of how they are financed, are more sensitive to the comparative production structures of the two sectors. The strongest effect is on the allocation of the private productive factors across the two sectors, but in general normalized government investment will be significantly more expansionary as the capital intensity of the public sector increases. These translate to somewhat larger aggregate welfare effects.

Calibrating the model to approximate the US, we find that public investment involves a dramatic intertemporal tradeoff. Consumption and welfare decline in the short run, as resources are attracted toward government investment, but improve sharply over time as the investment bears fruit and productivity is enhanced. For example, doubling the rate of public investment from 4% to 8% involves short-run welfare losses of between 2.5% - 3.2% but intertemporal welfare gains of between 7.7% - 10.9%, depending upon the relative sectoral capital intensities.

A similar tradeoff exists for the tax on capital income. The normalized reduction in this tax leads to immediate welfare losses of around 2.9%, as resources are diverted away from consumption and leisure toward private capital accumulation and labor. But in the long run, the benefits of more capital generate intertemporal welfare gains of just over 2%, with some minor variation across the different technologies. The equivalent reductions in the tax on labor income and consumption have milder effects over time. One unexpected result is that the intertemporal benefits of the normalized reduction in the tax on labor income exceed those of the tax on capital income. This is because the normalization is taken with respect to the government’s intertemporal deficit and the tax on capital has substantial growth effects. If instead, we normalize the tax cuts so as to have the same impact on the current deficit, this requires a relatively greater cut in the capital income tax so that its intertemporal effects dominate those of the equivalent labor income tax.

The remainder of the paper proceeds as follows. Section 2 sets out the structure of the
model, while its equilibrium dynamics are characterized in Section 3. Section 4 outlines the steady-state equilibrium in the centrally planned economy, which serves as a natural benchmark. Section 5 calibrates the model and considers the numerical effects of a number of policy changes. Section 6 concludes, while technical details of the solution are provided in the Appendix.

2. The Model

We begin by outlining the structure of the economy.

2.1 Private Firms

There are \( M \) identical private firms indexed by \( j \). We shall assume that the representative private firm produces private output \( Y_j \) in accordance with the Cobb-Douglas production function

\[
Y_j = \alpha_Y (K_j)^{\sigma} [L_j]^{1-\sigma} K_G^\sigma, \quad 0 < b < 1, \sigma > 0 \quad (1a)
\]

where \( K_j \) denotes the capital employed by the firm, \( L_j \) denotes the labor employed by the firm, and \( K_G \) denotes the aggregate stock of public capital. This production function is constant returns to scale in the two private factors, while public capital, provides a positive externality, and behaves as a pure public good, not subject to congestion.\(^5\) Summing over the \( M \) firms yields the aggregate production function describing output in the private sector

\[
Y = \alpha_Y (MK_j)^{\sigma} [ML_j]^{1-\sigma} K_G^\sigma \quad (1b)
\]

where \( Y = MY_j \) denotes aggregate private output.

We assume that the representative private firm maximizes profit, so that the equilibrium rates of return in the private sector satisfy

\[
r = b \frac{\partial Y_j}{\partial K_j} = b \frac{Y_j}{K_j} = b \frac{Y}{MK_j}; \quad (2a)
\]

\(^5\) This assumption is a polar one, since almost all public services are subject to some congestion. Fisher and Turnovsky (1998) develop a simple one-sector non-scale model in which productive public capital is subject to congestion. It would be straightforward to apply their parameterization of congestion to the present two-sector economy and is a direction in which the present model could be fruitfully extended.
\[
\frac{w}{L_j} = \frac{\partial Y_j}{\partial L_j} = \frac{Y_j}{L_j} = \frac{Y}{ML_j} \tag{2b}
\]

where \( r \) = gross return to capital, \( w \) = (before tax) wage rate.

### 2.2 Public Firm

We shall assume that there is one public firm, indexed by \( P \), that hires labor and private capital, paying the market wage and return to capital. The output of this public firm is described by the production function

\[
J = \alpha_j [K_p]^{\theta} [L_p]^{1-\theta} K_G^n \tag{3}
\]

which like (1b) is of the Cobb-Douglas form. However, the different exponents allow for potentially different factor productivities, and therefore equilibrium factor intensities, in the public sector from those in the private sector. Intuitively, we are assuming that the production of the diverse range of outputs produced in the private sector (consumption goods as well as private capital) involves a different technology than does the production of infrastructure produced in the public sector. In this respect, we shall permit the production function of the public sector to be either more or less capital intensive than is the representative production function in the private sector.

The public firm’s optimization problem is to produce a specified output of public capital, \( J \), chosen by the public policy maker, in accordance with the production function (3), at minimum cost. Formally, this is expressed by

\[
\min \ rK_p + wL_p + \lambda \left[ J - \alpha_j [K_p]^{\theta} [L_p]^{1-\theta} K_G^n \right] \tag{4}
\]

where \( \lambda \) measures the marginal cost of a unit of public investment. Performing the optimization yields the cost minimizing conditions

\[
r = \lambda \frac{\partial J}{\partial K_p} = \lambda \frac{J}{K_p} \tag{5a}
\]

\[
w = \lambda \frac{\partial J}{\partial L_p} = \lambda (1 - \theta) \frac{J}{L_p} \tag{5b}
\]
Note that the extent $\lambda$ deviates from unity reflects the extent to which the public firm deviates from profit maximization in its productive activities. Given $J$, public capital accumulates at the rate

$$\dot{K}_G = J - \delta_G K_G$$

(6)

where $\delta_G$ denotes the rate of depreciation of public capital.

2.3 Representative Consumer

The representative consumer is endowed with one unit of time that can be allocated to leisure, $l$, leaving $(1-l)$ available for work, $?$, of which is to employment in the private sector and $(1-\theta)$ to employment in the public sector. There are $N$ agents, all identical, and thus we can drop the subscript on labor. Agent $i$ also owns $K_i$ units of private capital, which he rents out at the rental rate, $r$, a fraction $\phi$ being allocated to the private sector, and $1-\phi$ to the public sector.

The representative agent’s welfare is specified by the intertemporal isoelastic utility function:

$$\max \int_0^\infty \frac{1}{\gamma} C_i^\gamma I_i^{1-\gamma} e^{-\beta t} dt; \quad \rho > 0; \ -\infty < \gamma \leq 1; \ 1 > \gamma (1+\rho)$$

(7a)

where $C_i$ denotes the agent’s consumption, at time $t$, $1/(1-\gamma)$ equals the intertemporal elasticity of substitution, and $\gamma$ measures the impact of leisure on the agent’s welfare. The remaining constraints on the coefficients are imposed to ensure that the utility function is concave in the quantities $C_i$ and $l$.

The agent’s objective is to maximize (7a) subject to his accumulation equation,

$$\dot{K}_i = [(1-\tau_k)R - n - \delta_K] K_i + (1-\tau_w)(1-l)W - (1+\tau_c) C_i - T_i$$

(7b)

where $\tau_k$ = tax on capital income, $\tau_w$ = tax on wage income, $\tau_c$ = consumption tax, $T_i = T/N$ = agent’s share of lump sum taxes (transfers). Equation (7b) assumes that private capital depreciates at the rate $\delta_K$, so that with the growing population, the net after tax private return to capital is $(1-\tau_k)R - n - \delta_K$.

Performing the optimization yields:

$$C_i^{\gamma-1} I_i^{\rho \gamma} = \mu_i (1+\tau_c)$$

(8a)
\[ \rho C_i^t l_i^{\rho(t-1)} = \mu_i (1 - \tau_i) w \]  
\[ (1 - \tau_K) r - n - \delta_K = \beta - \frac{\bar{\mu}}{\mu} \]

Equation (8a) equates the marginal utility of consumption to the individual’s tax adjusted shadow value of wealth, \( \mu_i \), while (8b) equates the marginal utility of leisure to its opportunity cost, the after tax real wage, valued at the shadow value of wealth. The third equation is the standard Keynes-Ramsey consumption rule, equating the rate of return on consumption to the after-tax rate of return on capital. Finally, in order to ensure that the agent’s intertemporal budget constraint is met, the following transversality condition must be imposed:

\[ \lim_{t \to \infty} \mu_i K_t e^{-\beta t} = 0 \]  

2.4 Aggregate Relationships

Aggregating over the firms and households, full employment in the labor market and private capital market is described by

\[ N(1 - l) = ML_j; \quad N(1 - \theta)(1 - l) = L_p \text{ and thus } N(1 - l) = ML_j + L_p \]  
\[ \phi NK_i = \phi K = MK_j; \quad (1 - \phi)NK_i = (1 - \phi)K = K_p \text{ and thus } K = NK_i = MK_j + K_p \]

Equation (9a) asserts that the total supply of labor must be allocated either to one of the private firms or to the public firm, while (9b) describes an analogous allocation condition for private capital. Using these relationships, the aggregate production function for the private sector and the public production function may be expressed as:

\[ Y = \alpha_f (\phi K)^{\beta} [\theta N (1 - l)]^{1-b} K_G^\theta \]  
\[ J = \alpha_j [(1 - \phi)K]^d [N(1 - \theta)(1 - l)]^{1-d} K_G^d \]

The government runs a balanced budget in accordance with

\[ \tau_K rK + \tau_w N(1 - l)w + \tau_C C + T = rK_p + wL_p = r(1 - \phi)K + wN(1 - \theta)(1 - l) \]
That is, the government finances its production costs, given by the right hand side of (11), from the aggregate tax revenues earned on capital income, labor income, consumption, or lump-sum taxes, where \( C \equiv NC \), denotes aggregate consumption.\(^6\)

We assume that the government ties its current rate of investment to aggregate private output in accordance with

\[
J = gY
\]  

(12)

where \( g \) is a chosen policy parameter. We should emphasize that (12) represents a specification of government policy, not any kind of resource constraint on the government, which is specified by (11). It represents the notion that the government wishes to maintain a balance between the rate of public investment and the size of the economy.

Using (2a) and (2b), together with the full employment conditions (9a) and (9b), we may express the government’s flow budget constraint as

\[
T = \left[ \frac{b}{\phi} (1-\phi) + \frac{1-b}{\theta} (1-\theta) - \tau_c \frac{b}{\phi} - \tau_w \frac{1-b}{\theta} - \tau_c \frac{C}{Y} \right] Y
\]  

(13a)

so that \( T \) represents the amount of lump-sum taxation (or transfers) necessary to finance the primary deficit and is therefore a measure of current fiscal imbalance. Defining

\[
V = \int_0^\infty \int_T(t) e^{\nu} \left[ \frac{b}{\phi} (1-\phi) + \frac{1-b}{\theta} (1-\theta) - \tau_c \frac{b}{\phi} - \tau_w \frac{1-b}{\theta} - \tau_c \frac{C}{Y} \right] e^{-\nu t} dt \]  

(13b)

where \( s(1-\tau_k) = r(1-\tau_k) - \delta_k \) is the implied equilibrium of interest, \( V \) measures the present discounted value of the lump-sum taxes or transfers necessary to balance the government budget over time, and thus is a measure of the intertemporal imbalance of the government’s budget.

Aggregating (1c) over the \( N \) agents, and recalling (4) and (6), implies market clearing in the private goods market

\[
\dot{K} = Y - C - \delta_k K
\]

\(^6\) All private agents in the economy, whether they be employed by the private or public firms, pay taxes. The model satisfies the conditions for Ricardian equivalence, so the abstraction of government bonds involves no loss of generality.
Specifically, this equation asserts that private output can be costlessly transformed into productive private capital or consumption, and the growth rate of private capital may be written as

\[
\frac{\dot{K}}{K} = \left(1 - \frac{C}{Y}\right)\frac{Y}{K} - \delta_k
\]  

(14a)

Likewise, substituting (7b) into (5), the growth rate of public capital may be written as:

\[
\frac{\dot{K}_G}{K_G} = g - \frac{Y}{K_G} \delta_G.
\]  

(14b)

Note also the fact that the government is cost-minimizing implies

\[
rK_p + wL_p = \lambda dJ + \lambda(1 - d)J = \lambda J = \lambda gY
\]  

(15)

Thus, the cost of producing the public capital exceeds the value of production if and only if \( \lambda > 1 \). In this case the government is an inefficient producer and is in effect being subsidized by the private sector. In the case that the public firm is maximizing profit, \( \lambda = 1 \) and its expenditures reduce to the conventional expression \( gY \).

3. Equilibrium Dynamics

Our objective is to analyze the dynamics of the aggregate economy about a stationary growth path. To derive the balanced growth we first impose the condition that along a balanced growth path the output-private capital is assumed to be constant, so that private output and private grow at the same constant rate, while the allocation of labor remains constant. We then note further that the policy specification (12) imposes the further condition that public capital must also grow at the same constant rate. Taking percentage changes of the aggregate production functions, (10a) and (10b), the long run (common) equilibrium growth of output, private and public capital is:

\[
\psi = \frac{1 - b}{1 - b - \sigma} n
\]  

(16a)

where the productive elasticities in the two sectors must satisfy the restriction:
Given the arbitrary returns to scale in the two sectors, condition (16b) is necessary in order for the growth rates in the two sectors to be consistent with maintaining a long-run constant ratio of public capital to output. Using (16b) we can write the two production functions in the form

\[ Y = \alpha_y (\phi K)^{\theta} [(1 - l) K_G^\phi]^{1-b} \]  

\[ J = \alpha_j [(1 - \phi) K]^d [N(1 - \theta)(1 - l) K_G^\eta]^{1-d} \]

Expressed in this way we can interpret the restriction (16b) as asserting that the externality associated with public capital operates as an increase in the efficiency of the labor input. This is a weak condition that we shall henceforth impose.

### 3.1 Transitional Dynamics

To analyze the transitional dynamics of the economy about its balanced growth path, we express the system in terms of the stationary variables: (i) the fraction of time devoted to leisure, \( l \) and (ii) the scale-adjusted per capita quantities:

\[ k = \frac{K}{N^{(1-b)/(1-b-\sigma)}} \quad k_G = \frac{K_G}{N^{(1-b)/(1-b-\sigma)}} \quad y = \frac{Y}{N^{(1-b)/(1-b-\sigma)}} \quad j = \frac{J}{N^{(1-b)/(1-b-\sigma)}} \quad c = \frac{C}{N^{(1-b)/(1-b-\sigma)}} \]

Using this notation, the scale-adjusted private and respectively public output can be written as:

\[ y = \alpha_y (\phi k)^{\theta} [(1 - l)]^{1-b} k_G^\sigma \]  

\[ j = \alpha_j [(1 - \phi) k]^d [(1 - \theta)(1 - l)]^{1-d} k_G^\eta \]

The optimality conditions then enable the dynamics to be expressed in terms of these scale-adjusted variables as follows. First by using the no arbitrage condition which implies the equality of returns to capital and labor in the private and public sector (2a), (2b), (5a) and (5b) and the labor and private capital allocations (6a) and (6b), together with (11) we can derive that:

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7 Under constant returns to scale, these scale adjusted per capita quantities reduce to the usual per capita quantities.
Then, substituting (17a) and (17b) into the policy relationship (12) yields:

\[
\frac{b}{1-b} \frac{\theta}{1-\theta} = \frac{d}{1-d} \frac{\phi}{1-\phi}
\]

(18a)

Equations (18a) and (18b) provide two equations that determine the instantaneous allocation of labor and private capital across the private and public production sectors. These solutions may be written as:

\[
\theta = \theta(k, k_G, l; g)
\]

(19a)

\[
\phi = \phi(k, k_G, l; g)
\]

(19b)

These conditions have straightforward intuitive interpretations. An increase in the rate of government investment, \( g \), given the current stock of productive factors requires that more labor and private capital be allocated to the public sector. The conditions also assert that an increase in the supply of any of the three factors of production raises the productivity of both sectors in proportion to an amount that depends upon the respective productive elasticity. To compensate for the relative change in productivity, and thus preserve the required balance between the rate of public investment and private output as specified by the policy rule (12), resources must move toward the sector in which the input has the smaller production elasticity (is less productive). It is evident from (18a) that over time the two private factors move together across sectors (\( \text{sgn}(\theta) = \text{sgn}(\phi) \)) enabling us to restrict our attention to say, labor.

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Note that the restriction (16b) implies \( \text{sgn}(\sigma - \eta) = \text{sgn}(d - b) \). Explicit expressions for most of the partial derivatives are provided in the Appendix.
The equilibrium dynamics of the economy can be expressed in terms of the stationary variables by the following system

\[ \frac{\dot{k}}{k} = \frac{\gamma}{k} - \frac{c}{k} \delta_k - \psi \]  
(20a)

\[ \frac{\dot{k}_G}{k_G} = g \frac{\gamma}{k_G} - \delta_G - \psi \]  
(20b)

\[ \rho \gamma \frac{\dot{y}}{l} + (\gamma - 1) \left( \frac{\dot{c}}{c} - n + \psi \right) = \beta - (1 - \tau_k) b \frac{y}{\phi k} + n + \delta_k \]  
(20c)

together with

\[ \frac{c}{y} = \left( \frac{1 - \tau_w}{1 + \tau_c} \right) \left( \frac{l}{1 - l} \right) \left( \frac{1 - b}{\rho} \right) \]  
(20d)

the efficient factor allocation conditions, \( \phi(.) \) and \( \theta(.) \) described by (19a), and (19b), and the production function for private output, (17a).

Equations (20a) and (20b) are the accumulation equation for scale-adjusted private and public capital, respectively. They are obtained by taking the time derivatives of \( k \) and \( k_G \), combining (14a) with (16a), and (14b) with (16a). Equation (20c) is obtained by taking the time derivative of the optimality condition (8a) and combining it with (8c), noting that the equilibrium rate of return on capital \( r = by/\phi k \). Equation (20d) is obtained by dividing (8b) by (8a), and noting that the equilibrium real wage is \( w = (1-b)Y/N\theta(1-l) \). It implies that the consumption to output ratio, given leisure, is increasing with a decrease in the tax on labor income and tax on consumption.

In the Appendix we show how this system can be reduced to an autonomous set of differential equations in \( k, k_G \), and \( l \), which then form the basis for our subsequent numerical work. This third order system has two sluggish variables, \( k \) and \( k_G \), and one jump variable, \( l \). To yield a well behaved dynamic behavior we require that the eigenvalues of this system consist of two stable and one unstable root, a property that we found to prevail over all of our wide-ranging simulations.

### 3.2 Steady State
The steady state to this economy, denoted by “∼”, is obtained by setting \( \dot{k} = \dot{k}_c = \dot{l} = 0 \) in (20a) – (20c) and can be summarized by:

\[
\begin{align*}
1 - \frac{\bar{c}}{\bar{y}} \frac{\bar{y}}{k} & = (\delta_k + \psi) \quad (21a) \\
\frac{\bar{y}}{k_G} & = (\delta_c + \psi) \quad (21b) \\
(1 - \tau_K) \frac{\bar{y}}{\phi_k} & = \beta + \delta_K - (\gamma - 1)\psi + n\gamma \quad (21c)
\end{align*}
\]

together with the production functions, (17a) and (17b), the sectoral allocations (18a, 18b), and the optimal consumption-leisure condition (20d). Equation (21a) describes the growth of private output, given consumption, necessary to provide the private capital to equip the growing labor force and replace depreciation. Equation (21b) is an analogous condition for public capital while equation (21c) equates the long-run net return to private capital to the rate of return on consumption. These 8 equations determine the steady-state equilibrium values for \( \bar{k}, \bar{k}_G, \bar{y}, \bar{j}, \bar{l}, \bar{c}, \bar{\theta}, \) and \( \bar{\phi} \) in terms of the fiscal policy instruments and other structural parameters.

From the steady-state equilibrium conditions we can show the following qualitative long-run responses to the fiscal shocks. By the nature of the non-scale model, the long-run growth rate is unaffected. The responses of the scale-adjusted per capita quantities are important because they describe the effects on the base levels on which the constant steady-state growth rate compounds. They represent the accumulated effects on the growth rates during the transition, and as our numerical results shall highlight, they can be substantial. These are all based on the assumption that these are financed by lump-sum taxes in accordance with (13).

**Increase in Government Investment**

\[
\frac{\partial \bar{l}}{\partial g} < 0 \quad (22a)
\]
An increase in government investment reduces the allocation of time devoted to leisure, increasing employment, with a greater fraction of both labor and private capital being devoted to the public sector. The relative reduction of labor in the private sector exceeds that of capital if and only if \( b > d \), i.e. the private sector is relatively more capital intensive. Public investment stimulates both public and private capital, having a greater effect on the former, thereby raising the long-run ratio of public to private capital. Similarly, because of the reallocation of employment it has a less stimulating effect on private output, raising the long-run output-capital ratio.

**Increase in Capital Income Tax**

\[
\frac{\partial \tilde{k}}{\partial \tau_k} < 0, \quad \frac{\partial \tilde{y}}{\partial \tau_k} < 0
\]

An increase in the capital income tax increases the time devoted to leisure, reducing the overall time allocated to work. The higher tax on capital income, by impinging directly on private investment, reduces the steady-state stock of private capital more than it does public capital or private output, both of which decline proportionately. The effect on the sectoral factor allocation depends critically upon the sectoral factor intensity. If \( b = d \) so that both sectors have the same
production function, sectoral private factor allocations remain unchanged. If \( b > d \) \((b < d)\) the decline in activity will be borne less (more) heavily by the private sector, with the capital in that sector declining in greater proportion.

**Increase in Labor Income Tax or Consumption Tax**

\[
\frac{\partial \tilde{t}}{\partial \tau_i} > 0, \quad i = w, c \tag{24a}
\]

\[
\frac{\partial \tilde{k}}{\partial \tau_i} = \frac{\partial \tilde{k}_s}{\partial \tau_i} = \frac{\partial \tilde{y}}{\partial \tau_i} < 0, \quad i = w, c \tag{24b}
\]

\[
\frac{\partial \theta}{\partial \tau_i} = \frac{\partial \phi}{\partial \tau_i} = 0, \quad i = w, c \tag{24c}
\]

Both these taxes decrease labor supply, but since neither impinges directly on either form of capital accumulation they lead to proportionate long-run changes in the two capital stocks and output, leaving the sectoral allocations unchanged as well.

**4. Socially Optimal Government Investment**

Because of the declining marginal physical product of public capital, benefits to increasing government expenditure are limited. This is because there is socially optimal level of public capital. To help our understanding of some of the numerical simulations we shall conduct below, it is useful to set out the steady-state equilibrium for the centrally planned economy in which the planner controls resources directly. The optimality conditions for such an economy comprise the production functions (17a), (17b), the efficient allocation condition (18a), the equilibrium growth conditions (21a), (21b), together with:

\[
\rho \frac{c}{y} = \left( \frac{l}{1-l} \right) \left( (1-b) + q(1-d) \frac{j}{y} \right) \tag{25a}
\]

\[
\frac{1-b}{\theta} = q \frac{1-d}{1-\theta} \frac{j}{y} \tag{25b}
\]
\[
\frac{b}{k} \frac{y}{k} + qd \frac{j}{k} - \delta_k = \frac{\sigma}{q} \frac{y}{k_g} + \eta \frac{j}{k_g} - \delta_g
\]  
(25c)

\[
\frac{b}{k} \frac{y}{k} + qd \frac{j}{k} - \delta_k = \beta + \psi (1 - \gamma) + \gamma n
\]  
(25d)

where \( q \) denotes the shadow price of public capital in terms of private capital. These 9 equations determine the steady-state solutions for \( \hat{c}, \hat{y}, \hat{k}, \hat{k}_g, \hat{j}, \hat{\theta}, \hat{\phi}, \) and \( \hat{q} \). Note that the implied optimal investment share, \( \hat{g} \) is obtained from the ratio \( \hat{g} = \hat{j}/\hat{y} \). In contrast to the decentralized economy, the after-tax private rates of return are replaced by the corresponding social rate of return. Thus in equation (25a) the after-tax real wage, which appears in (20d) is replaced by the social return to a unit of labor. Similarly, (25b) determines the socially optimal ratio of public capital to private output. Finally, equations (25c) and (25d) equate the long-run social rates of return of investment in the two types of capital to the rate of return on consumption.

5. **Numerical Analysis of Transitional Paths**

The complexity of the model limits study of its analytical properties. Further insights into the effects of fiscal policy can be obtained by carrying out numerical analysis of the model. We begin by characterizing a benchmark economy, calibrating the model using parameters representative of the US economy reported in Table 1a.

The elasticity on capital, \( b = 0.35 \), implies that 35% of final output accrues to private capital and the rest to labor, which grows at an annual rate of 1.5%. The elasticity \( s = 0.2 \) on public capital implies that public capital generates a significant externality in production. This parameter lies within the range of the consensus estimates (see Gramlich 1994). As a benchmark, we begin with the case where the private and the public production functions are identical, with the elasticity on private capital in production function for public capital \( d = 0.35 \). However, since one of the issues we wish to address concerns the relative sectoral capital intensities of the two production sectors, we also consider \( d = 0.2, d = 0.5 \), respectively. Given \( b, d, \sigma \), the elasticity of public capital externality in the production of public capital, \( \eta \) is determined by (16b).

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9 This is considerably less than the estimate of 0.39 suggested in Aschauer’s early work.
The value of $\gamma =-1.5$ implies an intertemporal elasticity of substitution in consumption of 0.4, consistent with the estimate by Ogaki and Reinhardt (1998), and well within the standard range of estimates. The elasticity of leisure $\rho =1.75$, which accords with the standard value in the business cycle literature, yields an equilibrium fraction of time devoted to leisure of about 0.75 which is consistent with the empirical evidence.

The benchmark tax on wage income, $t_w=0.28$ reflects the average marginal personal income tax rate in the US. Given the complex nature of capital income taxes, part of which may be taxed at a lower rate than wages and part of which at a higher rate, we have chosen the common rate, $t_k=0.28$ as the benchmark. The benchmark consumption tax is zero. On the expenditure side we focus on government investment. The choice $g=0.04$ is based on US evidence, suggesting that approximately 4% of output is allocated to government investment. We do not introduce government consumption explicitly, thereby implicitly assuming that it is a perfect substitute for private consumption and produced in the private sector. The annual depreciation rates $d_G=0.035$ and $d_K=0.05$ approximate the average depreciation rates for public and private capital for the US during recent years.\(^\text{10}\)

These parameters lead to the benchmark equilibria reported in Table 1b. Focusing on $d=0.35$, the implied fraction of time devoted to leisure, $l=0.75$, accords with the empirical evidence, as noted. Also, the total consumption-output ratio = 0.85, the private output-private capital ratio = 0.46, and the ratio of public to private capital = 0.33, are all plausible and generally consistent with recent empirical evidence for the United States.\(^\text{11}\) The benchmark equilibrium also implies that over 96% of labor and private capital is employed in the final output sector, which is also consistent with the available data.\(^\text{12}\) The table also reports the two stable eigenvalues, which for

\(^{10}\) Data on private and public stocks of capital have been obtained from the Table “Real Net Stock of Fixed Reproducible Tangible Wealth for the US” in the \textit{Survey of Current Business}, May 1997. Data on gross public and private investment have been obtained from the 2002 \textit{US Economic Report of the President} (see Tables B2 and B21). Using these data we have computed the average annual depreciation rates on private and public capital over the period 1990-1995 to be around 4.7% and 3.4%, respectively. A depreciation rate of 5% is a common benchmark in the real business cycle literature; see e.g. Cooley (1995). We also experimented with higher depreciation rates (double), with little effect on our conclusions, other than to yield a somewhat higher rate of convergence, but still consistent with the empirical evidence.

\(^{11}\) Specifically, the private consumption-output ratio =0.67 plus the public consumption-output ratio of 0.16 yields an overall consumption-output ratio of 0.83, the output-private capital ratio = 0.48, and the ratio of public to private capital = 0.30. These estimates have been computed from the sources cited in footnote 10.

\(^{12}\) In recent years the fraction of labor employed in the government sector is around 16% With about 20% of government expenditure being on public investment, this would suggest that around 3.2% of labor is employed in producing
the benchmark economy are approximately -0.031 and -0.102. These imply that the per capita output and capital converge at the asymptotic rate of around 2.3%, consistent with much empirical evidence. Finally, the steady-state growth rate, which by the non-scale nature of the economy is independent of policy, equals 2.17%. Table 1b also reports the equilibria when the public investment sector is labor intensive relative to the private sector \((d = 0.2)\) and relatively capital intensive \((d = 0.5)\). These fairly large changes in relative sectoral intensities have relatively small effects on the equilibrium. The most interesting aspect is that if the public sector is relatively labor intensive then in the long run it will employ 5.33% of the labor force but only 2.5% of the private capital stock, rather than 3.85% of both. If it is relatively capital intensive then the corresponding allocations will be 2.43% and 4.41% respectively.

Table 1c summarizes the first-best optimal allocation of resources that would be chosen by a central planner. For the chosen production and preference parameters, we see that the long-run equilibrium in the decentralized economy involves too much leisure and two little public investment, relative to the optimal fraction of government of government expenditure. If both sectors have the same technology then 89.4% of both factors should be employed in the private sector, with public investment being approximately 12% of private output, implying a ratio of public to private capital of around 65%. If the public sector is relatively capital intensive then only around 8.2% of the labor force should be employed in that sector, while 14.2% of the capital stock should be so allocated. The long-run rate of public investment should be significantly higher at 15.5%, with the long-run ratio of public to private capital increased to 81.5%.

Tables 2-4 describe the consequences of various policy changes on the economy. One of our primary concerns is the impact of the alternative policies on economic welfare. This is measured by the optimized utility of the representative agent

\[
W = \int_0^\infty Z(t)e^{-\beta t} dt = \int_0^\infty \frac{1}{\gamma} (C / N)^\gamma l^{pi} e^{-\beta t} dt
\]

(26)

government capital. Data enabling us to determine the allocation of private capital between the two sectors do not appear to be available.

\(^{13}\) This estimate is based on the measure of convergence proposed by Eicher and Turnovsky (1999).
where $Z(t)$ denotes instantaneous utility (short-run welfare) and $C/N$ and $I$ are evaluated along the equilibrium path. The welfare gains reported are calculated as the percentage change in the flow of income necessary to maintain the level of welfare unchanged in response to the policy shock. The short-run impact is measured by the change in $Z(t)$, while the long-run impact is summarized by the change in the overall intertemporal index, $W$.

The other key measure of economic performance, intertemporal fiscal balance, has been defined previously in equation (13b). Evaluating this measure along the balanced growth path, we find that in the benchmark case, $V=-0.947$, which implies a substantial intertemporal fiscal surplus. When the public production function is more labor intensive the fiscal surplus is somewhat smaller ($V=-0.926$), whereas when it is more capital intensive the fiscal surplus is higher ($V=-0.974$) due to the fact that the leisure is lower and the level of capital and consumption are higher, which provide higher tax revenues. In interpreting these figures, two things should be borne in mind. First, government consumption expenditure, which is around 16% of current income, is lumped in with private consumption, while second, our measure of $T$, and therefore $V$, is net of government transfers, which historically for the United States have been around 12% of current income. Once we account for these, the surplus is transformed to an intertemporal deficit. In any event, the results are insensitive to the arbitrarily chosen level of $V$; what is relevant is the change in $V$.

5.1 Uncompensated Normalized Fiscal Changes

Table 2 describes various basic policy changes from the benchmark economy, with the corresponding dynamic transition paths being illustrated in Figs. 1 and 2. These are uncompensated, meaning that they lead to changes in the government’s fiscal deficit. To make valid comparisons between alternative fiscal policies the shocks must be standardized for revenue changes and this can

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14 Since both $V$ and $Y$ are proportional to $\alpha$, set arbitrarily to 1, we interpret $V$ as being measured in units of income.

15 With the base income tax rate in our model being 28%, while government investment is 4% of income, this obviously implies a large government surplus. Adding in government consumption expenditure of 16% plus transfers of 12% augments government outlays to around 32% of current income, which now implies a deficit.

16 We have also estimate the value of $\varphi$ which if higher than 1 shows that the cost of producing in the public sector exceeds the value of the output being produced and the results point out that if the public sector production function is more capital intensive $\varphi=0.806$ and if it is more labor intensive $\varphi=1.121$. Therefore, if the public production function is more labor intensive, the government is an inefficient producer and is in effect being subsidized by the private sector.
be done in at least two natural ways, each of which provides some insight. First, they may be constrained to lead to the same change in the government’s current deficit $T$. This view may be found in the rhetoric of politicians, why in professing the need to balance the budget typically have only a short-term time horizon in mind.\textsuperscript{17} Alternatively, we may impose the constraint that they involve the same change in the government’s intertemporal deficit, $V$.\textsuperscript{18} It turns out that the comparisons under these two forms of normalization yield slight differences. We shall focus on the latter, but note some issues that arise when we normalize with respect to the short-term balance. One further consideration is that as we compare economies having different production characteristics, they also begin with different initial intertemporal deficits (see Table 1b) and this too needs to be taken into account when imposing the intertemporal normalization. The benchmark we consider is one where the private and public sectors have the same production function, parameterized by $b = d = 0.35$, given in Panel B of Table 2. This approximates the conventional one-sector production technology.\textsuperscript{19}

5.1.1 Increase in Government Investment

The base case we consider is one in which the government doubles its rate of investment from 4% to 8%, financed by lump-sum taxation. This has the effect of increasing the intertemporal budget deficit by 15.92% and this quantity change determines the normalization we impose upon all the other policy changes.

Not surprisingly a doubling of the rate of government investment leads to a dramatic long-run structural change in the economy. Steady-state private capital increases by 40.5%, output increases by 35.3%, while public capital increases by over 170%, raising the long-run ratio of public to private capital by 90%. The increase in productivity induces a reduction in leisure of 0.84 percentage points and the dramatic increase in the public investment sector induces a 3.56 percentage point reallocation of both labor and private capital to that sector.

\textsuperscript{17}Turnovsky (2003) analyzes various fiscal shocks normalizing them with respect to their effect on the short-run deficit.
\textsuperscript{18}This normalization is adopted by Devereux and Love (1994) in their analysis of the two-sector Lucas model and by Turnovsky and Chatterjee (2002).
\textsuperscript{19}In fact the results we obtain in this case area generally similar to those obtained by Turnovsky (2003) using an aggregate one-sector model.
The immediate effect of the lump-sum tax financed increase in government investment expenditure is to reduce the private agent’s wealth, inducing him to supply more labor, thereby raising the marginal productivity of both types of capital and raising output. The dynamic responses of this shock are shown in the four panels of Figure 1. The initial claim on resources by the government crowds out private investment and consumption, so that the growth of private capital is reduced below the growth rate of population; the scale-adjusted stock of private capital therefore initially declines. By contrast, public capital initially grows by over 8% (see (ii)), although this then declines steadily over time as the increase in its stock reduces the return to further public investment. As the new public capital is put in place, its productivity raises the return to private capital, thereby stimulating its growth rate. Over time, as more public capital is accumulated and its growth rate declines, this effect is mitigated, and the growth rate of private capital, after initially rising, begins to decline uniformly to its unchanged steady-state level. Fig. 1 (iii) illustrates the transitional paths of key economic variables relative to their respective initial steady-state levels. The reduction in initial consumption and leisure, with no public expenditure benefits, leads to an initial reduction in welfare of 3.97% (Fig. 1 (iv)). But as capital is accumulated and output and consumption grow, so does welfare. Asymptotically, instantaneous welfare rises to around 30% above the base level, and the overall intertemporal welfare gain, derived from (26) is about 8.6%, in terms of steady income flow.

5.1.2 Decrease in Tax on Capital

Row 2 reduces the tax on capital income. In order to increase the intertermporal deficit by the benchmark level of 15.92%, \( \tau_e \) must be reduced from 0.28 to 0.2029. This also has a dramatic long-run effect on the economy. It increases private capital by over 22% and public capital and output proportionately by around 10.6%, reducing both the output-private capital and public capital-private capital ratios by almost 10%. The increase in long-run capital raises the productivity of labor, causing a long-run substitution toward more labor and less consumption. As long as the production functions in the two sectors are identical \((b = d)\), the reallocation of resources occurs in a balanced way, so that the relative sizes of the two sectors remain unchanged; c.f. (23c).

The dynamics are illustrated in Fig. 2A. Upon impact, the lower capital tax stimulates the
accumulation of private capital, so that the growth rate of private capital immediately increases to around 3.6%. The increase in labor, all of which occurs virtually on impact, and the increase in private capital raises the growth rate of output, and the growth rate of public capital begins to increase as well, so that both $k$ and $k_g$ increase as indicated in Fig. 2A(i). With the incentive applying directly to private capital, $k$ increases relative to $k_g$ during the initial stages. But as private capital increases in relative abundance its productivity declines, inducing less investment in private capital and therefore a decline in its growth rate. This in turn reduces the productivity of public capital, the gradually increasing growth rate of which is reversed after around 10 periods. In the short run the lower capital tax reduces welfare by nearly 3%. This is because the lower capital income tax attracts resources toward investment and away from consumption, while the increase in labor productivity from the higher capital induces a substitution toward more labor and less leisure, both of which are welfare-reducing. However, the steady increase in consumption over time implies that instantaneous welfare rises steadily relative to the benchmark, increasing asymptotically by about 7%, more than offsetting the initial losses. The overall increase in intertemporal welfare, (26), is equivalent to a 2.07% increase in base income.

5.1.3 Decrease in Tax on Wage Income and Consumption

Rows 3 and 4 consider a decrease in the wage tax from 0.28 to 0.2175, and the introduction of a consumption subsidy of -0.0488, respectively, both of which decrease $V$ by the same fixed amount of 15.92%. These taxes both impinge on the consumption-leisure margin and are thus qualitatively similar. We therefore illustrate only $\tau_w$. In contrast to either $g$ or $\tau_k$ these taxes do not impinge directly on either form of capital and consequently the time paths for the two capital stocks, as illustrated by the phase diagram Fig. 2B(i), indicates a much more balanced accumulation path.

The contrasts in both these taxes with the capital income tax are quite striking. First, since both $\tau_w$ and $\tau_c$ fall more directly on consumers, their cuts have a significantly greater positive impact on labor supply. Second, because of their more balanced impacts both tax cuts lead to proportionate long-run increases in public capital, private capital, and output, these effects being substantially smaller than for the capital income tax cut. They also have effect on the long-run
allocations of private capital and labor between the two sectors, although they do have some temporary impacts, we shall discuss below. Moreover, whereas the capital income tax cut leads to short-run losses followed by large long-run gains, wage and consumption taxes lead to steadily increasing welfare gains.

The final column yields an interesting ranking among these four fiscal policies each of which increases the long-run deficit by the same proportionate amount. First, increasing government investment is superior by a substantial margin. This is hardly surprising, given that we are starting an initial government investment rate of around 4% (the recent US figure), whereas the optimal that would be provided in the first-best equilibrium is nearly 12%. Cutting the tax on labor income would be the next best, raising welfare by 2.26%, which is superior to cutting the tax on capital, raising welfare by 2.07%, while introducing the consumption subsidy is inferior and would raise welfare by only 1.39%.

The interesting observation here is the superiority of cutting the wage income tax over cutting the capital income tax. This would appear to run counter to conventional wisdom that has argued that capital income taxes are the most harmful. Here, the normalization of the tax revenues in terms of the intertemporal deficit matters. From Table 2 we see that whereas reducing $\tau_k$ to 0.2029 or $\tau_w$ to 0.2175 both increase the intertemporal deficit by 15.92%, the cut in the capital tax has a much smaller effect on the current deficit than does the wage income tax, increasing it by 9.8% rather than 13.3%. This reflects the fact that the capital income tax has a much greater effect on growth and thus on the long run.

If, instead, we were to normalize all changes so as to increase the short-run deficit by 13.48% (the effect of increasing $g$ from 0.04 to 0.08), then we would find that while the tax on labor income could be reduced only marginally more to 0.2167, the tax on capital income could be reduced substantially to 0.1747. These policies would lead to intertemporal welfare improvements of 2.29% and 2.76%, respectively, so that now cutting the tax on capital would be superior to cutting the tax on labor. But, at the same time this would be associated with a larger increase in the intertemporal

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20 Chamley (1986) originally showed that in the long-run the optimal tax on capital is zero. This result depends upon the absence of externalities and modifications of this proposition in the presence of externalities and growth has been addressed extensively in the recent growth literature.
deficit, suggesting a tradeoff between the welfare of the representative agent and the intertemporal solvency of the government.

5.1.4 The Role of Sectoral Factor Intensities

Panels A and C of Table 2 report the long-run responses to the policies we have been considering in the case where the public production function is relatively less capital intensive, \( b = 0.35 > 0.20 = \beta \), and relatively more capital intensive \( b = 0.35 < 0.50 = \beta \), respectively. Overall, the main qualitative effects are robust with respect to these substantial differences in technologies. All of the long-run welfare gains increase as the relative capital intensity of the public investment sector increases. This is because the more productive is private capital in the public sector the greater the benefits from growth stimulating policies.

While the quantitative effects of the various tax policies are also rather insensitive to the production technologies, that is not so in the case of direct government investment. Table 2 suggests that government investment is generally about 30% more expansionary if the public production function is capital intensive than if it is labor intensive. This translates to welfare gains ranging from 7.73% and 10.94% in these two cases. The most important impacts of the different production functions for the public capital is on the allocation of the private factors of production between the private and public sectors. Thus, whereas with identical production functions raising the public investment rate increases the allocation of both capital and labor in the public sector by 3.56%, these quantities are 2.46% and 4.46%, respectively if the public sector is relatively less capital intensive and 4.82% and 2.77% if it is relatively more capital intensive.

5.2 Compensated Changes

Many policy discussions focus on compensated fiscal changes, meaning that the policy change is accompanied by some other accommodating change so that the government deficit remains unchanged. As we noted before, the normalization can be with respect to the current deficit or the intertemporal deficit. We choose the latter, and Tables 3 and 4 describe a number of changes that hold \( V \) unchanged. Again we shall focus primarily on the case where the production functions
are common across the two sectors, referring to Panel B.

5.2.1 Alternatively-Financed Modes of Government Production Expenditure

To provide a common benchmark, Row 1 repeats the policy of an increase in government investment expenditure from 0.04 to 0.08, financed by lump-sum taxes. Rows 2-4 then compare this to the cases where the public investment is financed by the three distortionary tax rates, and where in each case the rate is set so as to maintain the present value of the fiscal deficit unchanged. The dynamics of capital, sectoral labor allocation, and instantaneous welfare are illustrated in Fig. 3 for the two cases of relative sectoral factor intensities.

The most interesting results are the welfare rankings. In the short run welfare declines by about 3% for the three cases of lump-sum tax, consumption tax, and wage tax financing. This is because the government investment stimulates employment, leading to substitution from consumption, while providing no initial direct benefits. Over time, as the government investment yields its productive payoff and output rises, instantaneous welfare in all cases rises steadily. As noted before, with lump sum tax financing it converges to a higher long-run level around 30% above that of the benchmark economy, with a present value of 8.58%, and as before this is best. With consumption tax, capital income tax, and wage tax financing the respective improvements in asymptotic welfare are 25.6%, 19.5%, and 21.3%, respectively, yielding the corresponding intertemporal welfare gains of 6.68%, 5.52%, and 4.71%, respectively. This long-run ranking reflects the rankings in 5.1 where, for example, although lowering the consumption tax was least beneficial, raising it is also the least harmful.

The time paths for the two capital stocks exhibit interesting differences across financing modes. In all cases government capital increases steadily. Private capital also increases in the long run, although in the case of capital income tax-financing it is modest. It is also subject to an initial decline during the transition, which increases in magnitude as we switch from consumption tax financing, to wage income tax financing, and to capital income tax financing.

Comparing Figs. 3A and 3C we see that these results are robust with respect to the nature of the sectoral production functions. The only difference is that the welfare gains are slightly higher
when the productivity of private capital in the public sector is higher.

Fig. 3B illustrates the time paths for the sectoral allocation of labor. As noted earlier, the sectoral allocation of capital follows a similar pattern and is not illustrated. We shall focus on the case \( b > d \), in which case the initial allocation is \( \theta = 94.77 \). In the case of lump-sum tax financing, wage income, or the consumption tax, the long-run effect of the increase in government investment is to reduce \( \theta \) by 4.71 percentage points, so that it converges to \( \theta = 90.06 \). In each case, the reallocation instantaneously adjusts to within 0.1 percentage points of the new equilibrium. During the initial phase of the transition a small fraction of the labor is reallocated back to the private sector, before converging back to the new equilibrium. In the case of capital income tax financing, employment in the private sector is reduced by 4.46 percentage points, so that it converges to \( \theta = 90.31 \). On impact, the labor allocation overshoots by about 0.20 percentage points and it then overshoots again slightly during the subsequent transition. The case where the public sector is relatively capital intensive is illustrated in Fig. 3B and is seen to be a mirror image.

5.2.2 Tax Substitution

Table 4 summarizes two forms of tax substitution. Row 1 introduces a 6% consumption tax, which permits income taxes to be reduced to around 24% (depending upon the factor intensity) to maintain the initial intertemporal deficit, \( V \), unchanged. This change in the tax structure raises long-run output and public capital by around 5% and private capital by 10.8%, relative to the benchmark economy. The shift from the wage tax to the consumption tax causes a slight reduction in long-run leisure, with a proportionately slightly larger reduction in the consumption-output ratio.

The dynamics are illustrated in Fig. 2B. The reduction in the tax on income favors private capital, the growth rate of which rises to nearly 2.9% on impact. This stimulates the productivity of public capital, the growth rate of which also rises in the short run, though more modestly. The tax switch causes an initial drop in consumption of 1% and an increase in labor supply of 1%, thus leading to an initial deterioration in welfare. However, the higher output level and resulting higher consumption throughout the transition causes current welfare to improve over time, doing so by 3.5% asymptotically, and a present value equivalent of nearly 1%.
Row 2 substitutes a higher capital income tax for a lower wage tax. The main point of interest here is that the qualitative net benefits of this depend upon the relative capital intensities of the two production functions. It will be mildly beneficial if the public production function is more capital intensive and mildly undesirable otherwise.

6. Conclusions

Recent research in fiscal policy has paid increasing attention to the role of public capital. Virtually all models that introduce public capital employ a one-sector technology, thereby assuming that public capital is produced in the private sector along with all other output by profit maximizing firms. In some circumstances, the government may be more appropriately thought of as operating its own production operation. Typically, government investment projects go out to contractors who agree to produce the specified amount of government investment – determined by policy makers – as efficiently as possible. In this paper, we have developed a two-sector “non-scale” production model in which there are two types of firms, conventional profit-maximizing private firms, and what we call “public firms”, whose objective is to produce a specified quantity of government investment goods at minimum cost. Furthermore, we assume the production functions of the two sectors do not in general coincide. Using this two-sector production set up we have characterized the equilibrium dynamics, and employed this equilibrium to analyze a variety of fiscal disturbances. Because of the complexity of the model our analysis has been carried out using simulations of a calibrated economy. We have obtained many results, of which the following merit comment.

1. Despite the fact that fiscal policy in such an economy has no effect on the long-run equilibrium growth rate, the slow rate of convergence implies that fiscal policy has a sustained impact on growth rates for substantial periods during the transition. These accumulate to substantial effects on the long-run equilibrium levels of crucial economic variables, including welfare.

2. As an example of the accumulated impacts of policy, an increase in government investment from 0.04 to 0.08 of private output raises the long-run level of private output by between 32% and 42%, and the ratio of public to private capital between 158% to over 200%, depending upon the relative capital intensities of the two production sectors.
3. We have ranked alternative fiscal policies for normalized changes on the government’s intertemporal deficit. For the calibrated economy, increasing the rate of government investment from 4% to 8% clearly dominates from an intertemporal welfare viewpoint other equivalent policy changes of (i) reducing the tax on capital income by around 8%, (ii) reducing the tax on labor income by around 6.5%, and (iii) introducing a consumption subsidy of just under 5%. The margin of superiority increases as the public production function becomes more capital intensive.

4. Interestingly, reducing the tax on labor income yields slightly higher welfare than does a comparable reduction in the tax on capital income. However, the conventional inferiority of capital income taxation applies if the revenues are normalized with regard to the current, rather than the intertemporal, government deficit.

5. Our numerical simulations suggest the following welfare ranking for the different modes of financing government investment. Lump-sum tax financing dominates consumption tax financing, which in turn dominates capital tax financing and finally wage tax financing.

The effects of tax policies are remarkably robust with respect to the relative capital intensities of the two productive sectors. In contrast, the effects of government investment are much more sensitive to this aspect. One conclusion of this is that one can continue to employ the one-sector Ramsey model analyze tax policy in the presence of public capital without seriously jeopardizing the analysis. However, one has to be more careful in analyzing the impact of public capital itself.
Appendix

In this Appendix we derive the linear approximation to the macrodynamic equilibrium employed in our simulations. We begin with the four equilibrium equations (20a) – (20d) written as

\[ \dot{k} = y - c - k(\delta_k + \psi) \]  
\[ \dot{k}_G = gy - k_G(\delta_G + \psi) \]  
\[ \rho \gamma \frac{\dot{i}}{l} + (\gamma - 1) \left( \frac{\dot{c}}{c} - n + \psi \right) = \beta - (1 - \tau_k) b \frac{y}{\phi k} + \delta_k + n \]  
\[ \frac{c}{y} = \left( \frac{1 - \tau_w}{1 + \tau_l} \right) \left( \frac{l}{\theta (1 - l)} \right) \left( \frac{1 - b}{\rho} \right) \]

Taking the time differential of (A.1d) yields

\[ \frac{\dot{c}}{c} = \frac{\dot{y}}{y} - \frac{\ddot{c}}{c} + \frac{\dot{i}}{l} + \frac{\dot{i}}{1 - l} \]

and taking the time derivative of the private sector production function, (17a), yields

\[ \frac{\dot{y}}{y} = b \frac{\dot{\phi}}{\phi} + b \frac{\dot{k}}{k} + (1 - b) \frac{\dot{\theta}}{\theta} - (1 - b) \frac{\dot{i}}{1 - l} + \sigma \frac{\dot{k}_G}{k_G} \]

Substituting into (A.2) implies

\[ \frac{\dot{c}}{c} = b \frac{\dot{\phi}}{\phi} + b \frac{\dot{k}}{k} - \frac{\phi}{k} - \frac{b}{\theta} + \frac{\ddot{i}}{1 - l} + \sigma \frac{\dot{k}_G}{k_G} + \frac{\dot{i}}{l} \]

which involves the contemporaneous changes in the sectoral allocations. To determine these, we take the time derivatives of the solutions (19a) and (19b)

\[ \frac{\dot{\theta}}{\theta} = \left( \frac{\partial \theta}{\partial k} \frac{\dot{k}}{k} + \frac{\partial \theta}{\partial k_G} \frac{\dot{k}_G}{k_G} + \frac{\partial \theta}{\partial l} \frac{\dot{l}}{l} \right) \frac{1}{\theta} \]

and analogously for \( \dot{\phi} \). Substituting these expressions into (A.3) we may express \( \dot{c} \) in terms of \( k, k_G, \dot{l} \)
\[
\dot{c} = \left( b \frac{\partial \phi}{\partial k} + b \frac{\partial \theta}{k \partial k} \right) \dot{k} + \left( b \frac{\partial \phi}{\partial k_G} + \frac{\sigma}{k_g} - b \frac{\partial \theta}{\partial k_G} \right) \dot{k}_G + \left( b \frac{\partial \phi}{\partial l} + \frac{1}{l} + b \frac{1}{1-l} \frac{\partial \theta}{\partial l} \right) \dot{l} \quad (A.4)
\]

Substituting (A.4) into (A.1c) yields the following relationship between \( \dot{i}, \dot{k}, \) and \( \dot{k}_G \)

\[
\dot{i} = \frac{1}{\Delta} \left[ \beta - (1-\tau_k) b \frac{y}{\phi k} + \delta_k + n - (\gamma - 1)(\psi - n) \right] - \frac{(\gamma - 1)}{\Delta} \left[ \left( b \frac{\partial \phi}{\partial k} + b \frac{\partial \theta}{k \partial k} \right) \dot{k} + \left( b \frac{\partial \phi}{\partial k_G} + \frac{\sigma}{k_g} - b \frac{\partial \theta}{\partial k_G} \right) \dot{k}_G \right] \quad (A.5)
\]

where

\[
\Delta \equiv (\gamma - 1) \left( b \frac{\partial \phi}{\partial l} + b \frac{1}{1-l} - b \frac{\partial \theta}{\partial l} + \frac{1}{l} \right) + \rho \gamma \frac{1}{l}
\]

To derive the linearized equilibrium, we linearize (A.1a), (A.1b), and (A.5) in turn. Thus, linearizing (A.1a) yields

\[
\dot{k} = \left[ b \frac{y}{\phi} \frac{\partial \phi}{\partial k} + b \frac{y}{k} (1-b) \frac{\partial \theta}{\partial k} - \frac{\partial c}{\partial k} - (\delta_k + \psi) \right] (k - \bar{k}) \quad (A.6a)
\]

\[
+ \left[ b \frac{y}{\phi} \frac{\partial \phi}{\partial k} + (1-b) \frac{y}{k} \frac{\partial \theta}{\partial k} + \sigma \frac{y}{k_g} - \frac{\partial c}{\partial k_g} \right] (k_G - \bar{k}_G) + \left[ b \frac{y}{\phi} \frac{\partial \phi}{\partial l} + (1-b) \frac{y}{\theta} \frac{\partial \theta}{\partial l} - (1-b) \frac{y}{1-l} \frac{\partial c}{\partial l} \right] (l - \bar{l})
\]

Denoting the 3 x 3 matrix by \( A \equiv (a_{ij}) \), we can identify this equation as

\[
\dot{k} = a_{11}(k - \bar{k}) + a_{12}(k_G - \bar{k}_G) + a_{13}(l - \bar{l}) \quad (A.7a)
\]

Similarly, linearizing (A.1b) yields

\[
\dot{k}_G = \left[ gb \frac{y}{\phi} \frac{\partial \phi}{\partial k} + gb \frac{y}{k} + g(1-b) \frac{y}{\theta} \frac{\partial \theta}{\partial k} \right] (k - \bar{k})
\]

\[
+ \left[ gb \frac{y}{\phi} \frac{\partial \phi}{\partial k_G} + g(1-b) \frac{y}{k} \frac{\partial \theta}{\partial k_G} + g \sigma \frac{y}{k_G} - (\delta_k + \psi) \right] (k_G - \bar{k}_G)
\]

\[
+ \left[ gb \frac{y}{\phi} \frac{\partial \phi}{\partial l} + (1-b) \frac{y}{\theta} \frac{\partial \theta}{\partial l} - g(1-b) \frac{y}{1-l} \frac{\partial c}{\partial l} \right] (l - \bar{l}) \quad (A.6b)
\]

which we can identify as
\[
\dot{k}_g = a_{21}(k - \tilde{k}) + a_{22}(k - \tilde{k}_g) + a_{23}(l - \tilde{l}) \quad (A.7b)
\]

Linearizing (A.5) is a little more complex, but in doing so, we evaluate the expression at the steady state, where \( \dot{k} = \dot{k}_g = 0 \). Following this procedure yields

\[
\hat{i} = \frac{1}{\Delta} \left\{ -(1 - \tau_k) \left[ (b - 1) \frac{\partial \phi}{\partial k} \frac{y}{\phi k^2} + (b - 1) \frac{y}{\phi k^2} + (1 - b) \frac{y}{\phi \theta k} \right] + (\gamma - 1) \left[ a_{11} \left( \frac{b \partial \phi}{\phi \partial k} + \frac{b}{k} - \frac{b \partial \theta}{\theta \partial k} \right) + a_{21} \left( \frac{b \partial \phi}{\phi \partial k} + \frac{\sigma}{k} - \frac{b \partial \theta}{\theta \partial k} \right) \right] k - \tilde{k} \right\}
\]

\[
\hat{k} = \frac{1}{\Delta} \left\{ -(1 - \tau_k) \left[ (b - 1) \frac{\partial \phi}{\partial k} \frac{y}{\phi k^2} + (1 - b) \frac{y}{\phi \theta k} + \sigma \frac{y}{\phi k} \right] + (\gamma - 1) \left[ a_{12} \left( \frac{b \partial \phi}{\phi \partial k} + \frac{b}{k} - \frac{b \partial \theta}{\theta \partial k} \right) + a_{22} \left( \frac{b \partial \phi}{\phi \partial k} + \frac{\sigma}{k} - \frac{b \partial \theta}{\theta \partial k} \right) \right] (k - \tilde{k}_g) \right\}
\]

\[
\hat{b} = \frac{1}{\Delta} \left\{ -(1 - \tau_k) \left[ (b - 1) \frac{\partial \phi}{\partial l} \frac{y}{\phi k^2} + (1 - b) \frac{y}{\phi \theta k} - (1 - b) \frac{y}{\phi k} \right] - (\gamma - 1) \left[ a_{13} \left( \frac{b \partial \phi}{\phi \partial k} + \frac{b}{k} - \frac{b \partial \theta}{\theta \partial k} \right) + a_{23} \left( \frac{b \partial \phi}{\phi \partial k} + \frac{\sigma}{k} - \frac{b \partial \theta}{\theta \partial k} \right) \right] (l - \tilde{l}) \right\}
\]

which we can identify as

\[
\dot{k}_g = a_{31}(k - \tilde{k}) + a_{32}(k - \tilde{k}_g) + a_{33}(l - \tilde{l}) \quad (A.7c)
\]

Note that these expressions all involve the partial derivatives

\[
\frac{\partial \theta}{\partial k}, \frac{\partial \phi}{\partial k}, \frac{\partial \theta}{\partial k_g}, \frac{\partial \phi}{\partial k_g}, \frac{\partial \theta}{\partial l}, \frac{\partial \phi}{\partial l} \quad \text{as well as} \quad \frac{\partial c}{\partial k}, \frac{\partial c}{\partial k_g}, \frac{\partial c}{\partial l}
\]

To evaluate these, we proceed in the following sequential manner. Taking differentials of the sectoral allocation equations (19a) and (19b) we obtain

\[
\frac{\partial \theta}{\partial k} = \left( \frac{\theta}{1 - \phi} \right) \frac{1}{\Gamma} (b - d) k, \quad \frac{\partial \phi}{\partial k} = \frac{\theta (1 - \theta)}{\phi (1 - \phi)} \frac{\partial \phi}{\partial k} \quad (A.8a)
\]
\[
\frac{\partial \theta}{\partial k_G} = -\frac{(\theta/(1-\phi))(\sigma - \eta)/k_G}{\Gamma}; \quad \frac{\partial \phi}{\partial k_G} = \frac{\theta(1-\theta)}{\phi(1-\phi)} \frac{\partial \phi}{\partial k_G}
\]  
(A.8b)

\[
\frac{\partial \theta}{\partial t} = -\frac{(\theta/(1-\phi))(b-d)/(1-l)}{\Gamma}; \quad \frac{\partial \phi}{\partial k} = \frac{\theta(1-\theta)}{\phi(1-\phi)} \frac{\partial \phi}{\partial k}
\]  
(A.8c)

where

\[
\Gamma \equiv \frac{1}{1-\theta} \left[ b + \frac{\phi}{1-\phi} d \right] + \frac{1}{1-\phi} \left[ (1-b) + \frac{\theta}{1-\theta} (1-d) \right]
\]  
(A.8d)

Having obtained these, the relevant partial derivatives of \(c\) are obtained by differentiating (A.1d) in conjunction with (17a). This leads to the expressions

\[
\frac{\partial c}{\partial k} = \frac{1-\tau_w}{1+\tau_c} \frac{1-b}{\rho} \frac{l}{1-l} \left( b \frac{y \partial \phi}{\phi \partial k} + b \frac{y \partial \theta}{\theta k} - b \frac{y \partial \theta}{\theta^2 \partial k} \right)
\]  
(A.9a)

\[
\frac{\partial c}{\partial k_g} = \frac{1-\tau_w}{1+\tau_c} \frac{1-b}{\rho} \frac{l}{1-l} \left( b \frac{y \partial \phi}{\phi \partial k_g} - b \frac{y \partial \theta}{\theta \partial k_g} + \sigma \frac{y}{\theta k_g} \right)
\]  
(A.9b)

\[
\frac{\partial c}{\partial l} = \frac{1-\tau_w}{1+\tau_c} \frac{1-b}{\rho} \frac{l}{1-l} \left[ \left( b \frac{y \partial \phi}{\phi(1-l) \partial l} - b \frac{y \partial \phi}{\theta^2(1-l) \partial k_g} + b \frac{y \partial \phi}{\theta(1-l)^2 \partial k_g} \right) + \frac{y}{\theta(1-l)} \right]
\]  
(A.9c)

Expressions (A.8) and (A.9) yield all the partial derivatives and substituting these partial derivatives into the elements \(a_{ij}\) yields the linearized dynamics of the equilibrium system.
References

Arrow, K.J. and M. Kurz, (1970), *Public Investment, the Rate of Return, and Optimal Fiscal Policy*, Johns Hopkins Press, Baltimore MD.


### Table 1a

**Base Parameter Values**

| Preference and population parameters | $\gamma = -1.5$, $\delta = 1.75$, $\beta = 0.04$, $n = 0.015$ |
| Production parameters                | $\alpha_y = 1$, $b = 0.35$, $\sigma = 0.2$, $\delta_k = 0.05$ , $\alpha_j = 1$, $\delta_G = 0.035$ |
| Private sector                       | $d = 0.2$, $0.35$, $0.5$; $\eta = 0.246$, $0.2$, $0.1538$ |
| Public sector                        | $g = 0.04$, $t_k = 0.28$, $t_w = 0.28$, $t_c = 0.0$ |

### Table 1b

**Base equilibria**

<table>
<thead>
<tr>
<th>l</th>
<th>c/y</th>
<th>kg/k</th>
<th>y/k</th>
<th>$\gamma$</th>
<th>f</th>
<th>?</th>
<th>eigenvalues</th>
<th>PVGovt deficit</th>
</tr>
</thead>
<tbody>
<tr>
<td>d=0.20</td>
<td>75.02</td>
<td>0.848</td>
<td>0.332</td>
<td>0.471</td>
<td>94.77</td>
<td>97.50</td>
<td>2.167</td>
<td>-0.1023 -0.0305 -0.9263</td>
</tr>
<tr>
<td>d=0.35</td>
<td>75.25</td>
<td>0.846</td>
<td>0.328</td>
<td>0.464</td>
<td>96.15</td>
<td>96.15</td>
<td>2.167</td>
<td>-0.1018 -0.0305 -0.9468</td>
</tr>
<tr>
<td>d=0.50</td>
<td>75.50</td>
<td>0.845</td>
<td>0.326</td>
<td>0.462</td>
<td>97.57</td>
<td>95.59</td>
<td>2.167</td>
<td>-0.1017 -0.0305 -0.9740</td>
</tr>
</tbody>
</table>

### Table 1c

**Centrally Planned Economy**

<table>
<thead>
<tr>
<th>l</th>
<th>c/y</th>
<th>kg/k</th>
<th>y/k</th>
<th>$\gamma$</th>
<th>f</th>
<th>?</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>d=0.20</td>
<td>64.59</td>
<td>0.779</td>
<td>0.580</td>
<td>0.325</td>
<td>86.92</td>
<td>93.47</td>
<td>2.167</td>
</tr>
<tr>
<td>d=0.35</td>
<td>64.93</td>
<td>0.769</td>
<td>0.652</td>
<td>0.311</td>
<td>89.38</td>
<td>39.38</td>
<td>2.167</td>
</tr>
<tr>
<td>d=0.50</td>
<td>65.26</td>
<td>0.760</td>
<td>0.815</td>
<td>0.298</td>
<td>91.82</td>
<td>85.82</td>
<td>2.167</td>
</tr>
</tbody>
</table>
Table 2
Uncompensated Fiscal Changes Normalized with Respect to Present Value of Fiscal Deficit V

<table>
<thead>
<tr>
<th></th>
<th>Δl%</th>
<th>Θ%</th>
<th>Φ%</th>
<th>Δ(c/y)%</th>
<th>Δ(y/k)%</th>
<th>Δ(k)%</th>
<th>Δ(k,γ)%</th>
<th>Δ(y)%</th>
<th>Δ(T)%</th>
<th>Short-run welf. gains</th>
<th>Long-run welf. gains</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Public production function less intensive in capital:</strong> b = 0.35, d = 0.2, σ = 0.2, η = 0.246:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Increase in g from 0.04 to 0.0778</td>
<td>-1.00</td>
<td>-4.46</td>
<td>-2.48</td>
<td>-0.42</td>
<td>-2.31</td>
<td>90.02</td>
<td>35.78</td>
<td>158.0</td>
<td>32.64</td>
<td>-13.94</td>
<td>-2.52</td>
</tr>
<tr>
<td>2. Decrease in τₖ from 0.28 to 0.2035</td>
<td>-0.39</td>
<td>-0.12</td>
<td>-0.06</td>
<td>-1.92</td>
<td>-9.66</td>
<td>-9.66</td>
<td>22.21</td>
<td>10.41</td>
<td>10.41</td>
<td>-9.72</td>
<td>-2.92</td>
</tr>
<tr>
<td>3. Decrease in τₜ from 0.28 to 0.2190</td>
<td>-1.55</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9.11</td>
<td>9.11</td>
<td>9.11</td>
<td>-13.32</td>
<td>0.07</td>
<td>2.19</td>
</tr>
<tr>
<td>4. Decrease in τₑ from 0.0 to -0.0481</td>
<td>-0.93</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5.45</td>
<td>5.45</td>
<td>5.45</td>
<td>-14.28</td>
<td>0.08</td>
<td>1.35</td>
</tr>
<tr>
<td><strong>B. Public and private production functions identical: b = 0.35, d = 0.35, σ = 0.2, η = 0.2:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Increase in g from 0.04 to 0.08</td>
<td>-0.84</td>
<td>-3.56</td>
<td>-3.56</td>
<td>-0.70</td>
<td>-3.70</td>
<td>92.6</td>
<td>40.5</td>
<td>170.5</td>
<td>35.3</td>
<td>-13.48</td>
<td>-2.82</td>
</tr>
<tr>
<td>2. Decrease in τₖ from 0.28 to 0.2029</td>
<td>-0.37</td>
<td>0</td>
<td>0</td>
<td>-1.95</td>
<td>-9.67</td>
<td>-9.67</td>
<td>22.42</td>
<td>10.58</td>
<td>10.58</td>
<td>-9.80</td>
<td>-2.95</td>
</tr>
<tr>
<td>3. Decrease in τₜ from 0.28 to 0.2175</td>
<td>-1.58</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9.37</td>
<td>9.37</td>
<td>9.37</td>
<td>-13.30</td>
<td>0.08</td>
<td>2.26</td>
</tr>
<tr>
<td>4. Decrease in τₑ from 0.0 to -0.0488</td>
<td>-0.94</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5.55</td>
<td>5.55</td>
<td>5.55</td>
<td>-14.29</td>
<td>0.08</td>
<td>1.39</td>
</tr>
<tr>
<td>**C. Public production function more intensive in capital: b = 0.35, d = 0.5, σ = 0.2, η = 0.1538: **</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Increase in g from 0.04 to 0.0882</td>
<td>-0.72</td>
<td>-2.77</td>
<td>-4.82</td>
<td>-0.96</td>
<td>-5.04</td>
<td>109.4</td>
<td>49.70</td>
<td>213.5</td>
<td>42.16</td>
<td>-13.44</td>
<td>-3.24</td>
</tr>
<tr>
<td>2. Decrease in τₖ from 0.28 to 0.2014</td>
<td>-0.36</td>
<td>0.06</td>
<td>0.10</td>
<td>-1.99</td>
<td>-9.75</td>
<td>-9.75</td>
<td>22.77</td>
<td>10.81</td>
<td>10.81</td>
<td>-9.82</td>
<td>-2.98</td>
</tr>
<tr>
<td>3. Decrease in τₜ from 0.28 to 0.2153</td>
<td>-1.63</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9.73</td>
<td>9.73</td>
<td>9.73</td>
<td>-13.24</td>
<td>0.09</td>
<td>2.37</td>
</tr>
<tr>
<td>4. Decrease in τₑ from 0.0 to -0.0498</td>
<td>-0.96</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5.69</td>
<td>5.69</td>
<td>5.69</td>
<td>-14.29</td>
<td>0.10</td>
<td>1.44</td>
</tr>
</tbody>
</table>
Table 3: Revenue Neutral Increase in government Investment from $g = 0.04$ to $0.08$
Normalized with Respect to Present Value of Fiscal Deficit $V$

<table>
<thead>
<tr>
<th>A. Public production function less intensive in capital: $b = 0.35, d = 0.20, \sigma = 0.2, \eta = 0.246$</th>
<th>( \Delta/ ) % points</th>
<th>( \theta/ ) % points</th>
<th>( \phi/ ) % points</th>
<th>( \Delta \left( \frac{c}{y} \right)/ % )</th>
<th>( \Delta \left( \frac{y}{k} \right)/ % )</th>
<th>( \Delta \left( \frac{k}{k} \right)/ % )</th>
<th>( \Delta(k)/ % )</th>
<th>( \Delta(y)/ % )</th>
<th>Short-run welf. gains percent</th>
<th>Long-run welf. gains percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Financed by $T$</td>
<td>-1.05</td>
<td>-4.71</td>
<td>-2.38</td>
<td>-0.45</td>
<td>-2.44</td>
<td>95.1</td>
<td>37.5</td>
<td>168.4</td>
<td>34.2</td>
<td>-2.74</td>
</tr>
<tr>
<td>2. Financed by $\tau_k$ from 0.28 to 0.3623</td>
<td>-0.60</td>
<td>-4.46</td>
<td>-2.25</td>
<td>1.68</td>
<td>10.3</td>
<td>120.6</td>
<td>8.33</td>
<td>139.0</td>
<td>19.5</td>
<td>0.48</td>
</tr>
<tr>
<td>3. Financed by $\tau_w$ from 0.28 to 0.3524</td>
<td>0.94</td>
<td>-4.71</td>
<td>-2.38</td>
<td>-0.45</td>
<td>-2.44</td>
<td>95.1</td>
<td>22.6</td>
<td>139.3</td>
<td>19.6</td>
<td>-2.89</td>
</tr>
<tr>
<td>4. Financed by $\tau_c$ from 0 to 0.0565</td>
<td>-0.01</td>
<td>-4.71</td>
<td>-2.38</td>
<td>-0.45</td>
<td>-2.44</td>
<td>95.1</td>
<td>29.6</td>
<td>153.0</td>
<td>26.5</td>
<td>-2.77</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Public and private production functions identical: $b = 0.35, d = 0.35, \sigma = 0.2, \eta = 0.2$</th>
<th>( \Delta/ ) % points</th>
<th>( \theta/ ) % points</th>
<th>( \phi/ ) % points</th>
<th>( \Delta \left( \frac{c}{y} \right)/ % )</th>
<th>( \Delta \left( \frac{y}{k} \right)/ % )</th>
<th>( \Delta(k)/ % )</th>
<th>( \Delta(y)/ % )</th>
<th>Short-run welf. gains percent</th>
<th>Long-run welf. gains percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Financed by $T$</td>
<td>-0.84</td>
<td>-3.56</td>
<td>-3.56</td>
<td>-0.70</td>
<td>-3.70</td>
<td>92.6</td>
<td>40.5</td>
<td>170.5</td>
<td>35.3</td>
</tr>
<tr>
<td>2. Financed by $\tau_k$ from 0.28 to 0.3614</td>
<td>-0.44</td>
<td>-3.56</td>
<td>-3.56</td>
<td>1.44</td>
<td>8.56</td>
<td>117.1</td>
<td>10.9</td>
<td>140.8</td>
<td>20.4</td>
</tr>
<tr>
<td>3. Financed by $\tau_w$ from 0.28 to 0.3532</td>
<td>1.14</td>
<td>-3.56</td>
<td>-3.56</td>
<td>-0.70</td>
<td>-3.70</td>
<td>92.6</td>
<td>25.0</td>
<td>140.7</td>
<td>20.3</td>
</tr>
<tr>
<td>4. Financed by $\tau_c$ from 0 to 0.0556</td>
<td>0.17</td>
<td>-3.56</td>
<td>-3.56</td>
<td>-0.70</td>
<td>-3.70</td>
<td>92.6</td>
<td>32.5</td>
<td>155.1</td>
<td>27.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Public production function more intensive in capital: $b = 0.35, d = 0.5, \sigma = 0.2, \eta = 0.1538$</th>
<th>( \Delta/ ) % points</th>
<th>( \theta/ ) % points</th>
<th>( \phi/ ) % points</th>
<th>( \Delta \left( \frac{c}{y} \right)/ % )</th>
<th>( \Delta \left( \frac{y}{k} \right)/ % )</th>
<th>( \Delta(k)/ % )</th>
<th>( \Delta(y)/ % )</th>
<th>Short-run welf. gains percent</th>
<th>Long-run welf. gains percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Financed by $T$</td>
<td>-0.60</td>
<td>-2.31</td>
<td>-4.03</td>
<td>-0.81</td>
<td>-4.22</td>
<td>91.6</td>
<td>42.1</td>
<td>172.2</td>
<td>36.1</td>
</tr>
<tr>
<td>2. Financed by $\tau_k$ from 0.28 to 0.3550</td>
<td>-0.25</td>
<td>-2.42</td>
<td>-4.23</td>
<td>1.15</td>
<td>6.69</td>
<td>113.4</td>
<td>14.6</td>
<td>144.5</td>
<td>22.3</td>
</tr>
<tr>
<td>3. Financed by $\tau_w$ from 0.28 to 0.3486</td>
<td>1.24</td>
<td>-2.31</td>
<td>-4.03</td>
<td>-0.81</td>
<td>-4.33</td>
<td>91.6</td>
<td>27.3</td>
<td>144.0</td>
<td>21.8</td>
</tr>
<tr>
<td>4. Financed by $\tau_c$ from 0 to 0.0506</td>
<td>0.32</td>
<td>-2.31</td>
<td>-4.03</td>
<td>-0.81</td>
<td>-4.22</td>
<td>91.6</td>
<td>34.7</td>
<td>158.0</td>
<td>29.0</td>
</tr>
</tbody>
</table>
Table 4: Tax Substitutions: Normalized with Respect to Present Value of Fiscal Deficit V

<table>
<thead>
<tr>
<th></th>
<th>$\Delta l$ % points</th>
<th>$\Theta$ % points</th>
<th>$\Phi$ % points</th>
<th>$\Delta \left( \frac{c}{y} \right)$ %</th>
<th>$\Delta \left( \frac{y}{k} \right)$ %</th>
<th>$\Delta \left( \frac{k_s}{k} \right)$ %</th>
<th>$\Delta(k)$ %</th>
<th>$\Delta(k_s)$ %</th>
<th>$\Delta(y)$ %</th>
<th>Short-run welf. gains percent</th>
<th>Long-run welf. gains percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\tau_c = 0.06$</td>
<td>-0.12</td>
<td>-0.06</td>
<td>-0.03</td>
<td>-1.00</td>
<td>-5.29</td>
<td>-5.29</td>
<td>10.80</td>
<td>4.93</td>
<td>4.93</td>
<td>-1.56</td>
<td>0.96</td>
</tr>
<tr>
<td>$\tau_k = \tau_w = 0.2400$</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. $\tau_w = 0.1660$</td>
<td>-2.22</td>
<td>0.20</td>
<td>0.10</td>
<td>3.01</td>
<td>20.12</td>
<td>20.12</td>
<td>-18.04</td>
<td>-1.55</td>
<td>-1.55</td>
<td>5.05</td>
<td>-0.13</td>
</tr>
<tr>
<td>$\tau_s = 0.4, \tau_c = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

Public production function less intensive in capital: $b = 0.35, d = 0.20, \sigma = 0.2, \eta = 0.246$

Public and private production function identical: $b = 0.35, d = 0.35, \sigma = 0.2, \eta = 0.2$

<table>
<thead>
<tr>
<th></th>
<th>$\Delta l$ % points</th>
<th>$\Theta$ % points</th>
<th>$\Phi$ % points</th>
<th>$\Delta \left( \frac{c}{y} \right)$ %</th>
<th>$\Delta \left( \frac{y}{k} \right)$ %</th>
<th>$\Delta(k)$ %</th>
<th>$\Delta(k_s)$ %</th>
<th>$\Delta(y)$ %</th>
<th>Short-run welf. gains percent</th>
<th>Long-run welf. gains percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\tau_c = 0.06$</td>
<td>-0.11</td>
<td>0</td>
<td>0</td>
<td>-1.02</td>
<td>-5.27</td>
<td>-5.27</td>
<td>10.84</td>
<td>5.00</td>
<td>5.00</td>
<td>-1.57</td>
</tr>
<tr>
<td>$\tau_k = \tau_w = 0.2399$</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2. $\tau_w = 0.1641$</td>
<td>-2.22</td>
<td>0</td>
<td>0</td>
<td>3.04</td>
<td>20.0</td>
<td>20.0</td>
<td>-17.84</td>
<td>-1.40</td>
<td>-1.40</td>
<td>5.10</td>
</tr>
<tr>
<td>$\tau_s = 0.4, \tau_c = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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</tr>
</tbody>
</table>

Public production function more intensive in capital: $b = 0.35, d = 0.5, \sigma = 0.2, \eta = 0.1538$

<table>
<thead>
<tr>
<th></th>
<th>$\Delta l$ % points</th>
<th>$\Theta$ % points</th>
<th>$\Phi$ % points</th>
<th>$\Delta \left( \frac{c}{y} \right)$ %</th>
<th>$\Delta \left( \frac{y}{k} \right)$ %</th>
<th>$\Delta(k)$ %</th>
<th>$\Delta(k_s)$ %</th>
<th>$\Delta(y)$ %</th>
<th>Short-run welf. gains percent</th>
<th>Long-run welf. gains percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\tau_c = 0.06$</td>
<td>-0.11</td>
<td>0.03</td>
<td>0.05</td>
<td>-1.02</td>
<td>-5.24</td>
<td>-5.24</td>
<td>10.87</td>
<td>5.06</td>
<td>5.06</td>
<td>-1.57</td>
</tr>
<tr>
<td>$\tau_k = \tau_w = 0.2398$</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>2. $\tau_w = 0.1621$</td>
<td>-2.34</td>
<td>-0.10</td>
<td>-0.18</td>
<td>3.03</td>
<td>19.77</td>
<td>19.77</td>
<td>-17.47</td>
<td>-1.13</td>
<td>-1.13</td>
<td>5.11</td>
</tr>
<tr>
<td>$\tau_s = 0.4, \tau_c = 0$</td>
<td></td>
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</tr>
</tbody>
</table>
Figure 1: Increase in Government Investment

(i) phase diagram

(ii) growth rates

(iii) transitional paths

(iv) welfare
Figure 2A: Decrease in Capital Income Tax

Figure 2B: Decrease in Wage Income Tax
Figure 3: Increase in Government Investment under Alternative Financing

A. Private sector more capital intensive (b=0.35, d=0.20)

B. Public sector more capital intensive (b=0.35, d=0.50)
Figure 4: Substitution of Consumption Tax for Uniform Income Tax

A. private sector more capital intensive (b=0.35, d=0.20)

B. public sector more capital intensive (b=0.35, d=0.50)

(i) phase diagram

(ii) sectoral labor allocation

(iii) welfare

(initial $\theta = 94.77$)

(initial $\theta = 97.57$)