FOURTH EDITION

SECTION 1.2 (ANSWERS OR HINTS)

#4 (a) ab.aa.baa.ab.aa, aa.aa.baa.aa and baa.aa.ab.aa are in L.

(b) b.aa.aa.ab.aa.aa.b is not in L

#5. $\bar{L} = \{\lambda, a, b, ab, ba\} \cup \{w \in \{a,b\}^* : |w| \geq 3\}$

#25 (a) If $L = \{a^n b^{n+1} : n \geq 0\}$ then $L \neq L^*$ because $\lambda \notin L$ but $\lambda \in L^*$

(b) If $L = \{w : n_a(w) = n_b(w)\}$ then $L = L^*$. Prove This.

#6 Hint: $L \cup \bar{L} = V^*$ and V^* is infinite.

#7 No. No matter what L is, we will have $\lambda \in L^*$. So $\lambda \notin \overline{L^*}$. But $\lambda \in (\bar{L})^*$. So we can never have $\overline{L^*} = (\bar{L})^*$.

#10 (a) TRUE (b) TRUE

#11 (a) $S \to bS$, $S \to Sb$, $S \to a$

(b) $S \to bS$, $S \to Sb$, $S \to a$, $S \to SS$

(c) $S \to AAA$, $A \to \lambda$

$A \to bA$, $A \to Ab$, $A \to a$

(d) $S \to AAA$, $A \to AA$

$A \to bA$, $A \to Ab$, $A \to a$

#12 $L(G) = \{(ab)^n : n \geq 0\}$ #13. $L(G) = \phi$

#25 (b) $L = \{w : n_a(w) = n_b(w)\}$. Since $L^* = \bigcup_{k=0}^{\infty} L^k$, $L \subseteq L^*$.
Now suppose $\varphi \in L^*$. Then $\varphi = \varphi_1 \varphi_2 \cdots \varphi_k$ where each $\varphi_i \in L$. So $n_a(\varphi) = n_a(\varphi_1) + n_a(\varphi_2) + \cdots + n_a(\varphi_k)$
$= n_b(\varphi_1) + n_b(\varphi_2) + \cdots + n_b(\varphi_k) = n_b(\varphi)$ bec. each $\varphi_i \in L$.
So $\varphi \in L$ bec. $n_a(\varphi) = n_b(\varphi)$. So $L^* \subseteq L$. Hence $L^* = L$.

14 (a) $S \to aSb \mid Sb \mid b$

(b) $S \to aSbb \mid \lambda$

(c) $S \to aaA, \quad A \to aAb \mid ab$

(d) $S \to aaaA, \quad A \to aAb \mid \lambda$

(e) $S \to AB, \quad A \to aAb \mid Ab \mid b, \quad B \to aBbb \mid \lambda$

(f) $S \to A \mid B, \quad A \to aAb \mid Ab \mid b, \quad B \to aBbb \mid \lambda$

(g) $S \to AAA, \quad A \to aAb \mid Ab \mid b$

(h) $S \to SA \mid \lambda, \quad A \to aAb \mid Ab \mid b$

15 (a) $S \to SS \mid aaa \mid \lambda$

(b) $S \to Saaa \mid aa \mid a$

(c) $S \to Sa^6 \mid a^5 \mid a^4 \mid a^3 \mid a^2$

(d) $S \to Sa^6 \mid a^5 \mid a^4 \mid a^2 \mid a \mid \lambda$

16. $S \to aSa \mid bSb \mid aa \mid bb$

17. $L(G) = \{\varphi a \varphi^T : \varphi \in \{a,b\}^*\}$ where $\overline{\varphi}^R = (\varphi$ with all a's replaced by b's and all b's replaced by a's$)^R$

18. (a) $S \to aA \mid AS, \quad A \to AA \mid aAb \mid bAa \mid \lambda$

(b) $S \to aS \mid AS \mid aA, \quad A \to AA \mid aAb \mid bAa \mid \lambda$

(c) $S \to abSa \mid aaSb \mid bSaa \mid SS \mid \lambda$

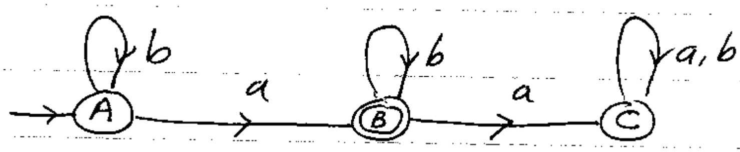
21. No. If $G_1 := S \to aSb \mid ab \mid \lambda$ and $G_2 := S \to aAb \mid ab, \; A \to aAb \mid \lambda$ then $\lambda \in L(G_1)$ but $\lambda \notin L(G_2)$. Actually $L(G_1) = \{a^n b^n : n \geq 0\}$ and $L(G_2) = \{a^n b^n : n \geq 1\}$

22. The only new production is $S \to SSS$, but we can simulate this from Ex. 1.12 by $S \Rightarrow S\underset{\sim}{S} \Rightarrow S\underset{\sim}{SS}$.

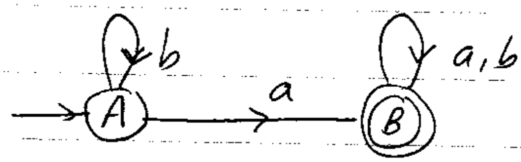
SECTION 2.1

#1. 0001 and 01001 will be accepted
0000110 will not be accepted

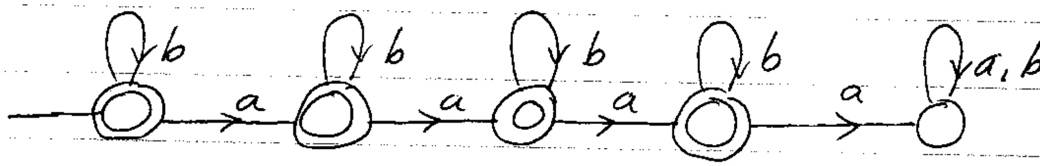
#2. (a)



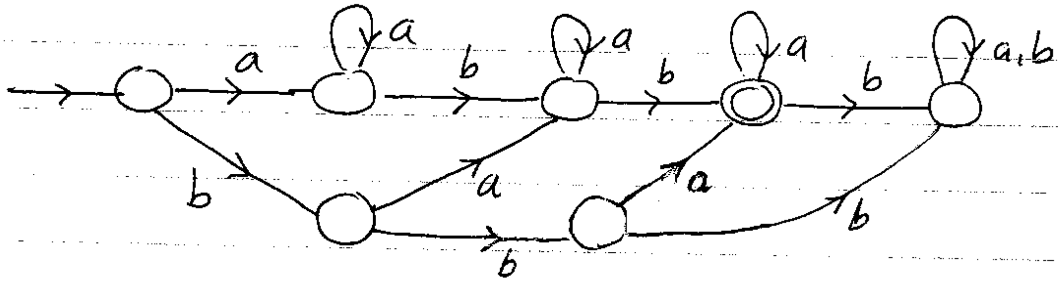
(b)



(c)



(d)



#3. Let $\varphi \in \bar{L}$. Then φ will be rejected by M. So when we input φ in M we will end up at q_0, q_1 or q_2. Since these are accepting states in M^c, φ will be accepted by M^c. So $\varphi \in L(M^c)$

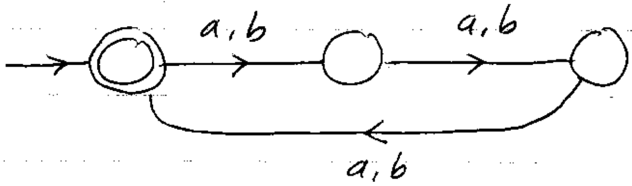
Now let $\varphi \in L(M^c)$. Then φ will be accepted by M^c. So when we enter φ in M^c, we will end up at q_0, q_1 or q_2. But these are rejecting states in M. So φ will be rejected by M. So $\varphi \notin L(M) = L$ So $\varphi \in \bar{L}$. Hence $\bar{L} = L(M^c)$.

SECTION 2.1

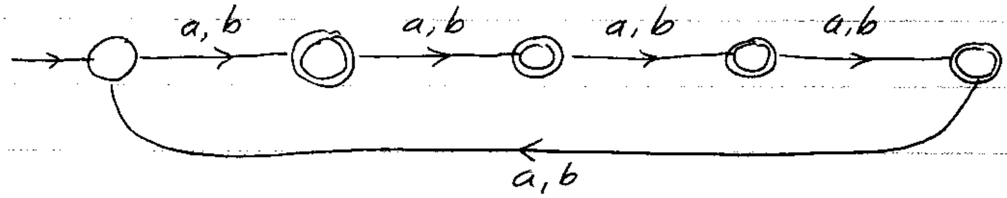
#4. Let $\varphi \in L(M)$. Then $\delta^*(q_0, \varphi) \in F$. Now

$$\begin{aligned} \varphi \in \overline{L(M)} &\iff \varphi \notin L(M) \\ &\iff \delta^*(q_0, \varphi) \notin F \\ &\iff \delta^*(q_0, \varphi) \in Q - F \\ &\iff \varphi \in L(\hat{M}) \end{aligned}$$

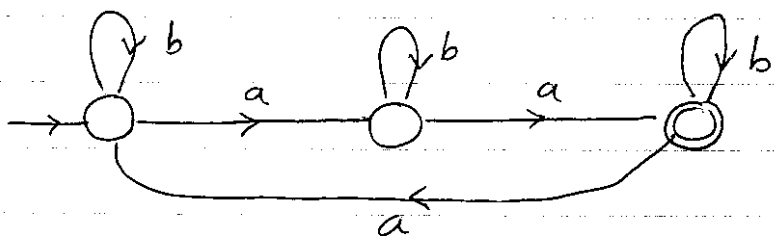
#7 (a)



(b)



(c)

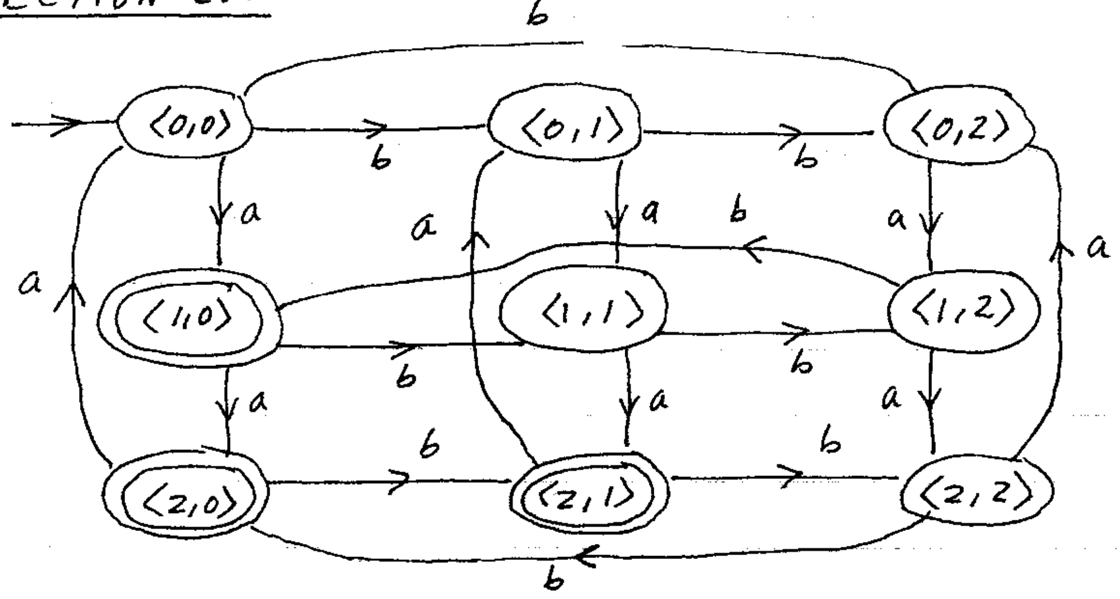


(d) Use the 9 states $\{\langle 0,0\rangle, \langle 0,1\rangle, \langle 0,2\rangle, \langle 1,0\rangle, \ldots, \langle 2,2\rangle\}$ $= \{0,1,2\} \times \{0,1,2\}$ to keep track of $n_a(\omega)$ & $n_b(\omega)$. The first component will be $n_a(\omega)$ (mod 3) and the second will be $n_b(\omega)$ (mod 3).

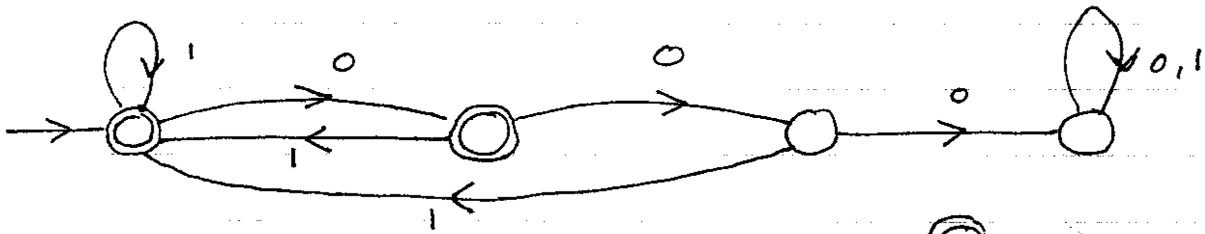
$\langle 0,0\rangle$ will be the starting state bec. $n_a(\lambda) = 0 = n_b(\lambda)$. $\langle 1,0\rangle$, $\langle 2,0\rangle$ & $\langle 2,1\rangle$ will be accepting states bec. $n_a(\omega) > n_b(\omega)$ (mod 3) for these states.

SECTION 2.1

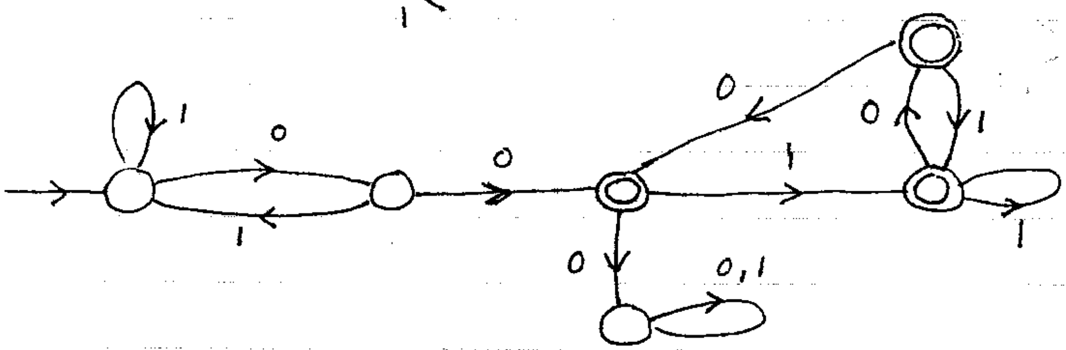
7(d)



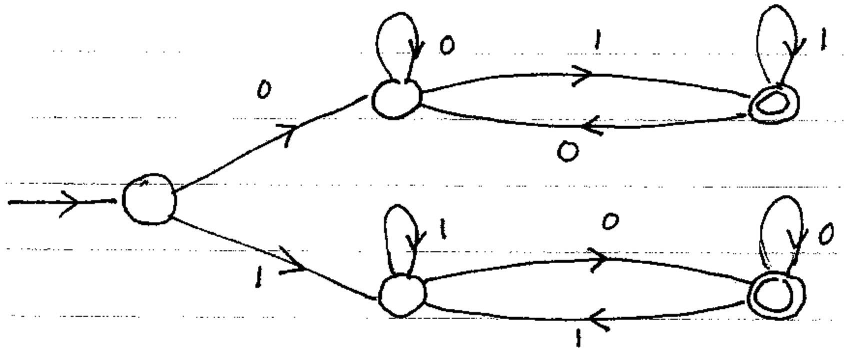
9(a)



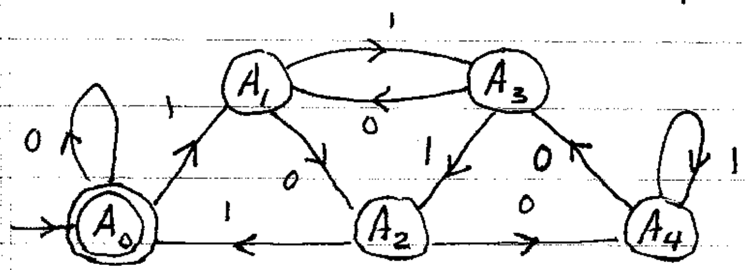
(b)



(c)



10.



Let A_i keep track of the binary value of φ (mod 5) where φ = input string so far. Initially $\varphi = \lambda$.

SECTION 2.1

#10 Example: 1 1 1 0 0 1

A_0 A_1 A_3 A_2 A_4 A_3 A_2

Successive inputs: $(\lambda)_2 = 0 \rightarrow A_0$

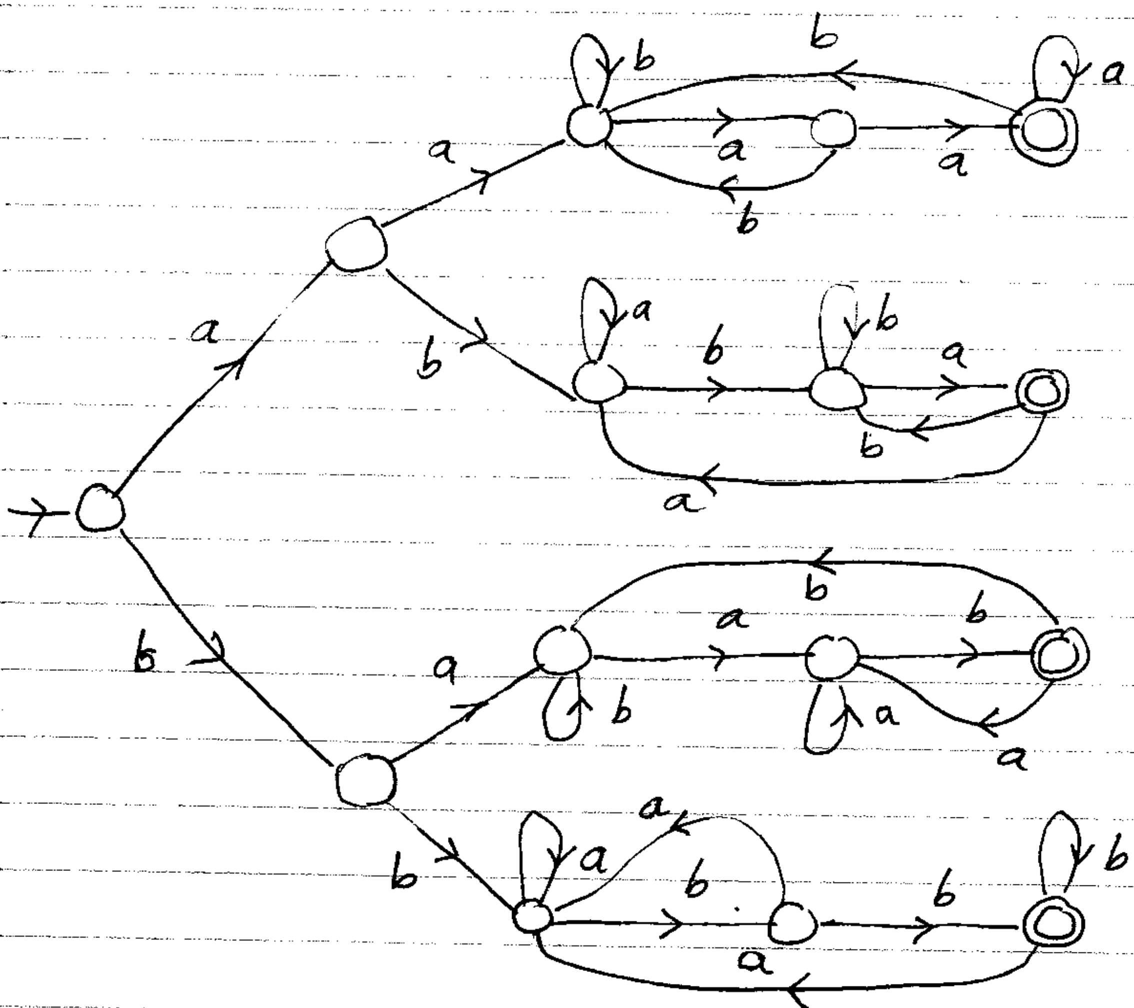
$(1)_2 = 1 \rightarrow A_1$ $(11)_2 = 3 \rightarrow A_3$ $7 = (111)_2 \rightarrow A_2$

$1110 = 14 \rightarrow A_4$ $(11100)_2 = 28 \rightarrow A_3$ $111001 = 57 \rightarrow A_2$

Note: $(\varphi 0)_2 = 2(\varphi)_2 + 0$

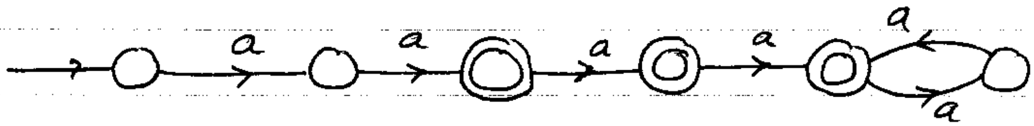
$(\varphi 1)_2 = 2 \cdot (\varphi)_2 + 1$

#11



SECTION 2.2

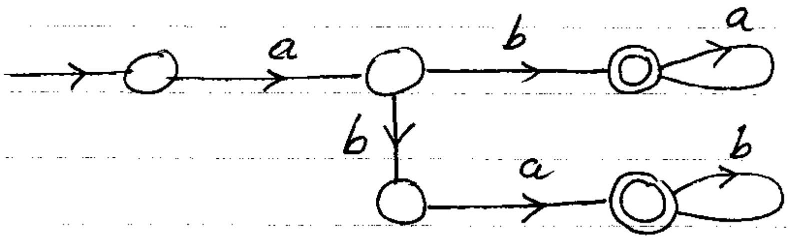
#2



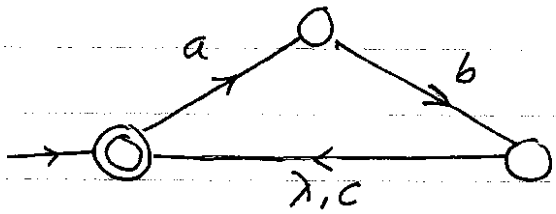
#4 $\delta^*(q_0, a) = \{q_0, q_1, q_2\}$
$\delta^*(q_1, \lambda) = \{q_0, q_1, q_2\}$

#5 $\delta^*(q_0, 1010) = \{q_0, q_2\}$
$\delta^*(q_1, 00) = \emptyset$

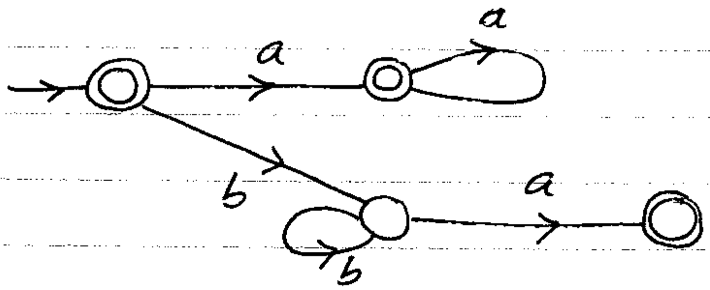
#7



#8

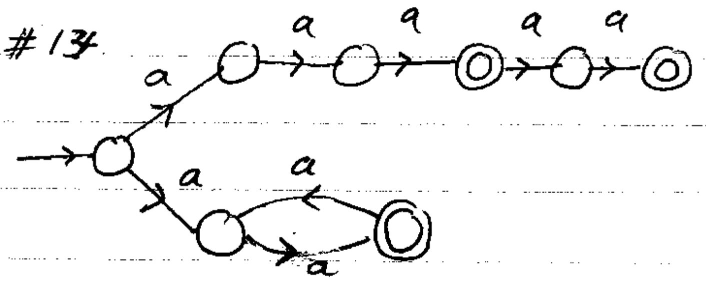


#11

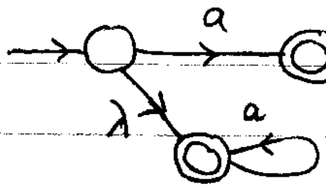


#12 01001 , 000

#13

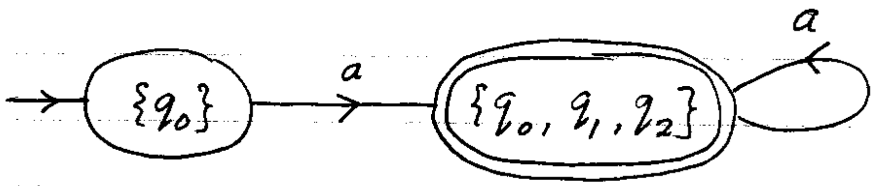


#16

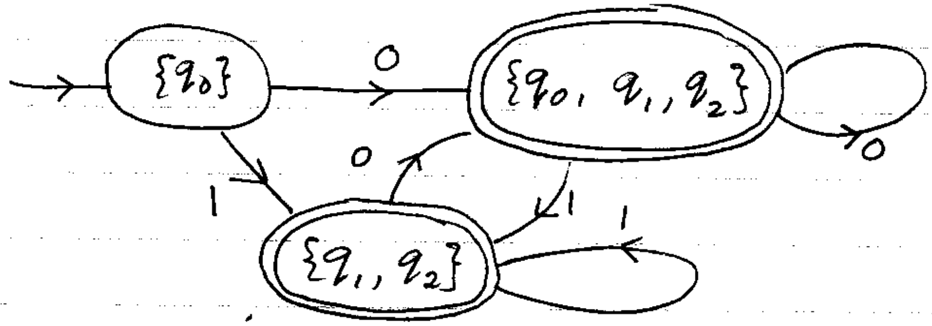


SECTION 2.3

#1



#3

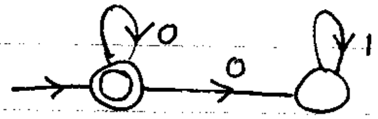


#5 Yes; <u>Hint</u>: $L(M) = \{\varphi \in \Sigma^* : \delta^*(q_0, \varphi) \cap F \neq \emptyset\}$

$\overline{L(M)} = \{\varphi \in \Sigma^* : \delta^*(q_0, \varphi) \cap F = \emptyset\}$

#6 No. The complement of $L(M)$ can include strings which are rejected by M, not because they drive M into a non-accepting state, but because M lacks transitions to process them.

M:



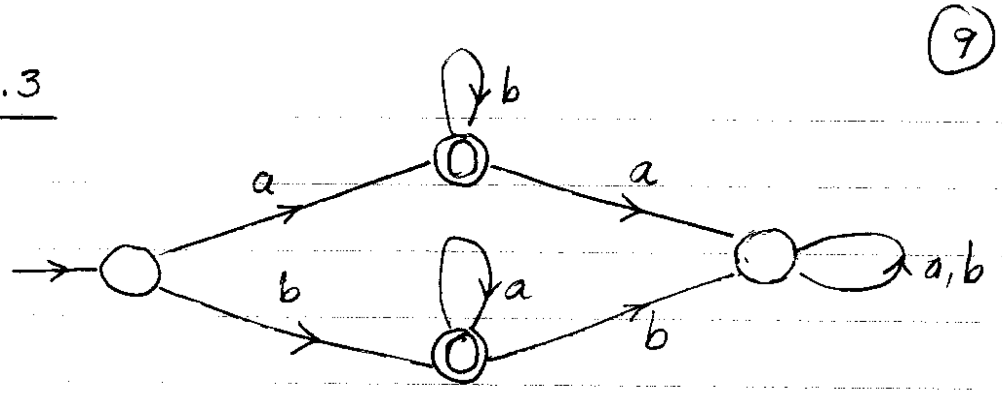
$\Sigma = \{0, 1\}$, $\quad 1 \notin L(M)$

So $1 \in \overline{L(M)}$ but $1 \notin \{w \in \Sigma^* : \delta^*(q_0, w) \cap Q - F \neq \emptyset\}$ because $\delta^*(q_0, 1) = \emptyset$.

#7 (a) Hint: Let $M = \langle Q, \Sigma, \delta, q_0, F \rangle$. Change M to M' by adding λ-transitions from all accepting states in M to a new accepting state in M' and make all the accepting states in M, non-accepting in M'. Then check that $L(M') = L(M)$.

SECTION 2.3

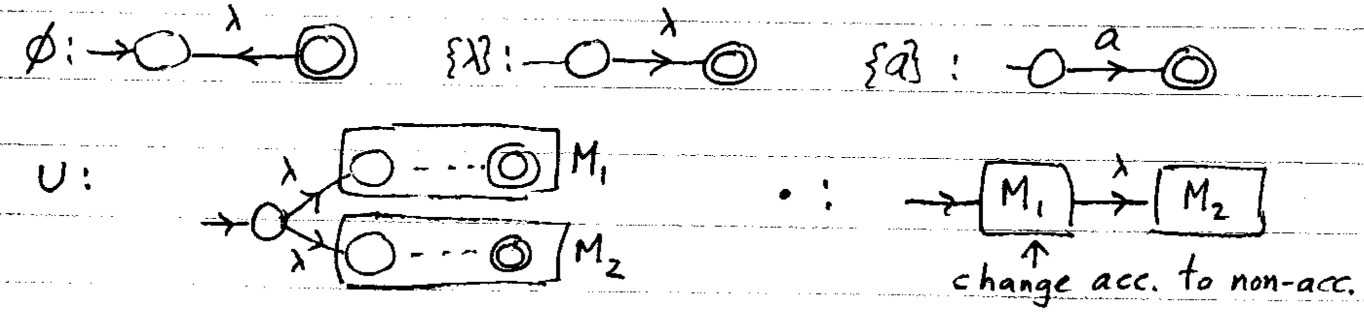
#7(b) No.



#10 Yes. First introduce a new initial state q_0' and add λ-transitions from it to all the initial states of M to get a new machine M'. In M', q_0' will be the only initial state. Now M' will be an nfa, so we can convert it into a dfa M''. The dfa M'' will have only one initial state and will be equivalent to our original M which had multiple initial states.

#11 (a) One way is to just show that every finite language can be described by a reg. expression.
Ex. $\{ab, baa, abab\} = (ab + baa + abab)$

(b) We can also show that we can find an nfa which accepts any finite language by starting with simple nfa's & using union & concatenation.



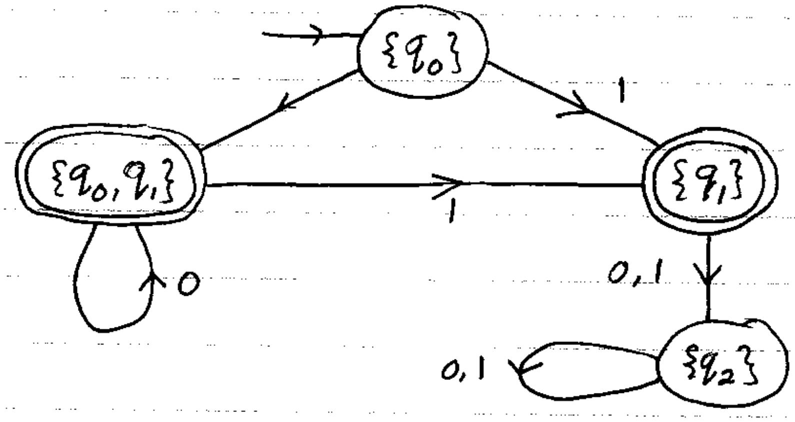
SECTION 2.4

#1 P_0 : $\underbrace{\{\{q_0\}, \{q_2\}, \emptyset\}}_{\text{non-accepting}}$, $\underbrace{\{\{q_1\}, \{q_0, q_1\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}}_{\text{accepting states}}$

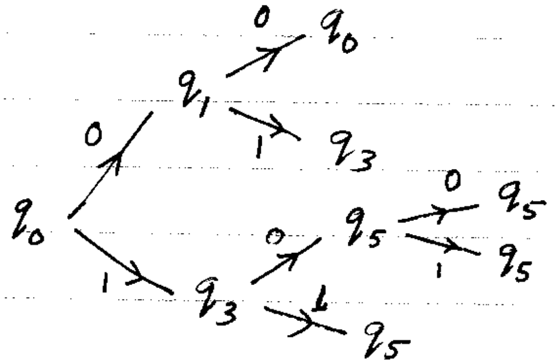
P_1 : $\{\{q_0\}\}, \{\{q_2\}, \emptyset\}, \{\{q_1\}, \{q_1, q_2\}\}, \{\{q_0, q_1\}, \{q_0, q_1, q_2\}\}$

P_2 : $= P_1$

Reduced dfa:

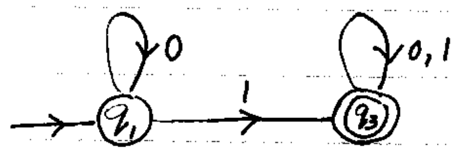


#4. (a)



q_2 & q_4 are inaccessible

(b) P_0 : $\{q_0, q_1\}$ $\{q_3, q_5\}$ M^R:



P_1 : $\{q_0, q_1\}$ $\{q_3, q_5\} = P_0$

#6 The conjecture is true. Suppose not. Then we can find a minimal DFA M for L such that $\hat{M}$ is not minimal for $\bar{L}$. Now minimize $\hat{M}$ to get a smaller DFA N for $\bar{L}$. By switching accepting & non-acc. states in N, we will get a DFA $\hat{N}$ for $\bar{\bar{L}} = L$, contradicting the minimality of M. So result is true.