SECTION 3.1

#1. $b, ab, ba, bb, aab, aba, abb, baa, bab, bba, bbb$

#2. Yes.

#3 (a) see page 12 for detailed solution

(b) $(\underline{1} + \underline{01})^* (\underline{0} + \underline{1}^*) . \lambda^*$

(c) $(\underline{1} + \underline{01})^* (\underline{0} + \underline{\lambda}) . \underline{\lambda}$

#4 $\underline{aaaa}^* . (\underline{bb})^*$

#5 $(\underline{aa})^* (\underline{bb})^* \; + \; \underline{a} (\underline{aa})^* \underline{b} (\underline{bb})^*$

#6 (a) $\underline{aaaaa}^* . (\underline{\lambda} + \underline{b} + \underline{bb} + \underline{bbb})$

(b) $(\underline{\lambda} + \underline{a} + \underline{aa} + \underline{aaa}) . (\underline{\lambda} + \underline{b} + \underline{bb} + \underline{bbb})$

#7 (a) $\{\lambda\}$ (b) ϕ

#8 Set of all strings that consists of a "b" surrounded by an even no. of a's on both sides or an odd number of a's on both sides.

$\{a^m b a^n \; : \; m - n \equiv 0 \pmod 2\}$

#9 $(\underline{ba} + \underline{a})^* . \underline{b} . (\underline{b} + \underline{a})^*$

#10 We split $L = \{a^n b^m : n.m \geq 3, n \geq 1 \; \& \; m \geq 1\}$ into three pieces according to the conditions

$n \geq 3, \; m = 1$ to get $\{a^n b : n \geq 3\}$

$n \geq 2, \; m = 2$ " $\{a^n bb : n \geq 2\}$

$n \geq 1, \; m \geq 3$ " $\{a^n b^m : n \geq 1, m \geq 3\}$

Then answer is $\underline{aaaa}^*\underline{b} + \underline{aaa}^*\underline{bb} + \underline{aa}^*\underline{bbbb}^*$

SECTION 3.1 (Number 3 redone)

#3 (a) Let $R_1 = (\underline{1}+\underline{01})^* \cdot (\underline{0}+\underline{1}^*)$ and
$R_2 = (\underline{1}+\underline{01})^* \cdot (\underline{0}+\lambda)$
be the expression from Example 3.6.

Clearly $L(R_2) \subseteq L(R_1)$ because $\underline{0}+\underline{\lambda} \subseteq \underline{0}+\underline{1}^*$
$(\lambda \in \underline{1}^*)$.

Now let $\varphi \in L(R_1)$. Then
$\varphi = \alpha.\beta$ where $\alpha \in L((\underline{1}+\underline{01})^*)$ and
$\beta = 0$ or $\beta \in L(\underline{1}^*)$

But if $\beta = 0$, then $\varphi \in L(R_2) = L\left((\underline{1}+\underline{01})^* \cdot (\underline{0}+\underline{\lambda})\right)$
And if $\beta \in L(\underline{1}^*)$, then $\beta = 1^n$ for some
$n \geq 0$. So

$$\begin{aligned} \varphi &= \alpha.1^n \quad \text{with} \quad \alpha \in L((\underline{1}+\underline{01})^*) \\ &= 1\text{'s } \& \ (01)\text{'s followed by } n \ 1\text{'s} \\ &\in L\ (\underline{1}+\underline{01})^* = L((\underline{1}+\underline{01})^* \cdot \underline{\lambda}) \\ &\subseteq L\ (\underline{1}+\underline{01})^* \cdot (\underline{0}+\underline{\lambda}) \end{aligned}$$

$\therefore$ $L(R_1) \subseteq L(R_2)$. Hence $L(R_1) = L(R_2)$

(b) $(\underline{01}+\underline{1})^* \cdot (\underline{\lambda}+\underline{0})$ and $(\underline{1}+\underline{01})^* \cdot (\underline{\lambda}+\underline{0}+\underline{1})$
are two other expressions which are equivalent to $(\underline{1}+\underline{01})(\underline{0}+\underline{1}^*)$.

#12 $L = \underline{a}\,\underline{b}\underline{b}\underline{b}\underline{b}^*(\underline{a}+\underline{b})(\underline{a}+\underline{b})^*$

$L =$ set of all strings which consists of an even no. of a's followed by an odd no. of b's

$= \{a^{2n} b^{2m+1} : n \geq 0, m \geq 0\}$

$L^c =$ set of all strings of the form $a^n b^m$ with n odd or m even, or of strings with a "b" in front of an "a"

$= \{a^{2n+1} b^m : n \geq 0, m \geq 0\} \cup \{a^n b^{2m} : n \geq 0, m \geq 0\}$
$\cup \{\text{anything} . ba . \text{anything}\}$

An expression for L^c is now easily seen to be

$$\underline{a}(\underline{a}\underline{a})^*\underline{b}^* + \underline{a}^*(\underline{b}\underline{b})^* + (\underline{a}+\underline{b})^* . \underline{b}\underline{a} . (\underline{a}+\underline{b})^*$$

#13. $\underline{a}\underline{a}(\underline{a}+\underline{b})^*\underline{a}\underline{a} + \underline{a}\underline{b}(\underline{a}+\underline{b})^*\underline{a}\underline{b} + \underline{b}\underline{a}(\underline{a}+\underline{b})^*\underline{b}\underline{a} + \underline{b}\underline{b}(\underline{a}+\underline{b})^*\underline{b}\underline{b}$.

#14. A silly question. The answer is $(\underline{a}+\underline{b})^*$.

#15. $(\underline{1} + \underline{0}\underline{1})^* . \underline{0}\underline{0} . (\underline{1} + \underline{1}\underline{0})^*$

#16 (a) $(\underline{b}+\underline{c})^*\underline{a}(\underline{b}+\underline{c})^*$

(b) Look at the four cases: no a's, one a, two a's and three a's. With these cases we get:

$(b+c)^* + (b+c)^* . a . (b+c)^* + (b+c)^* . a . (b+c)^* . a . (b+c)^*$
$+ (b+c)^* . a . (b+c)^* . a . (b+c)^* . a . (b+c)^*$

(c) Look at six cases: …a…b…c…, …a…c…b…, …b…a…c…, …b…c…a…, …c…a…b…, …c…b…a… .
Now insert $(a+b+c)^*$ for the dots.

Section 3.1 exactly 2 00's

$(1+01)^*.(000+00(1+10)^*.100).(1+10)^*$ (14)

\# 17. (a) $(0+1)^* . 01$ (f) $(0+\lambda)(1+00+000)^*(0+\lambda)$

(b) $(\lambda+0+1) + (0+1)^*.(00+10+11)$

(c) $(1^*01^*01^*)^* + 1^*$ is one answer

$(1+01^*0)^*$ is another answer

(d) $(0+1)^*(000 + 00(0+1)^*.00)(0+1)^*$

(e) $(01+1)^*(\lambda+0+00+000+00.(10+1)^*100)(10+1)^*$

\# 20 (a) Clearly $L(r_1^*) \subseteq L((r_1^*)^*)$. Now let $\varphi \in L((r_1^*)^*)$. Then

φ = a string of strings from r_1^*

= a string of strings of strings from r_1

= a string of strings from r_1

$\in L(r_1^*)$

$\therefore L((r_1^*)^*) \subseteq L(r_1^*)$

Thus $L((r_1^*)^*) = L(r_1^*)$ i.e. $(r_1^*)^* \equiv r_1^*$

(b) Again clearly $L((r_1+r_2)^*) \subseteq L(r_1^*(r_1+r_2)^*)$ because $\lambda \in L(r_1^*)$.

Now let $\varphi \in L(r_1^*(r_1+r_2)^*)$. Then

$\varphi = \alpha . \beta$ with $\alpha \in L(r_1^*)$ & $\beta \in L((r_1+r_2)^*)$

$= \alpha_1\alpha_2\cdots\alpha_m . \beta_1\beta_2\cdots\beta_n$ with the α_i's in $L(r_1)$ and β_j's in $L(r_1+r_2)$

$= \alpha_1\alpha_2\cdots\alpha_m\beta_1\cdots\beta_n$ with the α_i's and β_j's in $L(r_1+r_2)$

$\in L((r_1+r_2)^*)$

$\therefore L(r_1^*(r_1+r_2)^*) \subseteq L((r_1+r_2)^*)$.

So $L(r_1^*(r_1+r_2)^*) = L((r_1+r_2)^*)$ i.e. $r_1^*(r_1+r_2)^* \equiv (r_1+r_2)^*$.

#20 (c) First observe that $L(r_1^* r_2^*) \supseteq L(r_1 + r_2)$ bec. $\lambda \in L(r_1^*)$ and $\lambda \in L(r_2^*)$. So $L((r_1^* r_2^*)^*) \supseteq L((r_1+r_2)^*)$

Now let $\varphi \in L((r_1^* r_2^*)^*)$. Then

$\varphi =$ a string of things which are made of a string from r_1^* followed by a string from r_2^*

$=$ a string of things which are made up of strings of things in r_1 followed by strings of things in r_2.

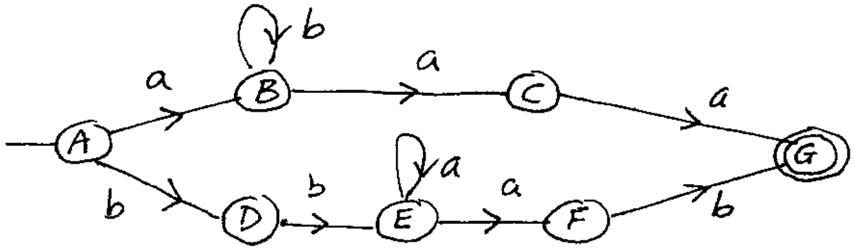
$=$ a string of things from either r_1 or r_2

$\in L((r_1 + r_2)^*)$.

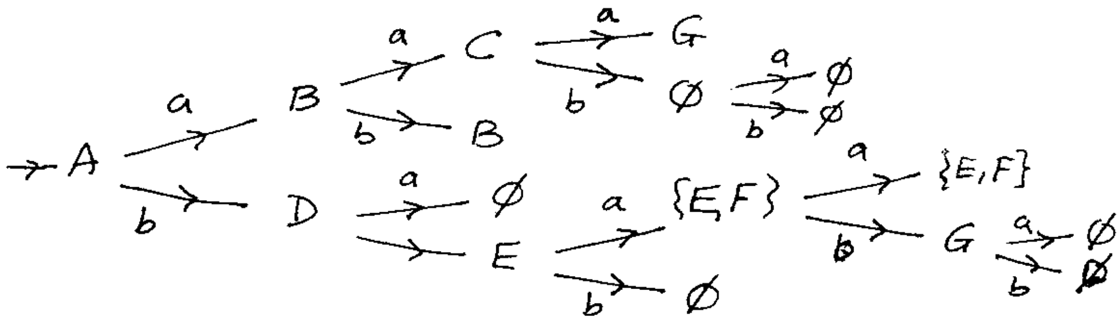
So $L((r_1^* r_2^*)^*) \subseteq L((r_1+r_2)^*)$. Thus $L((r_1^* r_2^*)^*) = L((r_1+r_2)^*)$ i.e. $(r_1+r_2)^* \equiv (r_1^* r_2^*)^*$

d) false. $(\underline{a}\cdot\underline{b})^* \neq \underline{a}^*\cdot\underline{b}^*$ because $abab \in (\underline{a}\cdot\underline{b})^*$ but $abab \notin \underline{a}^*\cdot\underline{b}^*$.

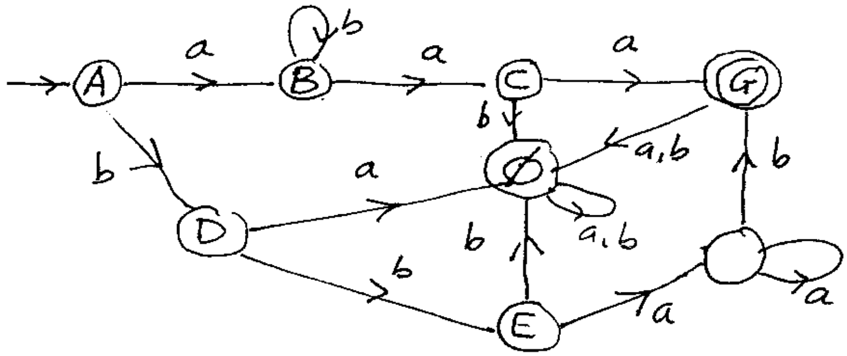
#1 $M =$



#2. (a) First convert M into a dfa M_D

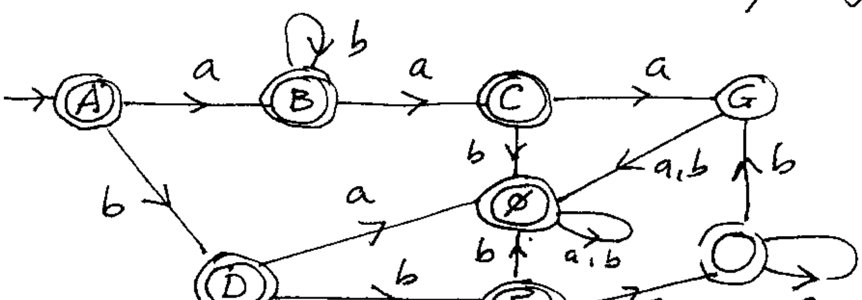


$M_D =$

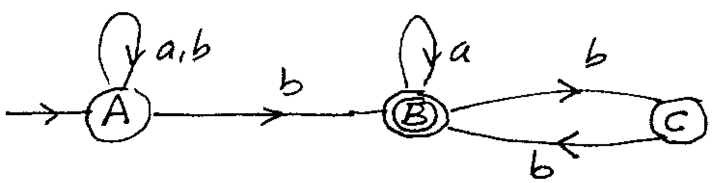


(b) Then switch accepting & non-accepting states

$M_D^C =$

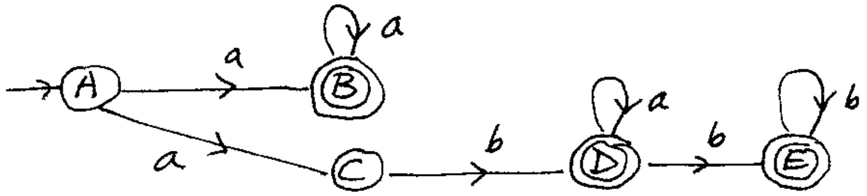


#3.

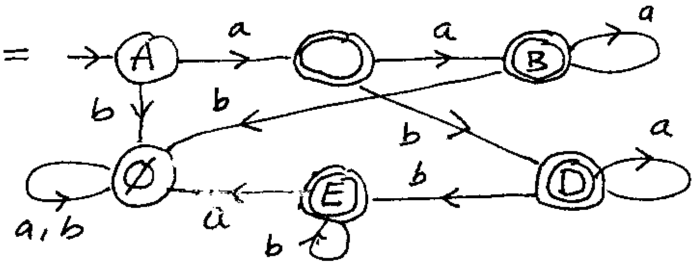


#4. First find an nfa and then convert it into a dfa

(a) M =
nfa :

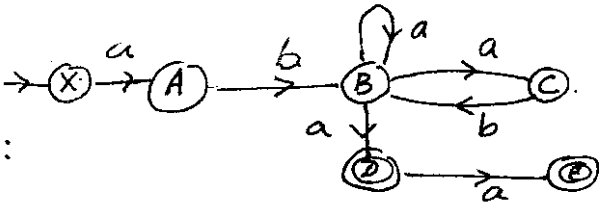


M_D =

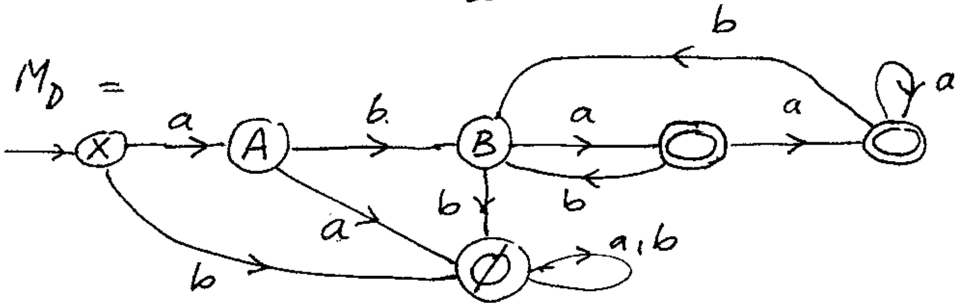


= dfa

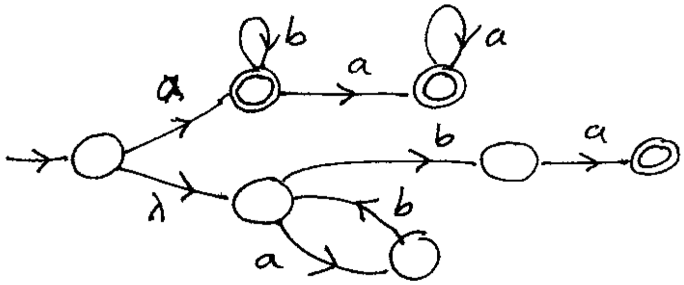
(b) M =
nfa:



dfa: M_D =

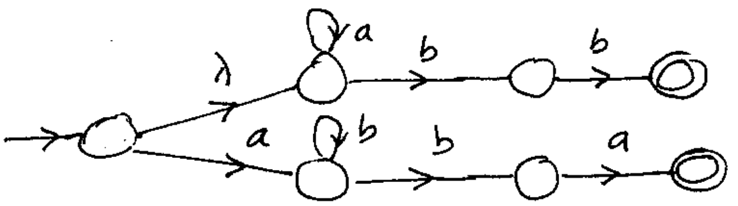


#5. (a) nfa,
M =



Now convert M into a dfa M_D.

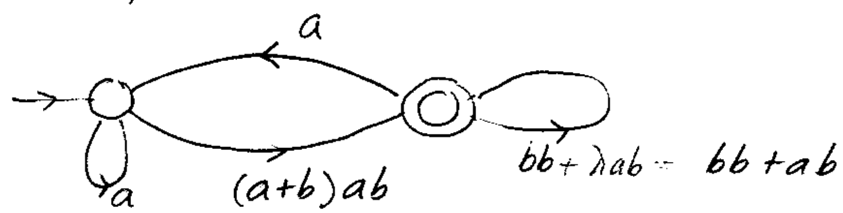
#7 nfa M =



Now convert M into a dfa M_D and then minimize M_D by using the Partition Algorithm.

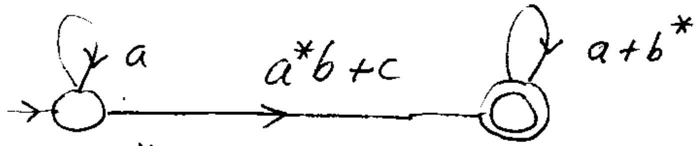
SECTION 3.2 p. 88

#8 (a)



(b) $$\underbrace{a}_{r_1}{}^{*} \cdot \underbrace{(a+b)ab}_{r_2} \cdot \Big(\big(\underbrace{bb+ab}_{r_4} \big) + \underbrace{a}_{r_3} \cdot \underbrace{a^*(a+b)ab}_{r_1^* \cdot r_2} \Big)^*$$

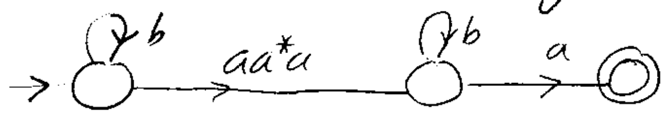
#9



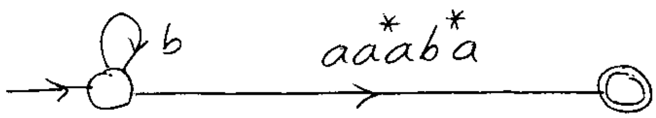
$$L(M) = \underbrace{a}_{r_1}{}^{*} \cdot \underbrace{(a^*b+c)}_{r_2} \cdot \underbrace{(a+b^*)}_{r_4}{}^{*}$$

#10 (a) Since this is an easy example, you can instantly see that $L(M) = b^*aa^*ab^*a$. But let's see how the algorithm proceeds

M_1



M_2



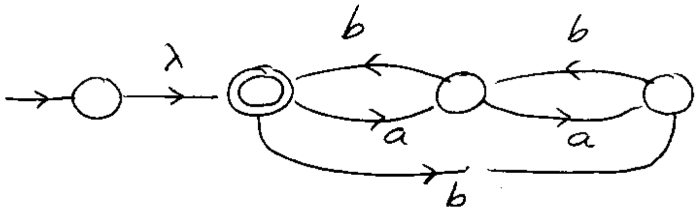
$$L(M) = \underbrace{b}_{r_1}{}^{*} \underbrace{aa^*ab^*a}_{r_2} \cdot \underbrace{(\emptyset)}_{r_4+r_3r_1^*r_2}{}^{*} \leftarrow r_1^* r_2 \big(\underbrace{r_4}_{\emptyset} + \underbrace{r_3 r_1^* r_2}_{=\emptyset} \big)^*$$

bec. $r_3 = \emptyset$

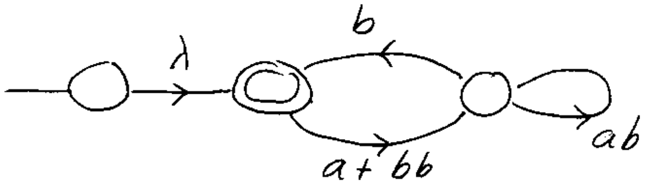
$$= b^*aa^*ab^*a \cdot \lambda = b^*aa^*ab^*a.$$

SECTION 3.2 p. 89

\# 10(b) M_0 :



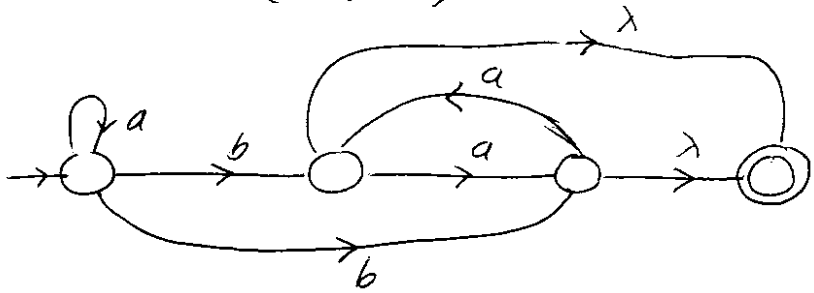
M_1 :



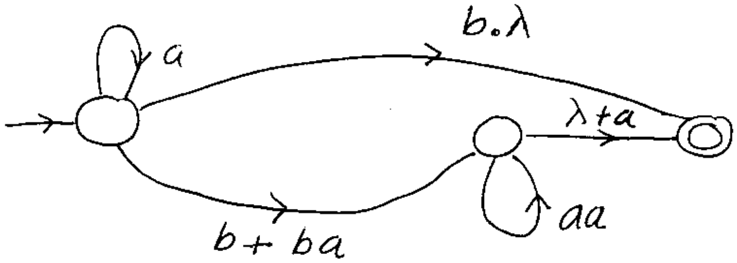
M_2 :

$$L(M) = \lambda.\left((a+bb).(ab)^*.b\right)^*$$

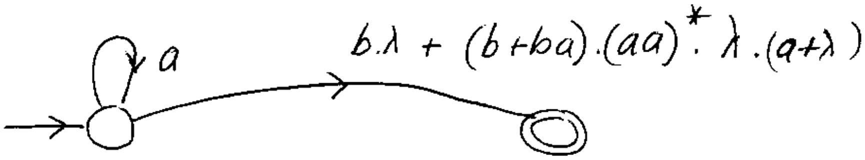
(c) M_0 :



M_1 :



M_2 :



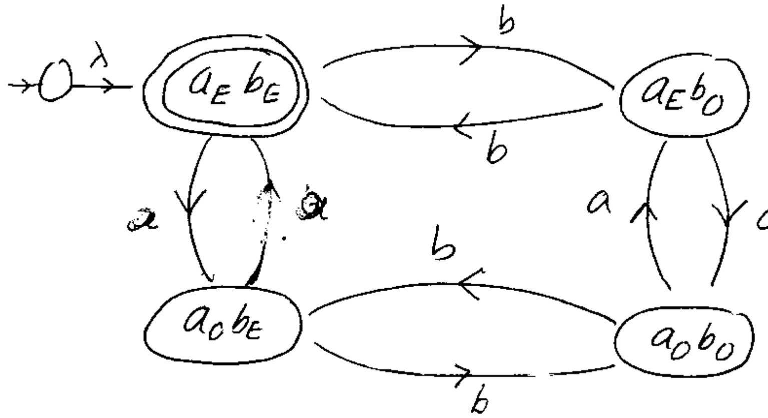
$$L(M) = a^*.\left(b + (b+ba).(aa)^*.\lambda.(a+\lambda)\right)$$

SECTION 3.2 p. 89

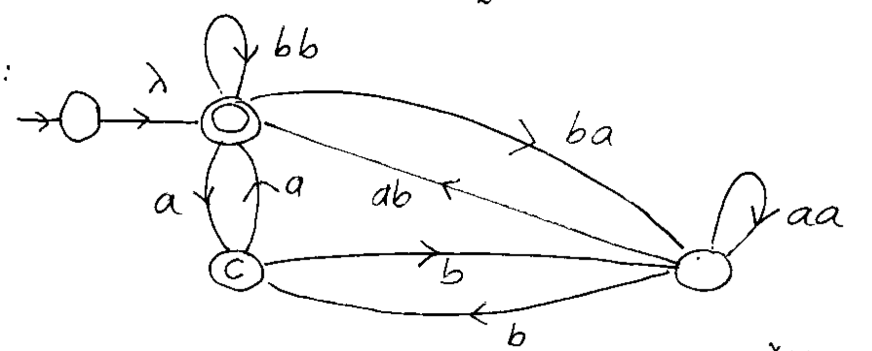
#13 (a) First find an nfa and then find the regular expression from your nfa.

E = even
O = odd

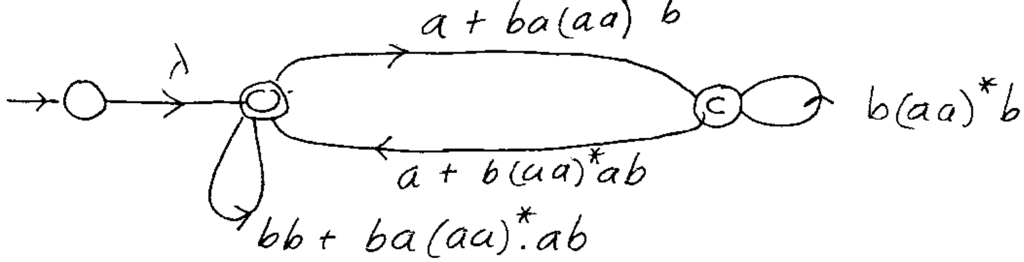
M_0:



M_1:

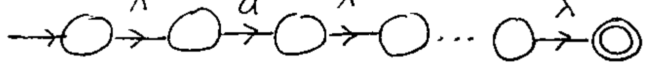


M_2:

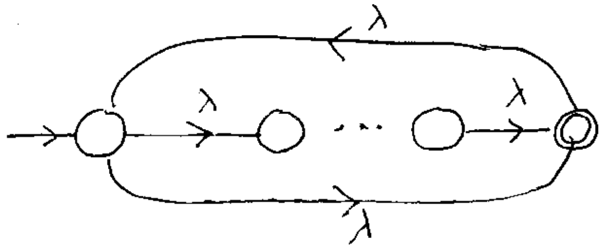


$$L(M) = \lambda\cdot\left(bb + ba\cdot(aa)^*\cdot ab + \left(a+ba(aa)^*b\right)\cdot\left(b(aa)^*b\right)^*\cdot\left(a+b(aa)^*ab\right)\right)^*$$

#18 (a) $L(a\emptyset)$:



(b) $L(\emptyset^*)$

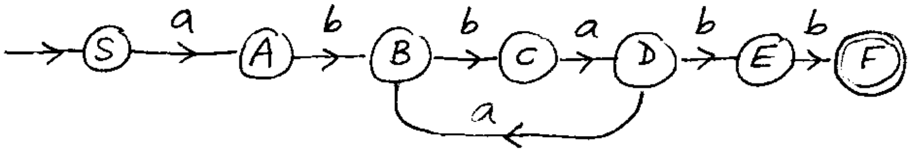


.... = no connection.
= dead wire

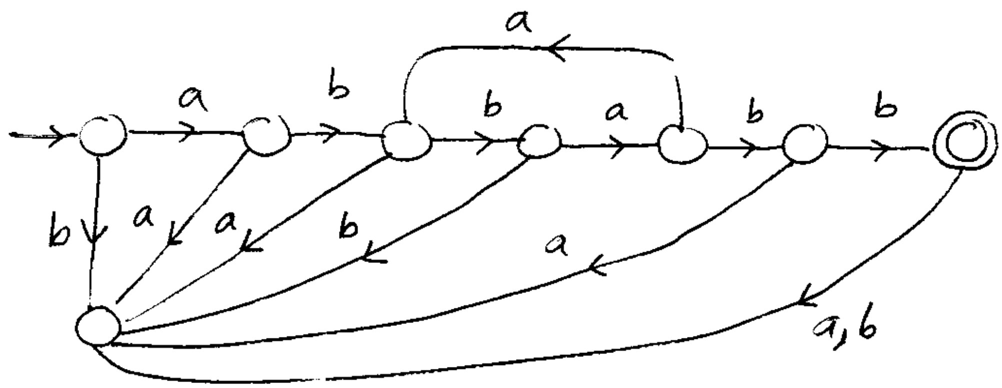
SECTION 3.3 p. 96

\# 1. First find an nfa that accepts $L(G)$ and then convert your nfa into a dfa.

nfa :



dfa :



\# 2. $S \to aA$, $A \to aA$, $A \to B$, $B \to abB$, $B \to aB$, $B \to \lambda$.

\# 3. The simplest thing to do is to find $L(G)$ in Exercise 1 and then find a left-linear grammar for $L(G)$.

$L(G) = abba \cdot (aba)^* bb$

Left-Lin. Grammar is $S \to Abb$, $A \to Aaba \mid abba$

\# 4. (a) RLG : $S \to aaA$, $A \to aA \mid B$, $B \to bB \mid bbb$

(b) LLG : $S \to Bbbb$, $B \to Bb \mid A$, $A \to aa \mid Aa$

\# 6. RLG : $S \to aaB$, $B \to bB$, $B \to ab$, $B \to abS$

$S \to \lambda$.

Note : After you see the scheme, you will then realize that the 3rd production is not needed. So a better ans. is : $S \to aaB \mid \lambda$, $B \to abS \mid bB$.

SECTION 3.3 p. 99

#7. Let $L_i = \{\varphi \in \{a,b\}^* : \varphi$ has exactly i a's$\}$ for $i = 0, 1, 2$ & 3. Find regular grammars G_i for L_i and then find a grammar G which gives the union.

G: $S \to S_0/S_1/S_2/S_3$, $S_0 \to bS_0/\lambda$,
$S_1 \to bS_1/aA$, $A \to bA/\lambda$,
$S_2 \to bS_2/aB$, $B \to bB/aC$, $C \to bC/\lambda$
$S_3 \to bS_3/aD$, $D \to bD/aE$, $E \to bE/aF$
$F \to bF/\lambda$.

#10. $S \to Aab$, $A \to Ab$, $A \to aa$, $A \to Saa$, $S \to \lambda$
As in exercise 5 you don't really need the 3rd production.

#11 Let $L_1 = \{a^n b^m$: n & m are even$\}$
$L_2 = \{a^n b^m$: n & m are odd$\}$
Then $L = L_1 \cup L_2$. Find Regular grammars for L_1 & L_2 & then do the union thing:
$S_2 \to aaS_2/aB$, $B \to bbB/b$
$S \to S_1/S_2$ $\quad$ $S_1 \to aaS_1/A$, $A \to bbA/\lambda$

#12 $S \to aA/bB/\lambda$, $A \to aS/bC$, $B \to bS/aC$, $C \to bA/aB$.

#13 Hint: Find the corresponding nfa then convert.
(a) $S \to \lambda/bB/aD$, $B \to bS/aC$, $C \to aB/bD$
$D \to aS/bC$