1. (4 pts.) (a) Find a parametric equation for the line through
\[ \mathbf{a} = \begin{bmatrix} -3 \\ 5 \end{bmatrix} \] and parallel to \( \mathbf{b} = \begin{bmatrix} 23 \\ -5 \end{bmatrix} \).

(b) Find a parametric equation for the line through \( \mathbf{a} \) and \( \mathbf{b} \), where
\[ \mathbf{a} = \begin{bmatrix} -3 \\ 5 \end{bmatrix} \] and \( \mathbf{b} = \begin{bmatrix} 23 \\ -5 \end{bmatrix} \).

2. (6 pts.) Using complete sentences and appropriate notation, define each of the items below.

(a) Linear Combination

(b) Span\( \{\mathbf{v}_1, \ldots, \mathbf{v}_m\} \)

(c) Linear Independent
3. (2 pts.) Write the general solution of the equation
\[ x_1 - 6x_2 + 8x_3 = 25 \]
in parametric form.

4. (2 pts.) The general solution of a certain matrix equation
\[ Ax = b \] with \( b \neq 0 \) is given in parametric vector form as follows:
\[
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 
\end{bmatrix} = \begin{bmatrix}
  -3 \\
  12 \\
  -5 
\end{bmatrix} + x_2 \begin{bmatrix}
  -5 \\
  1 \\
  0 
\end{bmatrix} + x_3 \begin{bmatrix}
  15 \\
  0 \\
  1 
\end{bmatrix}, \text{where } x_2 \text{ and } x_3 \text{ are arbitrary real numbers.}
\]
Give the solution to the corresponding homogeneous equation, \( Ax = 0 \).

5. (4 pts.) Suppose \( A \) is a 5 \( \times \) 3 matrix with 2 pivot elements.
(a) Are the columns of \( A \) linearly independent? Explain.

(b) Does the matrix equation \( Ax = b \) have a solution for every \( b \) in \( \mathbb{R}^5 \)? Explain.

6. (2 pts.) After asserting whether the following proposition is always true or false in at least one case, give a brief justification for or provide a counterexample to it:

If \( \{v_1, v_2, v_3, v_4\} \) is a linear independent set of vectors in \( \mathbb{R}^5 \), then \( \{v_2, v_3\} \) is also linearly independent.