1. (18 pts.) Fill in the following table with the information requested concerning domain, range, and period.

<table>
<thead>
<tr>
<th>Function Name</th>
<th>Domain (in radians)</th>
<th>Range</th>
<th>Period (in radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin(θ)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cos(θ)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tan(θ)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>cot(θ)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sec(θ)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>csc(θ)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. (7 pts.) If the point (2, -5) is on the terminal side of an angle θ, obtain the exact value of each of the six trigonometric functions of θ.

\[
\begin{align*}
\sin(\theta) &= \\
\cos(\theta) &= \\
\tan(\theta) &= \\
\cot(\theta) &= \\
\sec(\theta) &= \\
\csc(\theta) &= 
\end{align*}
\]
3. (6 pts.) Carefully sketch $y = \sin(x)$ through two periods that are symmetric about the origin. Use radian measure and label carefully.

4. (6 pts.) Carefully sketch $y = \cos(x)$ through two periods that are symmetric about the origin. Use radian measure and label carefully.

5. (6 pts.) Carefully sketch $y = \tan(x)$ through two periods. Use radian measure and label carefully.

6. (7 pts.) Carefully sketch $y = \sec(x)$ through two periods. Use radian measure and label carefully.
7. (5 pts.) Suppose $\sec \theta = 4$ and $\tan \theta < 0$. What is the exact value of each of the remaining trigonometric functions?

- $\cos(\theta) = \frac{\sec(\theta)}{2} = \frac{4}{2} = 2$
- $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\sin(\theta)}{2}$
- $\sin(\theta) = \frac{\tan(\theta)}{\sqrt{1 + \tan^2(\theta)}} = \frac{\frac{\sin(\theta)}{2}}{\sqrt{1 + \left(\frac{\sin(\theta)}{2}\right)^2}}$
- $\csc(\theta) = \frac{1}{\sin(\theta)}$ (exact value depends on $\sin(\theta)$)
- $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \frac{2}{\frac{\sin(\theta)}{2}}$

8. (4 pts.) If $\theta = -240^\circ$, what is the radian measure of $\theta$ as an exact multiple of $\pi$??

$\theta = -\frac{4\pi}{3}$

9. (4 pts.) If $\theta = \frac{7\pi}{6}$ in radian measure, what is the value of $\theta$ in degrees??

$\theta = -300^\circ$

10. (4 pts.) If $s = 5$ meters is the length of an arc of a circle of radius $r = 3$ meters subtended by a central angle $\theta$, what is the exact value of $\theta$ in degrees??

$\theta = \frac{s}{r} = \frac{5}{3}$

11. (4 pts.) If $\theta = 61^\circ50'23''$, convert $\theta$ to a decimal in degrees rounded to two decimal places.

$\theta = 61.8395^\circ$

12. (4 pts.) If $\theta = 30.421^\circ$, convert $\theta$ to $D^\circM'S''$ form with the answer rounded to the nearest second.

$\theta = 30^\circ25'12.6''$
13. (6 pts.) Using DeMoivre’s Theorem, find all the complex fourth roots of i. [Don’t confuse fourth roots with fourth powers!!]

14. (6 pts.) Write the equation of a sine function that has all the given characteristics:

Amplitude = 4π  Period = 3  Phase Shift: -(3/4)

15. (6 pts.) Carefully sketch $y = -4\cos(2x + \pi)$ through one period. You will need the amplitude, period, and phase shift to do this. Label very carefully.

16. (7 pts) Obtain the exact value of $\sin(\frac{9\pi}{8})$. Show all the uses of appropriate identities and formulas. [Hints: Reference angle? Quadrant??]

$\sin(\frac{9\pi}{8}) =$
17. (5 pts.) Find the exact value of $\sin^{-1}(\sin(-7\pi/5))$.

$\sin^{-1}(\sin(-7\pi/5)) = $ 

18. (5 pts.) Write $\sin(\cos^{-1}(u) + \sin^{-1}(v))$ as an algebraic expression containing $u$ and $v$.

$\sin(\cos^{-1}(u) + \sin^{-1}(v)) = $ 

19. (5 pts.) Find the exact value of $\tan(2 \cdot \tan^{-1}(3/4))$.

$\tan(2 \cdot \tan^{-1}(3/4)) = $ 

20. (5 pts.) Carefully sketch the graph of $y = \cos^{-1}(x)$. Label very carefully.

21. (5 pts.) Carefully sketch the graph of $y = \tan^{-1}(x)$. Label very carefully.
22. (10 pts.) Establish the following identity.

\[
\frac{1 - \cos(\alpha)}{\sin(\alpha)} = \frac{\sin(\alpha)}{1 + \cos(\alpha)}
\]

23. (15 pts.) Very carefully complete the following derivation of a couple of half angle identities by giving the information requested and performing the computations needed.

(a) Write down the trigonometric identity giving \( \cos(x + y) \) in terms of sums of products of sines and cosines of \( x \)'s and \( y \)'s.

\[
 \cos(x + y) =
\]

(b) Uniformly replace each instance of a \( y \) by \( x \) in the identity above to obtain an identity for \( \cos(2x) \). Use exponentiation to clean up the products appearing.

\[
 \cos(2x) = \cos(x + x) =
\]

(c) Using the Pythagorean identity connecting \( \sin^2(x) \) and \( \cos^2(x) \), replace \( \sin^2(x) \) in the identity for \( \cos(2x) \) and then solve for \( \cos^2(x) \) in terms of what remains.

\[
 \cos^2(x) =
\]

(d) Uniformly replace \( x \) by \( \theta / 2 \) in the identity for \( \cos^2(x) \) that you obtained in part (c) above in order to obtain an identity for \( \cos^2(\theta / 2) \). Clean up the algebra.

\[
 \cos^2(\theta / 2) =
\]

(e) Modify steps (c) and (d) appropriately to obtain a corresponding identity for \( \sin^2(\theta / 2) \). Show your work neatly.

\[
 \sin^2(\theta / 2) =
\]
24. (10 pts.) To measure the height of the top of a distant object, a surveyor takes two sightings of the top of the object 5000 feet apart. The first sighting, which is nearest the object, results in an angle of elevation of 45°. The second sighting, which is most distant from the object, results in an angle of elevation of 30°. If the transit used to make the sightings is 5 feet tall, what is the height of the object. You may assume the object is on a level plane with the base of the transit.

25. (15 pts.) Sketch the given curve in polar coordinates. Do this as follows: (a) Carefully sketch the auxiliary curve, a rectangular graph on the coordinate system provided. (b) Then translate this graph to the polar one.

**Equation:** \( r = 1 + 2 \cdot \sin(\theta) \)

(a) 

(b)
26. (10 pts.) Very carefully sketch the graph of the equation $(x + 1)^2 = -4(y - 2)$ below.

27. (5 pts.) Use the Law of Sines to solve the triangle with $\alpha = 115^\circ$, $\gamma = 30^\circ$, and $c = 3$. You may assume that the standard labelling scheme is used.

28. (10 pts.) Very carefully sketch the graph of the equation $(y + 1)^2 - (x + 2)^2 = 1$ below.