MAP 2302

REVIEW

<u>Directions</u>: If possible, express the solution of each problem as a function(s) in closed form. If this is not possible, resort to a series solution. If neither closed form nor series solutions can be found, write "NOT ABLE TO SOLVE."

1.) $y'' + xy' - 2y = 0$, $y(0) = 1$, $y'(0) = 0$

2.) $\frac{dy}{dx} = \frac{-y}{x}$

3.) $\frac{d^2y}{dx^2} + y = \tan^2 x$

4.) $\frac{d^2y}{dx^2} + \tan y \frac{dy}{dx} + xy = 0$

5.) $\frac{dx}{dt} - 6x + 3y = 8e^t$; $x(0) = -1$

$\frac{dy}{dt} - 2x - y = 4e^t$; $y(0) = 0$

6.) $\frac{dy}{dx} = \frac{2x+2y}{2x-2y}$, $y(1) = 3$

7.) $y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$

8.) $y''' - 4y'' + y' + 6y = 3xe^x + 2e^x - \sin x$

$y(0) = \frac{33}{40}$, $y'(0) = 0$, $y''(0) = 0$

9.) $2xy'' + y' + 2y = 0$, $y(0) = 1$, $y'(0) = 0$

10.) $x^2y'' - 2xy' - 10y = 0$, $y(0) = 5$, $y'(0) = 4$

11.) $x^2y'' + xy' + (x^2 - 1)y = 0$, $y(0) = 1$, $y'(0) = 2$

12.) $xy'' + y' + xy = 0$, $y(0) = 1$, $y'(0) = 2$

Page 2

13.) $\frac{dy}{dx} - \frac{y}{x} = \frac{-y^2}{x}$

14.) $(2y + 3x^2y^3)dx + (3x + 5x^3y^2)dy = 0$

15.) $x^2dy + y^2dx = x^2ydy - xy^2dx$

16.) $y^{(4)} - 10y'' + 9y = 0,\ y(0) = 1,\ y'(0) = -1$

17.) $(ye^x + 2e^x + y^2)dx + (e^x + 2xy)dy = 0$

18.) $\frac{\partial z}{\partial x} = x^2$

19.) $(x - 2y + 1)dx + (4x - 3y - 6)dy = 0$

20.) $xy'' + y' + 2y = 0,\ y(1) = 2,\ y'(1) = 4$

21.) $x^2(x-2)^2y'' + 2(x-2)y' + (x+1)y = 0,\ y(0) = 3,\ y'(0) = 0$

22.) $y' = \sin x \tan y$

23.) $\frac{d^2x}{dt^2} + (x^2+1)\frac{dx}{dt} + x^3 = 0$

1.) Assume a series solution of the form $y=\sum_{n=0}^{\infty} C_n X^n$

2.) Solve it as a separable, linear, or homogeneous.

3.) Variation of parameters

4.) NOT ABLE TO SOLVE (non-linear)

5.) Laplace transform

6.) Homogeneous

7.) NOT ABLE TO SOLVE

8.) Undetermined coefficients, variation of parameters, or Laplace transform

9.) Assume a series solution of the form $y=\sum_{n=0}^{\infty} C_n X^{n+r}$, Use method of Frobenius

10.) Cauchy-Euler equation

11.) Bessel's equation of order 1; one solution is $J_1(x)=\sum_{n=0}^{\infty}(-1)^n \frac{1}{n!(n+1)!}\left(\frac{x}{2}\right)^{2n+1}$

12.) Bessel's equation of order 0; one solution is $J_0(x)=\sum_{n=0}^{\infty}(-1)^n \frac{1}{n!n!}\left(\frac{x}{2}\right)^{2n}$

13.) Bernoulli, Separable

14.) NOT ABLE TO SOLVE; the alert student would try to find an integrating factor of the form $x^m y^n$ ($x^{-9}y^{-13}$ works!)

15.) Separable

16.) Solve the auxiliary equation $m^4-10m^2+9=0$, Laplace transform

17.) Exact

18.) NOT ABLE TO SOLVE; a good student might integrate partially with respect to x to obtain the solution $z=\frac{1}{3}x^3+\phi(y)$

19.) Make substitutions $x=X+3$ and $y=Y+2$

20.) Assume a series solution of the form $y=\sum_{n=0}^{\infty} C_n(x-1)^n$

21.) NOT ABLE TO SOLVE ($x_0=0$ is an irregular singular point)

22.) Separable

23.) NOT ABLE TO SOLVE (non-linear)