Section 7.2 The Equivalence of NPDAs and Context-Free Grammars.

(1) Let $G$ be the following context-free grammar, with $V = \{S, B, C\}$, $T = \{a, b, c\}$.

\[
S \to aSbbB \mid C \mid \lambda \\
B \to b \mid \lambda \\
C \to Cc \mid c
\]

(a) Find an npda $M$ such that $L(M) = L(G)$.
(b) Give a left-most derivation from $G$ of the string: $aaccbbbb$.
(c) Give the corresponding sequence of instantaneous descriptions for $M$.

(2) Let $M$ be the following npda, with $\Gamma = \{S, 0, 1, z\}$, $F = \{q_f\}$.

\[
\delta(q_0, \lambda, z) = \{(q_1, Sz)\} \\
\delta(q_1, a, S) = \{(q_1, 11S)\} \\
\delta(q_1, a, 1) = \{(q_1, 11)\} \\
\delta(q_1, a, 0) = \{(q_1, \lambda), (q_1, 010)\} \\
\delta(q_1, b, 1) = \{(q_1, \lambda)\} \\
\delta(q_1, b, S) = \{(q_1, \lambda), (q_1, 0S)\} \\
\delta(q_1, \lambda, 0) = \{(q_1, 000), (q_1, \lambda)\} \\
\delta(q_1, \lambda, z) = \{(q_f, \lambda)\}
\]

Find a context-free grammar $G$ such that $L(G) = L(M)$.

(3) (OPTIONAL) Let $M'$ be the following npda, with $Q' = \{q'_0, q'_1, q'_2\}$, $F' = \{q'_1, q'_2\}$, $\Sigma' = \{a, b\}$, and $\Gamma' = \{z', a, b\}$.

\[
\delta'(q'_0, a, z') = \{(q'_0, a z'), (q'_2, \lambda)\} \\
\delta'(q'_0, a, a) = \{(q'_0, aa), (q'_1, \lambda)\} \\
\delta'(q'_0, b, a) = \{(q'_0, ba)\} \\
\delta'(q'_1, b, b) = \{(q'_1, \lambda)\} \\
\delta'(q'_1, a, a) = \{(q'_1, \lambda)\} \\
\delta'(q'_2, a, a) = \{(q'_2, a)\}
\]

Find an npda $M$ such that $L(M) = L(M')$ and $M$ is in pre-standard form.

(4) Let $M$ be the following npda with $F = \{q_f\}$.

\[
\delta(q_0, a, z) = \{(q_0, az)\} \\
\delta(q_0, a, a) = \{(q_0, aa)\} \\
\delta(q_0, a, b) = \{(q_0, ab)\} \\
\delta(q_0, b, z) = \{(q_0, bz)\} \\
\delta(q_0, b, a) = \{(q_0, ba)\} \\
\delta(q_0, b, b) = \{(q_0, bb)\}
\]

(a) Find an npda $\tilde{M}$ such that $L(\tilde{M}) = L(M)$, and $\tilde{M}$ is in standard reduced form. Use the clause-template notation that I used in the example in the notes.
(b) Give the sequences of instantaneous descriptions for $M$ and $\tilde{M}$ that show that $M$ and $\tilde{M}$ accept the string: $abaaba$. 

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