

Theory of Algorithms. Spring 2000. Homework 10 Solutions.

Section 9.1

(2) Here is a Turing Machine with only two states which is an acceptor for the language $L(a(a+b)^*)$. Notice that the design of the Turing Machine employs the idea that **we don't have to read the whole string in order to accept it**. $F = \{q_f\}$. δ is given by one clause: $\delta(q_0, a) = (q_f, a, R)$.

(3) Regarding the Turing Machine presented in Example 9.7:

$q_0aba \vdash xq_1ba \vdash q_2xya \vdash xq_0ya \vdash xyq_3a \vdash \text{HALT}$.

Since q_3 is not an accept state, aba is rejected.

$q_0aaabbbb \vdash xq_1aabbbb \vdash xaq_1abbbb \vdash xaaq_1bbbb \vdash xaq_2aybbb \vdash xq_2aaybbb \vdash q_2xaaybbb \vdash xq_0aaybbb \vdash xxq_1aybbb \vdash xxaq_1ybbb \vdash x xayq_1bbb \vdash xxaq_2yybb \vdash xxq_2ayybb \vdash xq_2xayybb \vdash xxq_0ayybb \vdash xxxq_1yybb \vdash xxxyq_1ybb \vdash xxxyyq_1bb \vdash xxxyq_2yyb \vdash xxxq_2yyyb \vdash xq_2xyyyb \vdash xxxq_0yyyb \vdash xxxyq_3yyb \vdash xxxyyq_3yb \vdash xxxyyyq_3b \vdash \text{HALT}$.

Since q_3 is not an accept state, $aaabbbb$ is rejected.

(4) No.

(5) $L(ab^* + bb^*a(a+b)^*)$. Notice the $(a+b)^*$ part. This is because in the last δ -clause, M reads an a and halts and accepts without looking at the rest of the input string. Compare this with the fourth δ -clause in which M doesn't accept until it sees a blank, indicating the end of the string. **Make sure you understand this exercise!**

(7a) $F = \{q_f\}$.

$\delta(q_0, a) = (q_1, a, R)$ $\delta(q_1, b) = (q_2, b, R)$ $\delta(q_2, a) = (q_2, a, R)$
 $\delta(q_2, b) = (q_3, b, R)$ $\delta(q_3, \square) = (q_f, \square, R)$

(7b) $F = \{q_f\}$.

$\delta(q_0, a) = (q_1, a, R)$ $\delta(q_0, b) = (q_1, b, R)$ $\delta(q_0, \square) = (q_f, \square, R)$
 $\delta(q_1, a) = (q_0, a, R)$ $\delta(q_1, b) = (q_0, b, R)$

(7c) $F = \{q_f\}$.

$\delta(q_0, a) = (q_1, x, R)$ $\delta(q_1, a) = (q_1, a, R)$ $\delta(q_1, y) = (q_1, y, R)$ $\delta(q_1, b) = (q_2, y, L)$

$\delta(q_2, y) = (q_2, y, L)$ $\delta(q_2, a) = (q_2, a, L)$ $\delta(q_2, x) = (q_0, x, R)$

$\delta(q_0, y) = (q_3, y, R)$ $\delta(q_3, y) = (q_3, y, R)$ $\delta(q_3, b) = (q_4, b, R)$ $\delta(q_4, b) = (q_4, b, R)$

$\delta(q_4, \square) = (q_f, \square, R)$ (There are more b 's than a 's)

$\delta(q_1, \square) = (q_f, \square, R)$ (There are more a 's than b 's)

(7d) $F = \{q_f\}$.

$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) & \delta(q_0, b) &= (q_2, x, R) & \delta(q_0, x) &= (q_0, x, R) & \delta(q_0, \square) &= (q_f, \square, R) \\ \delta(q_1, a) &= (q_1, a, R) & \delta(q_1, b) &= (q_3, x, R) & \delta(q_1, x) &= (q_1, x, R) \\ \delta(q_2, a) &= (q_3, x, R) & \delta(q_2, b) &= (q_2, b, R) & \delta(q_2, x) &= (q_2, x, R) \\ \delta(q_3, a) &= (q_3, a, R) & \delta(q_3, b) &= (q_3, b, R) & \delta(q_3, x) &= (q_3, x, R) & \delta(q_3, \square) &= (q_4, \square, L) \\ \delta(q_4, a) &= (q_5, x, L) & \delta(q_4, b) &= (q_6, x, L) & \delta(q_4, x) &= (q_4, x, L) & \delta(q_4, \square) &= (q_f, \square, L) \\ \delta(q_5, a) &= (q_5, a, L) & \delta(q_5, b) &= (q_7, x, L) & \delta(q_5, x) &= (q_5, x, L) \\ \delta(q_6, a) &= (q_7, x, L) & \delta(q_6, b) &= (q_6, b, L) & \delta(q_6, x) &= (q_6, x, L) \\ \delta(q_7, a) &= (q_7, a, L) & \delta(q_7, b) &= (q_7, b, L) & \delta(q_7, x) &= (q_7, x, L) & \delta(q_7, \square) &= (q_0, \square, R)\end{aligned}$$

(7e) $F = \{q_f\}$.

$$\begin{aligned}\delta(q_0, a) &= (q_0, b, R) & \delta(q_0, b) &= (q_1, b, L) & \delta(q_1, b) &= (q_1, b, L) & \delta(q_1, \square) &= (q_2, \square, R) \\ \delta(q_2, b) &= (q_3, x, R) & \delta(q_3, b) &= (q_3, b, R) & \delta(q_3, y) &= (q_3, y, R) & \delta(q_3, a) &= (q_4, y, L) \\ \delta(q_4, y) &= (q_4, y, L) & \delta(q_4, b) &= (q_4, b, L) & \delta(q_4, x) &= (q_2, x, R) \\ \delta(q_2, y) &= (q_5, y, R) & \delta(q_5, y) &= (q_5, y, R) & \delta(q_5, \square) &= (q_f, \square, R)\end{aligned}$$

(7f) $F = \{q_f\}$.

$$\begin{aligned}\delta(q_0, \square) &= (q_f, \square, R) & \delta(q_0, a) &= (q_1, x, R) & \delta(q_1, a) &= (q_1, a, R) & \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, R) \\ \delta(q_2, b) &= (q_2, b, R) & \delta(q_2, z) &= (q_2, z, R) & \delta(q_2, a) &= (q_3, z, R) \\ \delta(q_3, a) &= (q_3, a, R) & \delta(q_3, w) &= (q_3, w, R) & \delta(q_3, b) &= (q_4, w, L) \\ \delta(q_4, w) &= (q_4, w, L) & \delta(q_4, a) &= (q_4, a, L) & \delta(q_4, z) &= (q_4, z, L) & \delta(q_4, b) &= (q_4, b, L) \\ \delta(q_4, y) &= (q_4, y, L) & \delta(q_4, x) &= (q_0, x, R) \\ \delta(q_0, y) &= (q_5, y, R) & \delta(q_5, y) &= (q_5, y, R) & \delta(q_5, z) &= (q_5, z, R) & \delta(q_5, w) &= (q_5, w, R) \\ \delta(q_5, \square) &= (q_f, \square, R)\end{aligned}$$

(7g) $F = \{q_f\}$.

$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) & \delta(q_1, a) &= (q_1, a, R) & \delta(q_1, y) &= (q_1, y, R) & \delta(q_1, b) &= (q_2, b, R) \\ \delta(q_2, b) &= (q_3, y, L) \\ \delta(q_3, y) &= (q_3, y, L) & \delta(q_3, a) &= (q_3, a, L) & \delta(q_3, x) &= (q_0, x, R) \\ \delta(q_0, y) &= (q_4, y, R) & \delta(q_4, y) &= (q_4, y, R) & \delta(q_4, \square) &= (q_f, \square, R)\end{aligned}$$

(8) $F = \{q_f\}$.

$$\delta(q_0, a) = (q_1, A, R) \quad \delta(q_0, b) = (q_1, B, R) \quad \delta(q_1, a) = (q_1, a, R) \quad \delta(q_1, b) = (q_1, b, R)$$

$$\delta(q_1, \bar{a}) = (q_2, \bar{a}, L) \quad \delta(q_1, \bar{b}) = (q_2, \bar{b}, L) \quad \delta(q_1, \square) = (q_2, \square, L)$$

$$\delta(q_2, a) = (q_3, \bar{a}, L) \quad \delta(q_2, b) = (q_3, \bar{b}, L) \quad \delta(q_3, a) = (q_3, a, L) \quad \delta(q_3, b) = (q_3, b, L)$$

$$\delta(q_3, A) = (q_0, A, R) \quad \delta(q_3, B) = (q_0, B, R) \quad \delta(q_0, \bar{a}) = (q_4, \bar{a}, L) \quad \delta(q_0, \bar{b}) = (q_4, \bar{b}, L)$$

$$\delta(q_4, A) = (q_4, A, L) \quad \delta(q_4, B) = (q_4, B, L) \quad \delta(q_4, \square) = (q_5, \square, R)$$

$$\delta(q_5, A) = (q_6, x, R) \quad \delta(q_5, B) = (q_7, x, R)$$

$$\delta(q_6, A) = (q_6, A, R) \quad \delta(q_6, B) = (q_6, B, R) \quad \delta(q_6, y) = (q_6, y, R) \quad \delta(q_6, \bar{a}) = (q_8, y, L)$$

$$\delta(q_7, A) = (q_7, A, R) \quad \delta(q_7, B) = (q_7, B, R) \quad \delta(q_7, y) = (q_7, y, R) \quad \delta(q_7, \bar{b}) = (q_8, y, L)$$

$$\delta(q_8, y) = (q_8, y, L) \quad \delta(q_8, A) = (q_8, A, L) \quad \delta(q_8, B) = (q_8, B, L) \quad \delta(q_8, x) = (q_5, x, R)$$

$$\delta(q_5, y) = (q_f, y, R)$$