1. **REASONING AND SOLUTION** Combining Equations 19.1 and 19.3, we have

\[ W_{AB} = E_{PE_A} - E_{PE_B} = q_0 (V_A - V_B) = (+1.6 \times 10^{-19} \text{ C})(0.070 \text{ V}) = 1.1 \times 10^{-20} \text{ J} \]

2. **REASONING** The change in the proton’s electric potential energy, \( E_{PE_A} - E_{PE_B} \), is equal to the work \( W_{AB} \) done by the electric force as the proton moves from point \( A \) to point \( B \) (see Equation 19.1). According to Equation 6.1, the work done by a constant force is the product of the magnitude \( F \) of the force, the magnitude \( s \) of the proton’s displacement, and the cosine of the angle between the force and displacement; \( W_{AB} = F s \cos \theta \). Since the magnitude of the electric force is equal to product of the proton charge \( q \) and the magnitude \( E \) of the electric field, the work can be written as \( W_{AB} = (qE) s \cos \theta \).

**SOLUTION**

a. The displacement of the proton is in the same direction as the electric force, so the angle between them is \( \theta = 0.0^\circ \):

\[ E_{PE_A} - E_{PE_B} = W_{AB} = (qE) s \cos \theta = (1.6 \times 10^{-19} \text{ C})(2.0 \times 10^4 \text{ N/C})(0.15 \text{ m}) \cos 0.0^\circ = 4.8 \times 10^{-16} \text{ J} \]

b. When the proton moves opposite the electric field, the angle between the proton’s displacement and the electric force is \( \theta = 180^\circ \), so that \( \cos 180^\circ = -1 \). The solution proceeds in the same manner as in part (a), except that the angle is 180°. Therefore, the change in the electric potential energy is \( -4.8 \times 10^{-16} \text{ J} \)

3. **REASONING** The number \( N \) of electrons that jumps from your hand (point \( A \)) to the door knob (point \( B \)) is equal to the total charge \( q \) that jumps divided by the charge \( -e \) of one electron: \( N = q/(-e) \), where \( e = 1.6 \times 10^{-19} \text{ C} \). We can determine \( q \) by using Equation 19.4, which relates the work \( W_{AB} \) done by the electric force to the difference in the electric potentials, \( V_B - V_A \), and the charge. The difference in the potentials is given as \( V_B - V_A = 2.0 \times 10^4 \text{ V} \).

**SOLUTION** The number of electrons that jumps from your hand to the door knob is
\[
N = \frac{q}{-e} = \frac{-W_{AB}}{V_B - V_A} = \frac{-1.5 \times 10^{-7} \text{ J}}{2.0 \times 10^4 \text{ V}} - \frac{1.6 \times 10^{-19} \text{ C}}{4.7 \times 10^7}
\]

### 4. REASONING AND SOLUTION

a. According to Equation 19.4, the work done by the electric force as the electron goes from point \( A \) (the cathode) to point \( B \) (the anode) is

\[
W_{AB} = -q(V_B - V_A) = -(1.6 \times 10^{-19} \text{ C})(+125 000 \text{ V}) = +2.00 \times 10^{-14} \text{ J}
\]

b. The only force that acts on the electron is the conservative electric force. Therefore, the total energy of the electron is conserved as it moves from point \( A \) to point \( B \):

\[
\frac{1}{2}mv_A^2 + E_{PE_A} = \frac{1}{2}mv_B^2 + E_{PE_B}
\]

Since the electron starts from rest, \( v_A = 0 \). The electric potential \( V \) is related to the electric potential energy \( E_{PE} \) by \( V = E_{PE}/q \) (see Equation 19.3). With these changes, the equation above gives the kinetic energy of the electron at point \( B \) (the anode) to be

\[
\frac{1}{2}mv_B^2 = -E_{PE_B} + E_{PE_A}
\]

\[
= -q(V_B - V_A) = -(1.60 \times 10^{-19} \text{ C})(125 000 \text{ V}) = 2.00 \times 10^{-14} \text{ J}
\]

### 5. SSM REASONING

The only force acting on the moving electron is the conservative electric force. Therefore, the total energy of the electron (the sum of the kinetic energy \( KE \) and the electric potential energy \( E_{PE} \)) remains constant throughout the trajectory of the electron. Let the subscripts \( A \) and \( B \) refer to the initial and final positions, respectively, of the electron. Then,

\[
\frac{1}{2}mv_A^2 + E_{PE_A} = \frac{1}{2}mv_B^2 + E_{PE_B}
\]

Solving for \( v_B \) gives

\[
v_B = \sqrt{v_A^2 - \frac{2}{m}(E_{PE_B} - E_{PE_A})}
\]

Since the electron starts from rest, \( v_A = 0 \). The difference in potential energies is related to the difference in potentials by Equation 19.4, \( E_{PE_B} - E_{PE_A} = q(V_B - V_A) \).
**SOLUTION**  The speed $v_B$ of the electron just before it reaches the screen is

$$v_B = \sqrt{\frac{-2q}{m} (V_B - V_A)} = \sqrt{\frac{2(-1.6 \times 10^{-19} \text{ C})}{9.11 \times 10^{-31} \text{ kg}} (25000 \text{ V})} = 9.4 \times 10^7 \text{ m/s}$$

6. **REASONING AND SOLUTION**  The only force that acts on the $\alpha$-particle is the conservative electric force. Therefore, the total energy of the $\alpha$-particle is conserved as it moves from point $A$ to point $B$:

$$\frac{1}{2}mv_A^2 + E_{PEA} = \frac{1}{2}mv_B^2 + E_{PEB}$$

Since the $\alpha$-particle starts from rest, $v_A = 0$. The electric potential $V$ is related to the electric potential energy $E_{PE}$ by $V = E_{PE}/q$ (see Equation 19.3). With these changes, the equation above gives the kinetic energy of the $\alpha$-particle at point $B$ to be

$$\frac{1}{2}mv_B^2 = E_{PEA} - E_{PEB} = q(V_A - V_B)$$

Since an $\alpha$-particle contains two protons, its charge is $q = 2e = 3.2 \times 10^{-19} \text{ C}$. Thus, the kinetic energy (in electron-volts) is

$$\frac{1}{2}mv_B^2 = q(V_A - V_B) = (3.2 \times 10^{-19} \text{ C})(+250 \text{ V} - (-150 \text{ V}))$$

$$= 1.28 \times 10^{-16} \text{ J} \left( \frac{1.0 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} \right) = 8.0 \times 10^2 \text{ eV}$$

7. **REASONING AND SOLUTION**  The power rating $P$ is defined as the work $W_{AB}$ done by the battery divided by the time $t$,

$$P = \frac{W_{AB}}{t}$$

The work done by the electric force as the charge moves from point $A$ (the positive terminal), through the electric motor, and to point $B$ (the negative terminal) is

$$W_{AB} = q(V_A - V_B) = (1300 \text{ C})(320 \text{ V}) = 4.2 \times 10^5 \text{ J} \quad (19.4)$$

The power rating is

$$P = \frac{W_{AB}}{t} = \frac{4.2 \times 10^5 \text{ J}}{8.0 \text{ s}} = 5.2 \times 10^4 \text{ W}$$
Since 746 W = 1 hp, the minimum horsepower rating of the car is

\[
\left(5.20 \times 10^4 \text{ W}\right) \frac{1 \text{ hp}}{746 \text{ W}} = 7.0 \times 10^1 \text{ hp}
\]

8. **REASONING AND SOLUTION** As the charge in the lightning bolt moves from point \(A\) to point \(B\), the change in its electric potential energy is

\[
E_{\text{PE}A} - E_{\text{PE}B} = q(V_A - V_B) = (29 \text{ C})(1.4 \times 10^8 \text{ V}) = 4.1 \times 10^9 \text{ J}
\]

If this electric energy were converted into heat \(Q\), the mass \(m\) of liquid water at 100 °C that is converted into steam is given by \(m = Q/L_v\) (see Section 12.8), where \(L_v\) is the latent heat of vaporization \((L_v = 22.6 \times 10^5 \text{ J/kg})\). The mass of water that is boiled away is

\[
m = \frac{Q}{L_v} = \frac{4.1 \times 10^9 \text{ J}}{22.6 \times 10^5 \text{ J/kg}} = 1.8 \times 10^3 \text{ kg}
\]

9. **REASONING** The only force acting on the moving charge is the conservative electric force. Therefore, the total energy of the charge remains constant. Applying the principle of conservation of energy between locations \(A\) and \(B\), we obtain

\[
\frac{1}{2}mv_A^2 + E_{\text{PE}A} = \frac{1}{2}mv_B^2 + E_{\text{PE}B}
\]

Since the charged particle starts from rest, \(v_A = 0\). The difference in potential energies is related to the difference in potentials by Equation 19.4, \(E_{\text{PE}B} - E_{\text{PE}A} = q(V_B - V_A)\). Thus, we have

\[
q(V_A - V_B) = \frac{1}{2}mv_B^2
\]

Similarly, applying the conservation of energy between locations \(C\) and \(B\) gives

\[
q(V_C - V_B) = \frac{1}{2}m(2v_B)^2
\]

Dividing Equation (1) by Equation (2) yields

\[
\frac{V_A - V_B}{V_C - V_B} = \frac{1}{4}
\]

This expression can be solved for \(V_B\).

**SOLUTION** Solving for \(V_B\), we find that

\[
V_B = \frac{4V_A - V_C}{3} = \frac{4(452 \text{ V}) - 791 \text{ V}}{3} = 339 \text{ V}
\]
10. **REASONING AND SOLUTION** The electron, starting from rest at point A, accelerates to point B. The only force acting on the electron is the conservative electric force. Therefore, the total energy of the electron remains constant. Applying the principle of conservation of energy between locations A and B, we obtain

\[
\frac{1}{2}mv_A^2 + E_{PEA} = \frac{1}{2}mv_B^2 + E_{PEB}
\]

Solving for \(v_B\), the speed of the electron at point B, yields

\[
v_B^{\text{electron}} = \sqrt{v_A^2 + \frac{2}{m}(E_{PEA} - E_{PEB})}
\]

Since the electron starts from rest, \(v_A = 0\). The difference in potential energies is related to the difference in potentials by Equation 19.4, \(E_{PEA} - E_{PEB} = q(V_A - V_B)\). Thus, the speed of the electron at point B is

\[
v_B^{\text{electron}} = \sqrt{\frac{2q(V_A - V_B)}{m}} = \sqrt{\frac{2(-1.6 \times 10^{-19} \text{ C})(V_A - V_B)}{9.11 \times 10^{-31} \text{ kg}}}
\]

Since the electron accelerated from point A to point B, the proton, being positively charged, accelerates from B to A. Using the conservation of energy, the speed of the proton at point A is

\[
v_A^{\text{proton}} = \sqrt{v_B^2 - \frac{2}{m}(E_{PEA} - E_{PEB})}
\]

Since, \(v_B = 0\) (the proton starts from rest), and \(E_{PEA} - E_{PEB} = q(V_A - V_B)\),

\[
v_A^{\text{proton}} = \sqrt{-\frac{2q(V_A - V_B)}{m}} = \sqrt{-\frac{2(+1.6 \times 10^{-19} \text{ C})(V_A - V_B)}{1.67 \times 10^{-27} \text{ kg}}}
\]

The ratio of the electron speed at point B to the proton speed at point A is

\[
\frac{v_B^{\text{electron}}}{v_A^{\text{proton}}} = \frac{\sqrt{\frac{2(-1.6 \times 10^{-19} \text{ C})(V_A - V_B)}{9.11 \times 10^{-31} \text{ kg}}}}{\sqrt{-\frac{2(+1.6 \times 10^{-19} \text{ C})(V_A - V_B)}{1.67 \times 10^{-27} \text{ kg}}}} = 42.8
\]

11. **REASONING** The gravitational and electric forces are conservative forces, so the total energy of the particle remains constant as it moves from point A to point B. Recall from Equation 6.5 that the
gravitational potential energy (GPE) of a particle of mass $m$ is $\text{GPE} = mgh$, where $h$ is the height of the particle above the earth’s surface. The conservation of energy is written as

$$\frac{1}{2}mv_A^2 + mgh_A + \text{EPE}_A = \frac{1}{2}mv_B^2 + mgh_B + \text{EPE}_B$$

(1)

We will use this equation several times to determine the initial speed $v_A$ of the negatively charged particle.

**SOLUTION**

When the negatively charged particle is thrown upward, it attains a maximum height of $h$. For this particle we have:

- $v_A = ?$
- $v_B = 0$ (at maximum height)
- $\text{EPE}_A = (-q)V_A$
- $\text{EPE}_B = (-q)V_B$
- $h_A = 0$ (ground level)
- $h_B = h$ (the maximum height)

Solving the conservation of energy equation, Equation (1), for $v_A$ and substituting in the data above gives

$$v_A = \sqrt{\frac{2}{m}[mgh + (-q)(V_B - V_A)]}$$

(2)

Equation (2) cannot be solved as it stands because the height $h$ and the potential difference $(V_B - V_A)$ are not known.

We now make use of the fact that a positively charged particle, when thrown straight upward with an initial speed of 30.0 m/s, also reaches the maximum height $h$. For this particle we have:

- $v_A = 30.0$ m/s
- $v_B = 0$ (at maximum height)
- $\text{EPE}_A = (+q)V_A$
- $\text{EPE}_B = (+q)V_B$
- $h_A = 0$ (ground level)
- $h_B = h$

Solving the conservation of energy equation, Equation (1), for the potential difference $(V_B - V_A)$ and substituting in the data above gives

$$V_B - V_A = \frac{1}{+q}\left[\frac{1}{2}m(30.0 \text{ m/s})^2 - mgh\right]$$

(3)

Substituting Equation (3) into Equation (2) gives, after some algebraic simplifications,

$$v_A = \sqrt{4gh - (30.0 \text{ m/s})^2}$$

(4)

Equation (4) cannot be solved because the height $h$ is still unknown.
We now make use of the fact that the uncharged particle, when thrown straight upward with an initial speed of 25.0 m/s, also reaches the maximum height $h$. For this particle we have:

\begin{align*}
v_A &= 25.0 \text{ m/s} \quad v_B = 0 \text{ (at maximum height)} \\
EPE_A &= qV_A = 0 \text{ (since } q = 0) \\
EPE_B &= qV_B = 0 \text{ (since } q = 0) \\
h_A &= 0 \text{ (ground level)} \\
h_A &= h
\end{align*}

Solving Equation (1) with this data for the maximum height $h$ yields

$$h = \frac{(25.0 \text{ m/s})^2}{2g} = \frac{(25.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 31.9 \text{ m}$$

Substituting $h = 31.9 \text{ m}$ into Equation (4) gives $v_A = 18.7 \text{ m/s}$.

12. **REASONING AND SOLUTION** First, we need to find the magnitude of the charge. Since $V = kq/r$, we have

$$q = \frac{Vr}{k} = \frac{(164 \text{ V})(0.20 \text{ m})}{9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 3.6 \times 10^{-9} \text{ C}$$

Thus, at a distance of 0.80 m the potential would be

$$V = \frac{kq}{r} = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.6 \times 10^{-9} \text{ C})}{0.80 \text{ m}} = 41 \text{ V}$$

13. **SSM REASONING AND SOLUTION** The electric potential $V$ at a distance $r$ from a point charge $q$ is given by Equation 19.6, $V = kq/r$. Solving this expression for $q$, we find that

$$q = \frac{rV}{k} = \frac{(0.25 \text{ m})(+130 \text{ V})}{9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = +3.6 \times 10^{-9} \text{ C}$$

14. **REASONING** The potential $V$ at a distance $r$ from a proton is $V = k(+e)/r$ (see Equation 19.6), where $+e$ is the charge of the proton. When an electron ($q = -e$) is placed at a distance $r$ from the proton, the electric potential energy is $EPE = -eV$, as per Equation 19.3.

**SOLUTION** The difference in the electric potential energies when the electron and proton are separated by $r_{\text{final}} = 5.29 \times 10^{-11} \text{ m}$ and when they are very far apart ($r_{\text{initial}} = \infty$) is
EPE_{\text{final}} - EPE_{\text{initial}} = \frac{(-e)ke}{r_{\text{final}}} - \frac{(-e)ke}{r_{\text{initial}}}

= -(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(1.60 \times 10^{-19} \text{ C})^2

\times \left( \frac{1}{5.29 \times 10^{-11} \text{ m}} - \frac{1}{\infty} \right) = -4.35 \times 10^{-18} \text{ J}

\hline
15. **REASONING AND SOLUTION** The initial and final separations of the charges are the same. Since the electric potential energy depends only on the separation of the charges, there is no change in the electric potential energy, and, hence, 

\text{no work is done}.

\hline
16. **REASONING AND SOLUTION** The initial and final electric potential energies of the configuration are, respectively,

\begin{align*}
EPE_A &= \frac{kq_1q_2}{r_A} \quad \text{and} \quad EPE_B = \frac{kq_1q_2}{r_B}
\end{align*}

Since \( EPE_B = 2 EPE_A \), we have

\begin{align*}
\frac{kq_1q_2}{r_B} &= 2 \frac{kq_1q_2}{r_A} \\
\Rightarrow \quad r_B &= \frac{r_A}{2} = \frac{0.74 \text{ m}}{2} = 0.37 \text{ m}.
\end{align*}

\hline
17. **SSM WWW REASONING** Initially, suppose that one charge is at C and the other charge is held fixed at B. The charge at C is then moved to position A. According to Equation 19.4, the work \( W_{\text{CA}} \) done by the electric force as the charge moves from C to A is \( W_{\text{CA}} = q(V_C - V_A) \), where, from Equation 19.6, \( V_C = \frac{kq}{d} \) and \( V_A = \frac{kq}{r} \).

From the figure at the right we see that \( d = \sqrt{r^2 + r^2} = \sqrt{2}r \). Therefore, we find that

\begin{align*}
W_{\text{CA}} &= q \left( \frac{\frac{kq}{\sqrt{2}r} - \frac{kq}{r}}{r} \right) = \frac{kq^2}{r} \left( \frac{1}{\sqrt{2}} - 1 \right)
\end{align*}
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**SOLUTION** Substituting values, we obtain

\[
W_{CA} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.0 \times 10^{-6} \text{ C})^2 (\frac{1}{\sqrt{2}} - 1)}{0.500 \text{ m}} = -4.7 \times 10^{-2} \text{ J}
\]

18. **REASONING** Each object contributes an electric potential at point A given by Equation 19.6 as \( V = kq/r \), where \( q \) is the charge and \( r \) is the distance between the object and point A. The total electric potential \( V_{\text{total}} \) at point A due to the three charges is the sum of the individual potentials. When a charge \( q_0 \) is placed at point A, the electric potential energy is \( \text{EPE} = q_0 V_{\text{total}} \), as per Equation 19.3.

**SOLUTION**
(a) The total electric potential at point A is

\[
V_{\text{total}} = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3}
\]

\[
= \left(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \left(\frac{+3.6 \times 10^{-6} \text{ C}}{2.0 \times 10^{-2} \text{ m}} + \frac{-20.0 \times 10^{-6} \text{ C}}{4.0 \times 10^{-2} \text{ m}} + \frac{+6.0 \times 10^{-6} \text{ C}}{3.0 \times 10^{-2} \text{ m}}\right)
\]

\[
= -1.1 \times 10^6 \text{ V}
\]

(b) When a \( +3.5 \times 10^{-6} \text{ C} \) charge is placed at A, the electric potential energy is

\[
\text{EPE} = q_0 V_{\text{total}} = \left(+3.5 \times 10^{-6} \text{ C}\right)(-1.1 \times 10^6 \text{ V}) = 3.8 \text{ J}
\]

19. **REASONING AND SOLUTION** Let \( s \) be the length of the side of the square and \( Q \) be the value of the unknown charge. The potential at either of the vacant corners is

\[
V = 0 = \frac{k(9q)}{s} + \frac{k(-8q)}{s} + \frac{kQ}{s\sqrt{2}}
\]

so

\[
Q = -\frac{q}{\sqrt{2}}
\]

20. **REASONING AND SOLUTION** Let the first spot where the potential is zero be a distance \( x \) to the left of the negative charge. Then,

\[
\frac{k(2q)}{d-x} = \frac{kq}{x} \quad \text{or} \quad x = \frac{d}{3}
\]
Let the second spot where the potential is zero be a distance \(x\) to the right of the negative charge. Then,

\[
\frac{k(2q)}{d+x} = \frac{kq}{x}
\]
or
\[
x = \frac{d}{x}
\]

---

21. **SSM REASONING** The only force acting on the moving charge is the conservative electric force. Therefore, the sum of the kinetic energy \(KE\) and the electric potential energy \(EPE\) is the same at points A and B:

\[
\frac{1}{2}mv_A^2 + EPE_A = \frac{1}{2}mv_B^2 + EPE_B
\]

Since the particle comes to rest at B, \(v_B = 0\). Combining Equations 19.3 and 19.6, we have

\[
EPE_A = qV_A = q\left(\frac{kq_1}{d}\right)
\]

and

\[
EPE_B = qV_B = q\left(\frac{kq_1}{r}\right)
\]

where \(d\) is the initial distance between the fixed charge and the moving charged particle, and \(r\) is the distance between the charged particles after the moving charge has stopped. Therefore, the expression for the conservation of energy becomes

\[
\frac{1}{2}mv_A^2 + \frac{kqq_1}{d} = \frac{kqq_1}{r}
\]

This expression can be solved for \(r\). Once \(r\) is known, the distance that the charged particle moves can be determined.

**SOLUTION** Solving the expression above for \(r\) gives

\[
r = \frac{kqq_1}{\frac{1}{2}mv_A^2 + \frac{kqq_1}{d}}
\]

\[
= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-8.00 \times 10^{-6} \text{ C})(-3.00 \times 10^{-6} \text{ C})}{\frac{1}{2} (7.20 \times 10^{-3} \text{ kg})(65.0 \text{ m/s})^2 + \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-8.00 \times 10^{-6} \text{ C})(-3.00 \times 10^{-6} \text{ C})}{0.0450 \text{ m}}}
\]

\[
= 0.0108 \text{ m}
\]

Therefore, the charge moves a distance of 0.0450 m – 0.0108 m = 0.0342 m.
22. **REASONING** Initially, the four charges are infinitely far apart. We will proceed as in Example 8 by adding charges to the square, one at a time, and determining the electric potential energy at each step. According to Equation 19.3, the electric potential energy $E_{PE}$ is the product of the charge $q$ and the electric potential $V$ at the spot where the charge is placed, $E_{PE} = qV$. The total electric potential energy of the group is the sum of the energies of each step in assembling the group.

**SOLUTION** Let the corners of the square be numbered clockwise as 1, 2, 3 and 4, starting with the top-left corner. When the first charge ($q = 5.0 \mu C$) is placed at a corner 1, the charge has no electric potential energy, $E_{PE1} = 0$. This is because the electric potential $V_1$ produced by the other three charges at corner 1 is zero, since they are infinitely far away.

Once $q$ is in place, the electric potential $V_2$ that it creates at corner 2 is

$$V_2 = \frac{kq}{r_{21}}$$

where $r_{21} = 0.50 \text{ m}$ is the distance between corners 2 and 1. When the second charge is placed at corner 2, its electric potential energy $E_{PE2}$ is

$$E_{PE2} = qV_2 = q\left(\frac{kq}{r_{21}}\right) = \left(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(5.0 \times 10^{-6} \text{ C}\right)^2 \frac{1}{0.50 \text{ m}} = 0.45 \text{ J}$$

The electric potential $V_3$ at the third corner is the sum of the potentials due to the two charges that are already in place on corners 1 and 2:

$$V_3 = \frac{kq}{r_{31}} + \frac{kq}{r_{32}}$$

where $r_{31} = 0.71 \text{ m}$, and $r_{32} = 0.50 \text{ m}$. When the third charge is placed at corner 3, the electric potential energy $E_{PE3}$ is

$$E_{PE3} = qV_3 = q\left(\frac{kq}{r_{31}} + \frac{kq}{r_{32}}\right) = kq^2\left(\frac{1}{r_{31}} + \frac{1}{r_{32}}\right)$$

$$= \left(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(5.0 \times 10^{-6} \text{ C}\right)^2 \left(\frac{1}{0.71 \text{ m}} + \frac{1}{0.50 \text{ m}}\right) = 0.77 \text{ J}$$

The electric potential $V_4$ at the fourth corner is the sum of the potentials due to the three charges that are already in place on corners 1, 2, and 3:
\[ V_4 = \frac{kq}{r_{41}} + \frac{kq}{r_{42}} + \frac{kq}{r_{43}} \]

where \( r_{41} = 0.50 \text{ m} \), \( r_{42} = 0.71 \text{ m} \), and \( r_{43} = 0.50 \text{ m} \). When the fourth charge is placed at corner 4, the electric potential energy \( E_{PE_4} \) is

\[
E_{PE_4} = qV_4 = q \left( \frac{kq}{r_{41}} + \frac{kq}{r_{42}} + \frac{kq}{r_{43}} \right) = kq^2 \left( \frac{1}{r_{41}} + \frac{1}{r_{42}} + \frac{1}{r_{43}} \right)
\]

\[
= \left( 9.0 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) \left( 5.0 \times 10^{-6} \text{ C} \right)^2 \left( \frac{1}{0.50 \text{ m}} + \frac{1}{0.71 \text{ m}} + \frac{1}{0.50 \text{ m}} \right) = 1.2 \text{ J}
\]

The electric potential energy of the entire array is

\[
E_{PE} = E_{PE_1} + E_{PE_2} + E_{PE_3} + E_{PE_4} = 0 + 0.45 \text{ J} + 0.77 \text{ J} + 1.2 \text{ J} = 2.4 \text{ J}
\]

23. **REASONING** The only force acting on each proton is the conservative electric force. Therefore, the total energy (kinetic energy plus electric potential energy) is conserved at all points along the motion. For two points, \( A \) and \( B \), the conservation of energy is

\[
\frac{1}{2}mv_A^2 + \frac{1}{2}mv_A^2 + E_{PE_A} = \frac{1}{2}mv_B^2 + \frac{1}{2}mv_B^2 + E_{PE_B}
\]

The electric potential energy of two protons (charge = +e) that are separated by a distance \( r \) is \( E_{PE} = ke^2/r \). By substituting this expression for \( E_{PE} \) into the conservation of energy, we will be able to determine the distance of closest approach.

**SOLUTION** When the protons are very far apart \( (r_A = \infty) \), so that \( E_{PE_A} = 0 \). At the distance \( r_B \) of closest approach, the speed of each proton is momentarily zero \( (v_B = 0) \). With these substitutions, the conservation of energy equation reduces to

\[
\frac{1}{2}mv_A^2 + \frac{1}{2}mv_A^2 = \frac{ke^2}{r_B}
\]

Solving for \( r_B \), the distance of closest approach is

\[
r_B = \frac{ke^2}{mv_A^2} = \frac{\left( 9.0 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) \left( 1.6 \times 10^{-19} \text{ C} \right)^2}{\left( 1.67 \times 10^{-27} \text{ kg} \right) \left( 1.5 \times 10^6 \text{ m/s} \right)^2} = 6.1 \times 10^{-14} \text{ m}
\]
24. The figure at the right shows two identical charges, \( q \), fixed to diagonally opposite corners of a square. The potential at corner A is caused by the presence of the two charges. It is given by

\[
(V_A)_0 = \frac{kq}{r} + \frac{kq}{r} = \frac{2kq}{r}
\]

Since both charges are the same distance from corner B, this is equal to the potential at corner B as well.

If a third charge, \( q_3 \), is placed at the center of the square, the potential at corner A (as well as corner B) becomes

\[
(V_A)_f = \frac{kq}{r} + \frac{kq}{r} + \frac{kq_3}{(d/2)}
\]

We can find \( d \) by applying the Pythagorean theorem to the geometry in the figure above:

\[
d = \sqrt{r^2 + r^2} = \sqrt{2}r \quad \text{so that} \quad \frac{d}{2} = \frac{r}{\sqrt{2}}
\]

The expression for \( (V_A)_f \) becomes

\[
(V_A)_f = \frac{kq}{r} + \frac{kq}{r} + \frac{kq_3\sqrt{2}}{r}
\]

From the problem statement, we know that the addition of \( q_3 \) causes the potential at A and B to change sign without changing magnitude.

In other words, \( (V_A)_f = -(V_A)_0 \), or

\[
\frac{kq}{r} + \frac{kq}{r} + \frac{kq_3\sqrt{2}}{r} = -\frac{2kq}{r}
\]

Solving for \( q_3 \) gives

\[
q_3 = -(2\sqrt{2})q = -(2\sqrt{2})(1.7 \times 10^{-6} \text{ C}) = -4.8 \times 10^{-6} \text{ C}
\]

25. **REASONING** Initially, the three charges are infinitely far apart. We will proceed as in Example 8 by adding charges to the triangle, one at a time, and determining the electric potential energy at each step. According to Equation 19.3, the electric potential energy \( \text{EPE} \) is the product of the charge \( q \) and the electric potential \( V \) at the spot where the charge is placed, \( \text{EPE} = qV \). The total electric potential energy of the group is the sum of the energies of each step in assembling the group.
SOLUTION  Let the corners of the triangle be numbered clockwise as 1, 2 and 3, starting with the top corner. When the first charge \( (q_1 = 8.00\,\mu\text{C}) \) is placed at a corner 1, the charge has no electric potential energy, \( \text{EPE}_1 = 0 \). This is because the electric potential \( V_1 \) produced by the other two charges at corner 1 is zero, since they are infinitely far away.

Once the 8.00-\( \mu\text{C} \) charge is in place, the electric potential \( V_2 \) that it creates at corner 2 is

\[
V_2 = \frac{kq_1}{r_{21}}
\]

where \( r_{21} = 5.00 \text{ m} \) is the distance between corners 1 and 2, and \( q_1 = 8.00 \,\mu\text{C} \). When the 20.0-\( \mu\text{C} \) charge is placed at corner 2, its electric potential energy \( \text{EPE}_2 \) is

\[
\text{EPE}_2 = q_2V_2 = q_2 \left( \frac{kq_1}{r_{21}} \right) = (20.0 \times 10^{-6} \,\text{C}) \left( \frac{8.99 \times 10^9 \,\text{N} \cdot \text{m}^2/\text{C}^2 \cdot 8.00 \times 10^{-6} \,\text{C}}{5.00 \,\text{m}} \right) = 0.288 \,\text{J}
\]

The electric potential \( V_3 \) at the remaining empty corner is the sum of the potentials due to the two charges that are already in place on corners 1 and 2:

\[
V_3 = \frac{kq_1}{r_{31}} + \frac{kq_2}{r_{32}}
\]

where \( q_1 = 8.00 \,\mu\text{C}, \, r_{31} = 3.00 \text{ m}, \, q_2 = 20.0 \,\mu\text{C}, \) and \( r_{32} = 4.00 \text{ m} \). When the third charge \( (q_3 = -15.0 \,\mu\text{C}) \) is placed at corner 3, its electric potential energy \( \text{EPE}_3 \) is

\[
\text{EPE}_3 = q_3V_3 = q_3 \left( \frac{kq_1}{r_{31}} + \frac{kq_2}{r_{32}} \right) = q_3 \frac{k}{r_{31}} \left( \frac{q_1}{r_{31}} + \frac{q_2}{r_{32}} \right) = (15.0 \times 10^{-6} \,\text{C}) \left( \frac{8.99 \times 10^9 \,\text{N} \cdot \text{m}^2/\text{C}^2 \cdot 8.00 \times 10^{-6} \,\text{C}}{3.00 \,\text{m}} + \frac{20.0 \times 10^{-6} \,\text{C}}{4.00 \,\text{m}} \right) = -1.034 \,\text{J}
\]

The electric potential energy of the entire array is given by

\[
\text{EPE} = \text{EPE}_1 + \text{EPE}_2 + \text{EPE}_3 = 0 + 0.288 \,\text{J} + (-1.034 \,\text{J}) = -0.746 \,\text{J}
\]

26. REASONING AND SOLUTION  The information about the electric field requires that
\[
\frac{kq_2}{(1.00 \text{ m})^2} = \frac{kq_1}{(4.00 \text{ m})^2} \quad \text{so} \quad q_2 = \frac{1}{16.0} q_1
\]

Let \( x \) be the distance of the zero-potential point from the negative charge. Then

\[
kq_1/(d + x) = kq_2/x
\]

if the point is to the right of \( q_2 \) and

\[
kq_1/(d - x) = kq_2/x
\]

if the point is to the left of \( q_2 \). Solving for \( x \) gives

\[
\begin{align*}
x &= 0.200 \text{ m to the right of the negative charge} \\
x &= 0.176 \text{ m to the left of the negative charge}
\end{align*}
\]

27. **REASONING AND SOLUTION** The electrical potential energy of the group of charges is

\[
\text{EPE} = kq_1q_2/d + kq_1q_3/(2d) + kq_2q_3/d = 0
\]

so

\[
q_1q_2 + \frac{1}{2}q_1q_3 + q_2q_3 = 0
\]

a. If \( q_1 = q_2 = q \), then

\[
q + \frac{1}{2}q_3 + q_3 = 0 \quad \text{or} \quad q_3 = -\frac{2}{3} q
\]

b. If \( q_1 = q \) and \( q_2 = -q \) then

\[
-q + \frac{1}{2}q_3 - q_3 = 0 \quad \text{or} \quad q_3 = -2q
\]

28. **REASONING** The only force acting on each particle is the conservative electric force. Therefore, the total energy (kinetic energy plus electric potential energy) is conserved as the particles move apart. In addition, the net external force acting on the system of two particles is zero (the electric force that each particle exerts on the other is an internal force). Thus, the total linear momentum of the system is also conserved. We will use the conservation of energy and the conservation of linear momentum to find the initial separation of the particles.

**SOLUTION** For two points, \( A \) and \( B \), along the motion, the conservation of energy is
ELECTRIC POTENTIAL ENERGY AND THE ELECTRIC POTENTIAL

\[
\frac{1}{2}m_1v_{1,A}^2 + \frac{1}{2}m_2v_{2,A}^2 + \frac{kq_1q_2}{r_A} = \frac{1}{2}m_1v_{1,B}^2 + \frac{1}{2}m_2v_{2,B}^2 + \frac{kq_1q_2}{r_B}
\]

Initial kinetic energy of the two particles
Initial electric potential energy
Final kinetic energy of the two particles
Final electric potential energy

Solving this equation for \(1/r_A\) and setting \(v_{1,A} = v_{2,A} = 0\) since the particles are initially at rest, we obtain

\[
\frac{1}{r_A} = \frac{1}{r_B} + \frac{1}{kq_1q_2}\left(\frac{1}{2}m_1v_{1,B}^2 + \frac{1}{2}m_2v_{2,B}^2\right)
\]

(1)

This equation cannot be solved for the initial separation \(r_A\), because the final speed \(v_{2,B}\) of the second particle is not known. To find this speed, we will use the conservation of linear momentum:

\[
\frac{m_1v_{1,A} + m_2v_{2,A}}{\text{Initial linear momentum}} = \frac{m_1v_{1,B} + m_2v_{2,B}}{\text{Final linear momentum}}
\]

Setting \(v_{1,A} = v_{2,A} = 0\) and solving for \(v_{2,B}\) gives

\[
v_{2,B} = -\frac{m_1}{m_2}v_{1,B} = -\frac{3.00 \times 10^{-3} \text{ kg}}{6.00 \times 10^{-3} \text{ kg}}(125 \text{ m/s}) = -62.5 \text{ m/s}
\]

Substituting this value for \(v_{2,B}\) into Equation (1) yields

\[
\frac{1}{r_A} = \frac{1}{0.100 \text{ m}} + \frac{1}{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(8.00 \times 10^{-6} \text{ C})^2}
\]

\[
\times \left[\frac{1}{2}(3.00 \times 10^{-3} \text{ kg})(125 \text{ m/s})^2 + \frac{1}{2}(6.00 \times 10^{-3} \text{ kg})(-62.5 \text{ m/s})^2\right]
\]

\[
r_A = 1.41 \times 10^{-2} \text{ m}
\]

29. **REASONING AND SOLUTION** Since all points on the equipotential surface are the same distance \(r\) from the point charge, the potential is given by Equation 19.6,

\[
V = \frac{kq}{r} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.0 \times 10^{-7} \text{ C})}{0.15 \text{ m}} = 18000 \text{ V}
\]

30. **REASONING AND SOLUTION** We know \(V = kq/r\), so
\[ r = \frac{\kappa q}{V} = \frac{\left(9.0 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) \left(2.0 \times 10^{-10} \text{ C} \right)}{12 \text{ V}} = 0.15 \text{ m} \]

31. **REASONING** The magnitude \( E \) of the electric field is given by Equation 19.7 as \( E = \frac{\Delta V}{\Delta s} \), where \( \Delta V \) is the potential difference between the two metal conductors of the spark plug, and \( \Delta s \) is the distance between the two conductors. We can use this relation to find \( \Delta V \).

**SOLUTION** The potential difference between the conductors is
\[ \Delta V = E\Delta s = \left(2.8 \times 10^6 \text{ V/m}\right) \left(0.75 \times 10^{-3} \text{ m}\right) = 2.1 \times 10^3 \text{ V} \]

32. **REASONING AND SOLUTION** From Equation 19.7 we know that \( E = -\frac{\Delta V}{\Delta s} \), where \( \Delta V \) is the potential difference between the two surfaces of the membrane, and \( \Delta s \) is the distance between them. If \( A \) is a point on the positive surface and \( B \) is a point on the negative surface, then \( \Delta V = V_A - V_B = 0.070 \text{ V} \). The electric field between the surfaces is
\[ E = -\frac{\Delta V}{\Delta s} = -\frac{V_B - V_A}{\Delta s} = \frac{V_A - V_B}{\Delta s} \]
\[ = \frac{0.070 \text{ V}}{8.0 \times 10^{-9} \text{ m}} = 8.8 \times 10^6 \text{ V/m} \]

33. **REASONING AND SOLUTION** As described in the problem statement, the charges jump between your hand and a doorknob. If we assume that the electric field is uniform, Equation 19.7 applies, and we have
\[ E = -\frac{\Delta V}{\Delta s} = -\frac{V_{\text{knob}} - V_{\text{hand}}}{\Delta s} \]

Therefore, solving for the potential difference between your hand and the doorknob, we have
\[ V_{\text{knob}} - V_{\text{hand}} = -E\Delta s = -\left(-3.0 \times 10^6 \text{ N} / \text{C}\right)\left(3.0 \times 10^{-3} \text{ m}\right) = +9.0 \times 10^3 \text{ V} \]

34. **REASONING AND SOLUTION** The ratio of the potentials of the two equipotential surfaces surrounding the positive point charge is
\[ \frac{V_A}{V_B} = \frac{kq/r_A}{kq/r_B} = \frac{r_B}{r_A} \]  

The equipotential surfaces around a single point charge are concentric spheres; therefore, the area of any equipotential surface around a point charge is equal to \(4\pi r^2\), where \(r\) is the distance from the point charge to the equipotential surface.

Since surface A has twice the area of surface B,

\[ 4\pi r_A^2 = 2(4\pi r_B^2) \quad \text{so that} \quad \frac{r_B^2}{r_A^2} = \frac{1}{2} \quad \text{and} \quad \frac{r_B}{r_A} = \frac{1}{\sqrt{2}} \]

Then, from Equation (1)

\[ \frac{V_A}{V_B} = \frac{r_B}{r_A} = \frac{1}{\sqrt{2}} = 0.707 \]

---

35. **REASONING AND SOLUTION**  
The drawing shows the electric field and the three points, A, B, and C, around the point P. We choose the upward direction as being positive. Thus, \(E = -3.0 \times 10^3 \text{ V/m}\), since the electric field points straight down.

a. The electric potential at point A can be determined from Equation 19.7 as

\[ E = \frac{-\Delta V}{\Delta s} = \frac{V_A - V_P}{\Delta s} = \frac{V_A - 135 \text{ V}}{8.0 \times 10^{-3} \text{ m}} \]

so that \(V_A = 159 \text{ V}\).

b. The electric potential at point B is found in a similar manner:

\[ E = \frac{-\Delta V}{\Delta s} = \frac{V_B - V_P}{\Delta s} = \frac{V_B - 135 \text{ V}}{-3.3 \times 10^{-3} \text{ m}} \]

so that \(V_B = 125 \text{ V}\).

c. \(\Delta V = 0\), since the path is perpendicular to the electric field, and no work is done in moving a charge along such a path: \(V_C = 135 \text{ V}\).
36. **REASONING** The electric field \( E \) that exists between two points in space is, according to Equation 19.7, proportional to the electric potential difference \( \Delta V \) between the points divided by the distance \( \Delta x \) between them: \( E = -\frac{\Delta V}{\Delta x} \).

**SOLUTION**

a. The electric field in the region from \( A \) to \( B \) is

\[
E = -\frac{\Delta V}{\Delta x} = -\frac{5.0 \text{ V} - 5.0 \text{ V}}{0.20 \text{ m} - 0 \text{ m}} = 0 \text{ V/m}
\]

b. The electric field in the region from \( B \) to \( C \) is

\[
E = -\frac{\Delta V}{\Delta x} = -\frac{3.0 \text{ V} - 5.0 \text{ V}}{0.40 \text{ m} - 0.20 \text{ m}} = 1.0 \times 10^1 \text{ V/m}
\]

c. The electric field in the region from \( C \) to \( D \) is

\[
E = -\frac{\Delta V}{\Delta x} = -\frac{1.0 \text{ V} - 3.0 \text{ V}}{0.80 \text{ m} - 0.40 \text{ m}} = 5.0 \text{ V/m}
\]

37. **SSM  REASONING** The total energy of the particle on the equipotential surface \( A \) is \( E_A = \frac{1}{2} m v_A^2 + q V_A \). Similarly, the total energy of the particle when it reaches equipotential surface \( B \) is \( E_B = \frac{1}{2} m v_B^2 + q V_B \). According to Equation 6.8, the work \( W \) done by the outside force is equal to the final total energy \( E_B \) minus the initial total energy \( E_A \), \( W = E_B - E_A \).

**SOLUTION** The work done by the outside force in moving the particle from \( A \) to \( B \) is,

\[
W = E_B - E_A = \left( \frac{1}{2} m v_B^2 + q V_B \right) - \left( \frac{1}{2} m v_A^2 + q V_A \right) = \frac{1}{2} m (v_B^2 - v_A^2) + q(V_B - V_A)
\]

\[
= \frac{1}{2} (5.00 \times 10^{-2} \text{ kg}) [(3.00 \text{ m/s})^2 - (2.00 \text{ m/s})^2]
\]

\[
+ (4.00 \times 10^{-5} \text{ C}) [7850 \text{ V} - 5650 \text{ V}] = 0.213 \text{ J}
\]

38. **REASONING AND SOLUTION** Let \( E_1 \) represent the magnitude of the electric field on the first equipotential surface, and \( E_2 \) represent the magnitude of the electric field when it has shrunk to one-half of its initial value. Then

\[
E_1 = \frac{kq}{r_1^2} \quad \text{and} \quad E_2 = \frac{kq}{r_2^2}
\]
Since \( E_2 = \frac{1}{2} E_1 \), we have

\[
\frac{kq}{r_2^2} = \frac{1}{2} \frac{kq}{r_1^2} \quad \text{so that} \quad \frac{r_2}{r_1} = \sqrt{2}
\]

Thus, the electric field shrinks to one-half its original value at a distance of \( r_2 = \sqrt{2} r_1 \).

The potential difference between the first equipotential surface and the locus of points where the electric field has dropped to one-half of its original magnitude is

\[
\Delta V = \frac{kq}{r_2} - \frac{kq}{r_1} = \frac{kq}{r_1} \left( \frac{1}{\sqrt{2}} - 1 \right)
\]

\[
= \left( \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \times 2.00 \times 10^{-6} \text{ C}}{1.60 \text{ m}} \right) \left( \frac{1}{\sqrt{2}} - 1 \right) = -3290 \text{ V}
\]

Since the potential difference between two successive surfaces is \( 1.00 \times 10^3 \text{ V} \), we have

\[
\text{Magnitude of } \frac{\Delta V \text{ from } r_1 \text{ to } r_2}{\Delta V \text{ between successive equipotential surfaces}} = \frac{3290 \text{ V}}{1.00 \times 10^3 \text{ V}} = 3.29
\]

Therefore, the number of equipotential surfaces crossed in going radially outward from \( r_1 \) to \( r_2 \) is \( 3 \).

### 39. REASONING AND SOLUTION

The capacitance, voltage, and charge are related by

\[
V = \frac{q}{C} = \frac{7.2 \times 10^{-5} \text{ C}}{6.0 \times 10^{-6} \text{ F}} = 12 \text{ V}
\]

### 40. REASONING AND SOLUTION

For an empty capacitor

\[
A = \frac{Cd}{\varepsilon_0} = \frac{(1.00 \text{ F})(1.00 \text{ m})}{8.85 \times 10^{-12} \text{ F/m}} = 1.13 \times 10^{11} \text{ m}^2
\]

In square miles, this area is

\[
A = \left( 1.13 \times 10^{11} \text{ m}^2 \right) \frac{1 \text{ mi}^2}{2.59 \times 10^6 \text{ m}^2} = 4.36 \times 10^4 \text{ mi}^2
\]
41. **REASONING AND SOLUTION** Equation 19.10 gives the capacitance for a parallel plate capacitor filled with a dielectric of constant \( \kappa \): 
\[
C = \kappa \varepsilon_0 A / d .
\]
Solving for \( \kappa \), we have
\[
\kappa = \frac{Cd}{\varepsilon_0 A} = \frac{(7.0 \times 10^{-6} \text{ F})(1.0 \times 10^{-5} \text{ m})}{(8.85 \times 10^{-12} \text{ F/m})(1.5 \text{ m}^2)} = 5.3
\]

42. **REASONING AND SOLUTION** The capacitance is given by
\[
C = \frac{k\varepsilon_0 A}{d} = \frac{5(8.85 \times 10^{-12} \text{ F/m})(5 \times 10^{-6} \text{ m}^2)}{1 \times 10^{-8} \text{ m}} = 2 \times 10^{-8} \text{ F}
\]

43. **REASONING AND SOLUTION** The total charge transferred is given by
\[
q = CV = (2.5 \times 10^{-8} \text{ F})(450 \text{ V})
\]
The number of electrons transferred is, then,
\[
\text{Number of electrons} = \frac{q}{e} = \frac{(2.5 \times 10^{-8} \text{ F})(450 \text{ V})}{1.60 \times 10^{-19} \text{ C}} = 7.0 \times 10^{13}
\]

44. **REASONING AND SOLUTION** The capacitance is
\[
C = \frac{q}{V} = \frac{2.7 \times 10^{-6} \text{ C}}{1.5 \text{ V}} = 1.8 \times 10^{-6} \text{ F}
\]
The plate separation is given by
\[
d = \frac{k\varepsilon_0 A}{C} = \frac{5.4(8.85 \times 10^{-12} \text{ F/m})(3.8 \text{ m}^2)}{1.8 \times 10^{-6} \text{ F}} = 1.0 \times 10^{-4} \text{ m}
\]

45. **REASONING** The charge that resides on the outer surface of the cell membrane is \( q = CV \), according to Equation 19.8. Before we can use this expression, however, we must first determine the capacitance of the membrane. If we assume that the cell membrane behaves like a parallel plate capacitor filled with a dielectric, Equation 19.10 \( C = \kappa \varepsilon_0 A / d \) applies as well.

**SOLUTION** The capacitance of the cell membrane is
\[
C = \frac{\kappa \varepsilon_0 A}{d} = \frac{(5.0)(8.85 \times 10^{-12} \text{ F/m})(5.0 \times 10^{-9} \text{ m}^2)}{1.0 \times 10^{-8} \text{ m}} = 2.2 \times 10^{-11} \text{ F}
\]
a. The charge on the outer surface of the membrane is, therefore,

\[ q = CV = (2.2 \times 10^{-11} \text{ F})(60.0 \times 10^{-3} \text{ V}) = 1.3 \times 10^{-12} \text{ C} \]

b. If the charge in part (a) is due to K\(^+\) ions with charge +e (\(e = 1.6 \times 10^{-19} \text{ C}\)), the number of ions present on the outer surface of the membrane is

\[
\text{Number of } K^+ \text{ ions} = \frac{1.3 \times 10^{-12} \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 8.1 \times 10^6
\]

46. **REASONING** The capacitance of a capacitor is defined by Equation 19.8 as \( C = \frac{q}{V} \), where \( q \) is the magnitude of the charge on one of the plates and \( V \) is the potential difference between the plates. The potential difference is related to the magnitude \( E \) of the electric field between the plates by \( E = \frac{V}{\Delta s} \) (Equation 19.7), where \( \Delta s \) is the distance between the plates. Eliminating \( V \) from these two equations gives \( C = \frac{q}{(E\Delta s)} \).

**SOLUTION** The capacitance of the parallel plate capacitor is

\[
C = \frac{q}{E\Delta s} = \frac{1.9 \times 10^{-5} \text{ C}}{(640 \text{ V/m})(11 \times 10^{-3} \text{ m})} = 2.7 \times 10^{-6} \text{ F}
\]

47. **REASONING AND SOLUTION**

a. We know that

\[
\text{Energy} = \frac{1}{2} CV^2 = \frac{1}{2}(750 \times 10^{-6} \text{ F})(330 \text{ V})^2 = 41 \text{ J}
\]

b. The power developed by the flash is

\[
P = \frac{\text{Energy}}{\text{Time}} = \frac{41 \text{ J}}{5.0 \times 10^{-3} \text{ s}} = 8200 \text{ W}
\]

48. **REASONING AND SOLUTION** The electric energy stored in the region between the metal spheres is

\[
\text{Energy} = \frac{1}{2} \kappa \varepsilon_0 E^2 (\text{Volume}) \tag{19.12}
\]

However, the magnitude \( E \) of the electric field is related to the potential difference \( \Delta V \) between the spheres and the distance \( \Delta s \) between them via Equation 19.7, \( E = \frac{\Delta V}{\Delta s} \). Thus,
49. **REASONING** According to Equation 19.11, the energy stored in a capacitor with a capacitance \( C \) and potential \( V \) across its plates is

\[
\text{Energy} = \frac{1}{2} CV^2
\]

Once we determine how much energy is required to operate a 75-W light bulb for one minute, we can then use the expression for the energy to solve for \( V \).

**SOLUTION** The energy stored in the capacitor, which is equal to the energy required to operate a 75-W bulb for one minute (\( = 60 \) s), is

\[
\text{Energy} = Pt = (75 \text{ W})(60 \text{ s}) = 4500 \text{ J}
\]

Therefore, solving Equation 19.11 for \( V \), we have

\[
V = \sqrt{\frac{2\text{Energy}}{C}} = \sqrt{\frac{2(4500 \text{ J})}{3.3 \text{ F}}} = 52 \text{ V}
\]

50. **REASONING AND SOLUTION** From Equations 19.9 and 18.4, we have

\[
A = \frac{q}{\kappa \varepsilon_0 E} = \frac{1.7 \times 10^{-7} \text{ C}}{(3.5) \left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right) \left(1.4 \times 10^7 \text{ N/C}\right)} = 3.9 \times 10^{-4} \text{ m}^2
\]

Since \( A = \pi r^2 \), the radius of the plates is

\[
r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{3.9 \times 10^{-4} \text{ m}^2}{\pi}} = 1.1 \times 10^{-2} \text{ m}
\]

51. **REASONING AND SOLUTION** The potential difference is constant so \( V_0 = q_0/C_0 = q/C = q/(\kappa C_0) \).

Thus,

\[
q = kq_0 = kC_0 V_0 = (4.0)(2.7 \times 10^{-6} \text{ F})(12 \text{ V}) = 1.3 \times 10^{-4} \text{ C}
\]
The original charge is \( q_0 = C_0V_0 = 3.2 \times 10^{-5} \text{ C} \). The surface charge \( Q \) is, therefore,

\[
Q = q - q_0 = 1.0 \times 10^{-4} \text{ C}
\]

52. **REASONING AND SOLUTION** The electron and proton experience forces of the same magnitude in the same electric field, since they have the same magnitude of charge. Newton's second law gives

\[
m_p a_p = m_e a_e
\]

where \( a_p \) and \( a_e \) are the accelerations of the proton and the electron. Kinematics gives

\[
a_p = 2x_p/t^2 \quad \text{and} \quad a_e = 2x_e/t^2
\]

so

\[
x_p = \frac{m_e x_e}{m_p} = \frac{9.11 \times 10^{-31} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} (0.0400 \text{ m}) = 2.18 \times 10^{-5} \text{ m}
\]

53. **SSM REASONING** According to Equation 19.10, the capacitance of a parallel plate capacitor filled with a dielectric is

\[
C = \kappa \epsilon_0 A/d,
\]

where \( \kappa \) is the dielectric constant, \( A \) is the area of one plate, and \( d \) is the distance between the plates.

From the definition of capacitance (Equation 19.8), \( q = CV \). Thus, the charge on a parallel plate capacitor that contains a dielectric is given by Equation 19.10 as

\[
q = \left( \kappa \epsilon_0 A/d \right)V.
\]

Since each dielectric occupies one-half of the volume between the plates, the area of each plate in contact with each material is \( A/2 \). Thus,

\[
q_1 = \frac{\kappa_1 \epsilon_0 A/2}{d}V = \frac{\kappa_1 \epsilon_0 A}{2d}V \quad \text{and} \quad q_2 = \frac{\kappa_2 \epsilon_0 A/2}{d}V = \frac{\kappa_2 \epsilon_0 A}{2d}V
\]

According to the problem statement, the total charge stored by the capacitor is

\[
q_1 + q_2 = CV
\]

where \( q_1 \) and \( q_2 \) are the charges on the plates in contact with dielectrics 1 and 2, respectively.

Using the expressions for \( q_1 \) and \( q_2 \) above, Equation (1) becomes

\[
CV = \frac{\kappa_1 \epsilon_0 A}{2d}V + \frac{\kappa_2 \epsilon_0 A}{2d}V = \frac{\kappa_1 \epsilon_0 A + \kappa_2 \epsilon_0 A}{2d}V = \frac{(\kappa_1 + \kappa_2) \epsilon_0 A}{2d}V
\]

This expression can be solved for \( C \).
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SOLUTION  Solving for C, we obtain

\[ C = \frac{\varepsilon_0 A (\kappa_1 + \kappa_2)}{2d} \]

54. REASONING AND SOLUTION  The electric potential is given by

\[ V = \frac{kq}{r} = \frac{\left(9.0 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) (1.0 \text{ C})}{1.0 \times 10^3 \text{ m}} = 9.0 \times 10^6 \text{ V} \]

55. REASONING AND SOLUTION  The potential at point A is

\[ V_A = \frac{kq}{r_A} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) (-3.00 \times 10^{-8} \text{ C})}{2.00 \text{ m}} = -135 \text{ V} \]

Similarly, the potential at point B is  \( V_B = kq/r_B = -89.9 \text{ V} \)

The difference in the potentials is

\[ V_B - V_A = +45 \text{ V} \]

Point B is at the higher potential.

56. REASONING AND SOLUTION

a. The change in the electric potential energy is

\[ E_{PE_A} - E_{PE_B} = W_{AB} = 5.80 \times 10^{-3} \text{ J} \]

b. The potential difference between the points is

\[ V_A - V_B = \frac{E_{PE_A} - E_{PE_B}}{q} = \frac{5.80 \times 10^{-3} \text{ J}}{1.80 \times 10^{-4} \text{ C}} = 32.2 \text{ V} \]

c. Point A has the higher potential.

57. SSM  REASONING  According to Equation 19.11, the energy stored in a capacitor with capacitance C and potential V across its plates is Energy = \( \frac{1}{2} CV^2 \).

SOLUTION  Therefore, solving Equation 19.11 for V, we have
\[ V = \sqrt{\frac{2\text{(Energy)}}{C}} = \sqrt{\frac{2(73 \text{ J})}{120 \times 10^{-6} \text{ F}}} = 1.1 \times 10^3 \text{ V} \]

58. **REASONING AND SOLUTION**

   a. The only force that acts on the particle is the conservative electric force. Therefore, the total energy of the particle is conserved as it moves from point A to point B:

   \[ \frac{1}{2}mv_A^2 + \text{EPE}_A = \frac{1}{2}mv_B^2 + \text{EPE}_B \]

   Since the particle starts from rest, \( v_A = 0 \). The electric potential \( V \) is related to the electric potential energy \( \text{EPE} \) by \( V = \frac{\text{EPE}}{q} \) (see Equation 19.3). With these changes, we can solve the equation above for the potential difference \( V_B - V_A \):

   \[ V_B - V_A = \frac{1}{q} \left( 0 - \frac{1}{2}mv_B^2 \right) = \frac{1}{-1.5 \times 10^{-6} \text{ C}} \left[ 0 - \frac{1}{2} \left( 2.5 \times 10^{-6} \text{ kg} \right) (42 \text{ m/s})^2 \right] = 1500 \text{ V} \]

   b. **Point B** is at the higher potential, because a negative charge accelerates from a lower potential to a higher potential.

59. **REASONING AND SOLUTION** The capacitance is \( C = \frac{q_0}{V_0} = \frac{q}{V} \). The new charge \( q \) is, therefore,

   \[ q = \frac{q_0 V}{V_0} = \frac{(5.3 \times 10^{-5} \text{ C})(9.0 \text{ V})}{6.0 \text{ V}} = 8.0 \times 10^{-5} \text{ C} \]

60. **REASONING AND SOLUTION** Let \( d = 2.0 \text{ m} \) be the separation between the charges, and let \( y \) be the distance from the negative charge to the point where the potential is zero. Then

   \[ \frac{kq}{y} = \frac{2kq}{\sqrt{y^2 + d^2}} \]

   so \( y = \pm \frac{d}{\sqrt{3}} = \pm 1.2 \text{ m} \). The points are located \( 1.2 \text{ m on either side of the negative charge} \).

61. **SSM REASONING AND SOLUTION** Equation 19.7 gives the result directly:
\[ E = -\frac{\Delta V}{\Delta s} = -\frac{V_B - V_A}{\Delta s} = -\frac{28 \text{ V} - 95 \text{ V}}{0.016 \text{ m}} = 4.2 \times 10^3 \text{ V/m} \]

The electric field points from high potential to low potential. Thus, it points from \( A \) to \( B \).

62. **REASONING AND SOLUTION** The charge on the empty capacitor is \( q_0 = C_0 V_0 \). With the dielectric in place, the charge remains the same. However, the new capacitance is \( C = \kappa C_0 \) and the new voltage is \( V \). Thus, \[ q_0 = CV = \kappa C_0 V = C_0 V_0 \]

Solving for the new voltage yields \[ V = V_0 / \kappa = (12.0 \text{ V}) / 2.8 = 4.3 \text{ V} \]

The potential difference is \( 12.0 - 4.3 = 7.7 \text{ V} \). The change in potential is a **decrease**.

63. **REASONING AND SOLUTION** Let point \( A \) be on the \( x \)-axis where the potential is 515 V. Let point \( B \) be on the \( x \)-axis where the potential is 495 V. From Equation 19.7, the electric field is \[ E = -\frac{\Delta V}{\Delta s} = -\frac{V_B - V_A}{\Delta s} = -\frac{495 \text{ V} - 515 \text{ V}}{2(6.0 \times 10^{-3} \text{ m})} = -1.7 \times 10^3 \text{ V/m} \]

The magnitude of the electric field is \( 1.7 \times 10^3 \text{ V/m} \). Since the electric field is negative, it points to the **left**, from the high toward the low potential.

64. **REASONING** The only force acting on each particle is the conservative electric force. Therefore, the total energy (kinetic energy plus electric potential energy) is conserved as the particles move apart. In addition, the net external force acting on the system of two particles is zero (the electric force that each particle exerts on the other is an internal force). Thus, the total linear momentum of the system is also conserved. We will use the conservation of energy and the conservation of linear momentum to find the final speed of each particle.

**SOLUTION** For two points, \( A \) and \( B \), along the motion, the conservation of energy is

\[
\frac{1}{2} m v^2_{1,A} + \frac{1}{2} m v^2_{2,A} + \frac{kq_1 q_2}{r_A} = \frac{1}{2} m v^2_{1,B} + \frac{1}{2} m v^2_{2,B} + \frac{kq_1 q_2}{r_B}
\]

Initial kinetic energy of the two particles
Initial electric potential energy
Final kinetic energy of the two particles
Final electric potential energy
Setting $v_{1,A} = v_{2,A} = 0$ since the particles are initially at rest, and letting $r_B = \frac{1}{2} r_A$, the conservation of energy equation becomes

$$\frac{1}{2} m v_{1,B}^2 + \frac{1}{2} m v_{2,B}^2 = -\frac{k q_1 q_2}{r_A}$$  \hspace{1cm} (1)

This equation cannot be solved for $v_{1,B}$ because the final speed $v_{2,B}$ of the second particle is not known. To find this speed, we will use the conservation of linear momentum:

$$m v_{1,A} + m v_{2,A} = m v_{1,B} + m v_{2,B}$$

Initial linear momentum \hspace{1cm} Final linear momentum

Setting $v_{1,A} = v_{2,A} = 0$ and solving for $v_{2,B}$ gives $v_{2,B} = -v_{1,B}$. Substituting this result into Equation (1) and solving for $v_{1,B}$ yields

$$v_{1,B} = \sqrt{-\frac{k q_1 q_2}{m r_A}}
= \sqrt{-\left(9.0 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2\right) \left(+5.0 \times 10^{-6} \text{ C}\right) \left(-5.0 \times 10^{-6} \text{ C}\right) \left(6.0 \times 10^{-3} \text{ kg}\right) \left(0.80 \text{ m}\right)} = 6.8 \text{ m/s}$$

This is also the speed of $v_{2,B}$.

65. **SSM WWW REASONING** If we assume that the motion of the proton and the electron is horizontal in the $+x$ direction, the motion of the proton is determined by Equation 2.8, $x = v_0 t + \frac{1}{2} a_p t^2$, where $x$ is the distance traveled by the proton, $v_0$ is its initial speed, and $a_p$ is its acceleration. If the distance between the capacitor places is $d$, then this relation becomes

$$\frac{1}{2} d = v_0 t + \frac{1}{2} a_p t^2$$

or

$$d = 2v_0 t + a_p t^2$$  \hspace{1cm} (1)

We can solve Equation (1) for the initial speed $v_0$ of the proton, but, first, we must determine the time $t$ and the acceleration $a_p$ of the proton. Since the proton strikes the negative plate at the same instant the electron strikes the positive plate, we can use the motion of the electron to determine the time $t$.

For the electron, $\frac{1}{2} d = \frac{1}{2} a_e t^2$, where we have taken into account the fact that the electron is released from rest. Solving this expression for $t$ we have $t = \sqrt{d / a_e}$. Substituting this expression into Equation (1), we have
\[ d = 2v_0 \sqrt{\frac{d}{a_e} + \left( \frac{a_p}{a_e} \right) d} \]  

(2)

The accelerations can be found by noting that the magnitudes of the forces on the electron and proton are equal, since these particles have the same magnitude of charge. The force on the electron is \( F = eE = eV/d \), and the acceleration of the electron is, therefore,

\[ a_e = \frac{F}{m_e} = \frac{eV}{m_e d} \]  

(3)

Newton's second law requires that \( m_e a_e = m_p a_p \), so that

\[ \frac{a_p}{a_e} = \frac{m_e}{m_p} \]  

(4)

Combining Equations (2), (3) and (4) leads to the following expression for \( v_0 \), the initial speed of the proton:

\[ v_0 = \frac{1}{2} \left( 1 - \frac{m_e}{m_p} \right) \sqrt{\frac{eV}{m_e}} \]

SOLUTION  Substituting values into the expression above, we find

\[ v_0 = \frac{1}{2} \left( 1 - \frac{9.11 \times 10^{-31} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} \right) \sqrt{\frac{(1.6 \times 10^{-19} \text{ C})(175 \text{ V})}{9.11 \times 10^{-31} \text{ kg}}} = 2.77 \times 10^6 \text{ m/s} \]

66. REASONING AND SOLUTION

a. Let \( d \) be the distance between the charges. The potential at the point \( x_1 = 4.00 \text{ cm} \) to the left of the negative charge is

\[ V = 0 = \frac{kq_1}{d - x_1} - \frac{kq_2}{x_1} \]

which gives

\[ \frac{q_1}{q_2} = \frac{d}{x_1} - 1 \]  

(1)

Similarly, at the point \( x_2 = 7.00 \text{ cm} \) to the right of the negative charge we have
\[ V = 0 = \frac{kq_1}{x_2 + d} - \frac{kq_2}{x_2} \]

which gives

\[ \frac{q_1}{q_2} = \frac{d}{x_2} + 1 \]  \hspace{1cm} (2)

Equating Equations (1) and (2) and solving for \(d\) gives \(d = 0.187\, \text{m}\).

b. Using the above value for \(d\) in Equation (1) yields \(\frac{q_1}{q_2} = 3.67\).