Possible Questions for Final Exam

1. Derive Lagrange equation from the minimal action principle

2. Show that adding $\frac{df}{dt}$ function will not change the equation of the motion. On which arguments f should be dependent in order to the above statment to be true?

3. Using the Lagrangian of electromagnetic interaction:

$$L = \frac{1}{2} m \dot{r}^2 - q \varphi (r, t) + \frac{q}{c} \vec{A} (r, t) \dot{\vec{r}}$$

a) Obtain the Lorentz force using the Hamilton principle

b) Calculate the generalized momenutm and the generalized force

4. Derive Newton's 3 rd law from the Lagrangian of interacting two particle system

5. Derive Hamilton's equations

6. Derive Posson Brackets

7. Show that the angular momentum is vector with respect to rotation and pseudovector with respect to space inversion

9. What are the definition of energy and momentum conservations in Hamiltonian formalism?

10. What is the main characteristics of complex vector space that makes it a Hilbert Space

11. Which sates can be called orthonormal

12. Express the inverse of the product of operators ABC through the inverse of the individual operators A, B and C.

13 Express the matrix element of the product of two operators AB through the matrix elements of operators A and B

14. Show that if two operators have a common eigenstate then they are commuting.

15. Show that eighenvalues of Hermitian operators are real

16. For orthonormal vector states construct an operator for which these states are eigenstates

17. Prove that $(AB)^\dagger = B^\dagger A^\dagger$

18. Show that operators of contineous symmetry transformation are unitarity operators. Then show that corresponding generators are Hermitean operators

19. Prove that operator of space inversion is a hermitean operator

20. Show that commuting operators may have common eigenstates

21. Prove the Uncertainity Priniple for two noncumtating operators A and B

22. Elaborate correspondence principle

23. Using time - translation and correspondence princples derive Schroedenger equation.

24. Based on Schroedinger equation show that total probability is conserved (is time independent)

25. Consider space translation symmetry for the quantum vector state and obtain the generator of this transformation. Obtain commutative properties of this generator with the operator of position / coordinate.

26. Introduce the wave function and obtain the expression of the operator of momentum in the basis of the coordinate wave functions.

27. Obtain the expression of p^2 operator in the coordinate space.

28. In the coordinate space calculate the commutator $[p, f(x)]$ where $f(x)$ is a contineous function of coordinate.

29. Obtain the Schroedinger equation in coordinate representation.

30. From Schroedinger equation derive continuity equation

31. Show why continuity equation is important for time independence of the total probability of the quantum state.

32. Discuss the time dependence of the eigenstate of stationary Hamiltonian (i.e. it is not explicitely time dependent)

33. Derive the time evolution for arbitrary stationary state.

34. Discuss the seven basic principles of Schroedinger equation for stationary states

35. Derive the virial theorem

36. Calculate the commutator of momentum and space inversion operator. Same for coordinate and space inversion operators. As well as discuss the properties of space inversion operator - such as whether it is herimtean and what eigenvalues it has.

37. Obtain eigenstates and eigenvalues of one dimensional infinite square well potential

38. Calculate the energy spectrum of Harmonic Oscilliator using canonical quantization prescription.

39. Calculate the ground state wave function for harmonic oscilliator

40. Demonstrate the virial theore for the case of HO.

41. Demonstrate the uncertainty priciple for x and p for the ground state of Harmonic oscilliator.

42. Derive the matrix elements of a and a^+

43. Obtain the requrent formula for $(| \psi >)_n$ expressed through the ground state wave function of HO.

44. Show that $a^n a^{+n} = n! + n a^+ a$

45. Calculate the expectation value of kinetic and potential energies of HO

46. Show that the angular momentum operator is a generator for the rotational operation as it is applied to the wave function of quantum system.

47. Show that L is hermitean operator

48. Show that $[L^2\ H] = 0$ and $[L_z\ H] = 0$ for spherically symmetric H.

49. Show that $[L^2\ F] = 0$ and $[L_z\ F] = 0$ for any function F which is a function of p^2, r^2 or $p \cdot r$

50. Calculate $[r_i\ L_j]$ and $[p_i\ L_j]$

51. Obtain the expression for L^2 and L_z in spherical coordinate representation.

52. Reproduce the 9 steps of determination of the quantum properties of the generators of rotation.

53. Obtain operator equations for L^2 and L_z in polar coordinates.

54. Show that the parity of the state defined by the magnitude of l.

55. Discuss the properties of the radial part of the wave function. Its asymptotics at $r \to 0$ and $r \to \infty$, for situation in which the potential energy disappears at infinity and increases at $r \to 0$ slower than $1/r^2$

56. Derive the free radial wave function for the $l = 0$ case

57. Using quantum mechanichal generalization of Rung - Lenz vector

$$\vec{A} = \frac{1}{2}(\hbar L \times p) - \frac{1}{2}(p \times \hbar L) + Ze^2 m \frac{\vec{r}}{r}$$

(a) show that it is hermitian operator

(b) calculate $[L_i\ A_j]$ and $[A_i\ A_j]$

(c) calculate $[L^2\ A^2]$, $[L_i\ A^2]$, $[A_i\ L^2]$, $[A_i\ A^2]$, $[H, A_i]$,
for Spherically symmetric hamiltonian (like coulomb interaction)

58. Calculate $[J_i, J_j]$, $[K_i, K_j]$, $[J_i, K_j]$, ...
Show that j = k, where j and k are eigenvalues of J^2 and K^2

59. Using A^2 derive energy spectrum of hydrogen atom

60. Reproduce the general derivation of the radial wave function of Hydrogen atom.
Then calculate the full wave function for n = 1, l = 0 and n = 2, l = 0 as well as n = 2 and l = 1