ALTERNATE METHOD FOR ANALYZING LOGICAL ARGUMENTS

A logical argument with *n* premises and conclusion *C* is valid if and only if the statement $H \to C$ is a tautology, where *H* is the conjunction of the *n* premises $(p_1 \wedge p_2 \wedge ... \wedge p_n)$. Now consider the following:

[T is used for true, and F for false.]

1) The only time the conditional statement $H \to C$ is false is when the hypothesis (antecedent) *H* is true, and the conclusion *C* is false. $(T \to T = T, F \to T = T, F \to F = F, \text{ but } T \to F = F)$

2) For the conjunction $H = p_1 \wedge p_2 \wedge ... \wedge p_n$ to be true, every premise $p_i,\ i = 1,...,n$ must be true.

Facts 1 and 2 allows for **non-consideration of any case where one (or more) premise is false, because** $F \to C = T, C = \text{conclusion}$**.** Thus, if there are **no** cases of a false conclusion when **every single premise is true**, then the argument is valid. Otherwise, the argument is not valid.

EXAMPLE ONE) Decide if the given argument is valid:

$p \vee \sim q$
$q \to r$
$\sim q \vee \sim r$
$\therefore\ p$

In the given table, the premise and conclusion columns are labeled.

Row						Premise	Premise	Premise	Conclusion
	p	q	r	$\sim q$	$\sim r$	$p \vee \sim q$	$q \to r$	$\sim q \vee \sim r$	p
1	T	T	T	F	F	T	T	F	T
2	T	T	F	F	T	T	F	T	T
3	T	F	T	T	F	T	T	T	T
4	T	F	F	T	T	T	T	T	T
5	F	T	T	F	F	F	T	F	F
6	F	T	F	F	T	F	F	T	F
7	F	F	T	T	F	T	T	T	F
8	F	F	F	T	T	T	T	T	F

In rows 1, 2, 5, and 6, there is at least one false premise. Those rows are now eliminated from the table.

Row						Premise	Premise	Premise	Conclusion
	p	q	r	$\sim q$	$\sim r$	$p \vee \sim q$	$q \to r$	$\sim q \vee \sim r$	p
3	T	F	T	T	F	T	T	T	T
4	T	F	F	T	T	T	T	T	T
7	F	F	T	T	F	T	T	T	F
8	F	F	F	T	T	T	T	T	F

We are left with all the cases of all true premises. Although rows 3 and 4 have a true conclusion, rows 7 and 8 do not. A true conclusion must follow from **every case** of no false premises. That does not happen with this argument. Therefore, the argument is **invalid**.

EXAMPLE TWO) Decide if the argument is valid:

Henry will take chemistry if and only if he takes physics. If Henry does not take chemistry, he will take calculus. Henry does not take calculus. Therefore, he will take physics.

Let *p* be the statement “Henry will take chemistry, let *q* be “Henry takes physics”, and *r* be he will take calculus.

In symbols, the argument is:

$p \leftrightarrow q$
$\sim p \rightarrow r$
$\sim r$
$\therefore\ q$

TABLE:

					Premise	Premise	Premise	Conclusion
	p	q	r	$\sim p$	$\sim r$	$p \leftrightarrow q$	$\sim p \rightarrow r$	q
1)	T	T	T	F	F	T	T	T
2)	T	T	F	F	T	T	T	T
3)	T	F	T	F	F	F	T	F
4)	T	F	F	F	T	F	T	F
5)	F	T	T	T	F	F	T	T
6)	F	T	F	T	T	F	F	T
7)	F	F	T	T	F	T	T	F
8)	F	F	F	T	T	T	F	F

ROWS WITH FALSE PREMISES REMOVED:

					Premise	Premise	Premise	Conclusion
	p	q	r	$\sim p$	$\sim r$	$p \leftrightarrow q$	$\sim p \rightarrow r$	q
2)	T	T	F	F	T	T	T	T

There are no cases of all true premises with a false conclusion. Therefore, the argument is **valid**.