

THE FUNDAMENTAL IDENTITIES

KNOW THESE TO THE POINT WHERE IF AWAKENED FROM A DEEP SLEEP IN THE MIDDLE OF THE NIGHT, YOU WOULD BE ABLE TO RECITE THEM ALL IN NO MORE THAN TWO MINUTES!

1) Reciprocal Identities:

$$\text{a) } \csc x = \frac{1}{\sin x} \text{ or } \sin x = \frac{1}{\csc x} \text{ or } (\csc x)(\sin x) = 1 \quad \text{b) } \sec x = \frac{1}{\cos x} \text{ or } \cos x = \frac{1}{\sec x} \text{ or } (\sec x)(\cos x) = 1$$

$$\text{c) } \cot x = \frac{1}{\tan x} \text{ or } \tan x = \frac{1}{\cot x} \text{ or } (\cot x)(\tan x) = 1$$

2) Quotient Identities:

$$\text{a) } \tan x = \frac{\sin x}{\cos x} \quad \text{b) } \cot x = \frac{\cos x}{\sin x}$$

3) Pythagorean Identities:

$$\text{a) } \sin^2 x + \cos^2 x = 1 \text{ or } \sin^2 x = 1 - \cos^2 x \text{ or } \cos^2 x = 1 - \sin^2 x$$

$$\text{b) } \tan^2 x + 1 = \sec^2 x \text{ or } \tan^2 x = \sec^2 x - 1 \text{ or } \sec^2 x - \tan^2 x = 1$$

$$\text{c) } 1 + \cot^2 x = \csc^2 x \text{ or } \cot^2 x = \csc^2 x - 1 \text{ or } \csc^2 x - \cot^2 x = 1$$

4) Co-function Identities:

$$\text{a) } \cos x = \sin\left(\frac{\pi}{2} - x\right)$$

$$\text{b) } \csc x = \sec\left(\frac{\pi}{2} - x\right)$$

$$\text{c) } \cot x = \tan\left(\frac{\pi}{2} - x\right)$$

5) Identities for Negatives:

$$\text{a) } \sin(-x) = -\sin x$$

$$\text{b) } \cos(-x) = \cos x$$

$$\text{c) } \tan(-x) = -\tan x$$

[NOTE: These are sometimes referred to as the even-odd identities because they tell us that sine and tangent are odd functions, while cosine is an even function.]

6) Sum and Difference Identities

$$\text{a) } \sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\text{b) } \sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\text{c) } \cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\text{d) } \cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\text{e) } \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\text{f) } \tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

7) Double-Angle Identities:

$$\text{a) } \sin 2x = 2 \sin x \cos x \quad \text{b) } \cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1 \quad \text{c) } \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

8) Half-Angle Identities: (The sign + or - depends on the quadrant of $\frac{x}{2}$)

$$\text{a) } \sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\text{b) } \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\text{c) } \tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$