Assignment 5

1. Here is a flow net (computed the way we have always done these things). The head is uniform on the top left half of the domain at 0 m and on the top right half of the domain at 1 m. As you can see from the head contours, the other boundaries are no-flow (head contours are perpendicular to boundaries). A thin no-flow boundary has also been placed inside the domain (i.e., \( \frac{\partial h}{\partial x} \bigg|_{x=24, \ -20<y<30} = 0 \) at roughly \( x = 24 \) m and from \( y \sim 20 \) m to the top). This could represent some sort of cut-off wall; slurry walls and sheet piling are in common use. The permeable material is assumed to be homogeneous and isotropic.

The stream lines are started with equal separation distance of 5 m; assuming that the infiltration rate on the right side is spatially uniform, each stream tube should carry the same flow.

Assume the hydraulic conductivity is \( 10^{-5} \) m s\(^{-1}\) and that the tics and labels on the x and y axes give the spatial scales in m. Compute the total flow through the domain per meter perpendicular to the plane of the cross-section. Give the result in m\(^3\) s\(^{-1}\).

2. Set up matrix equations to solve for the 4 unknown heads in Figure 2.3 of your book. Solve the matrix equation by inverting the coefficient matrix and multiplying it by the ‘knowns’ vector. Use Matlab or Excel. Compute the numerical solution on a spreadsheet and the analytical solution using Equation 2.6 in your text. Compare the 3 solutions.
3. Beginning with the 1-D Possion equation \( \frac{\partial^2 h}{\partial x^2} = -\frac{R}{T} \), write a general analytical solution for \( h(x) \). Using the boundary conditions \( \frac{dh}{dx}|_{x=0} = 10^{-3} \) and \( h(100) = 1 \) m, a transmissivity of 10 m² d⁻¹, and a recharge rate of 0.001 m d⁻¹, solve for the constants in your general expression and write the specific solution to this boundary value problem.

4. Solve the same problem as in #3 using a spreadsheet finite difference model (assume your cell spacing \( \Delta x \) is 1 m). Plot the results from both approaches on the same graph; use open symbols for the numerical solution and a solid line for the analytical solution.