1. [26] Evaluate each integral.

a) \( \int_0^1 \int_0^\pi \int_2^3 z^2(2r + 1) \sin \theta \, dz \, d\theta \, dr = \)

b) \( \int_0^1 \int_1^2 (x^2 - y^2) \, dx \, dy = \)

c) \( \int_0^1 \int_0^{e^2} \ln y \, dy \, dx = \)

d) \( \int_0^2 \int_0^\sqrt{7} 2x \, dx \, dy \, dz = \)
2. [10] Use polar coordinates to evaluate
\[ \int_0^1 \int_0^x \sqrt{x^2 + y^2} \, dy \, dx. \]
\(\int \sec^nx \, dx = \frac{\sec^{n-2}x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2}x \, dx,\]
\(\int \sec x \, dx = \ln|\sec x + \tan x| + C.\)

3. [12] Let \(F(x, y) = xy \vec{i} + yz \vec{j} + zx \vec{k}\). Find \(\text{div} \, F\), and \(\text{curl} \, F\).

4. [12] Evaluate the line integral \(\int_C (-y \, dx + x \, dy)\) along the curve shown on the figure.
5. [15] Let \( F(x, y) = (y \sin x - \cos y) \mathbf{i} + (x \sin y - \cos x) \mathbf{j} \). Show that \( F \) is conservative, and find a potential function \( \varphi \) for \( F \). Evaluate the line integral \( \int_{C} (y \sin x - \cos y) \, dx + (x \sin y - \cos x) \, dy \) along the curve \( C \) parametrized by \( r(t) = \tan(\pi t/4) \mathbf{i} + \tan^{-1} t \mathbf{j} \), \( 0 \leq t \leq 1 \).

6. [7] Use Green’s theorem to evaluate the line integral \( \int_{C} (2 \tan^{-1}(y/x)) \, dx + \ln(x^2 + y^2) \, dy \) along the curve \( C \) parametrized by \( r(t) = (3 + 2 \cos t) \mathbf{i} + (1 + 3 \sin t) \mathbf{j} \), \( 0 \leq t \leq 2\pi \).
7. [18] Let \( f(x, y) = xy \). Use the Lagrange multipliers to find the maximum and minimum values of \( f \) subject to the constraint: \( 2x^2 + 3y^2 = 1 \).
8. [Bonus, 10] State and prove the fundamental theorem of line integral.