Increasing Returns, Collateral, and Sovereign Debt

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Abstract

This paper examines how a poor country may invest in sectors with low productivity because of sovereign risk, and how collateral differences across sectors may exacerbate the problem. The paper models sovereign borrowing with a two-sector economy: one sector with increasing returns (IRS) and one sector with diminishing returns (DRS). Countries with incomes below a threshold will only invest in the DRS sector, and countries with incomes above a threshold will invest mostly in the IRS sector. A lender may only give enough loans for the borrower to invest in the DRS sector if the IRS sector does not provide sufficient collateral. If a sovereign borrower can increase collateralization in the DRS sector, it may be able to drastically increase its investment in the IRS sector. The effectiveness of foreign aid will have a non-monotonic relationship with increases in income. A twin-peak income distribution is also supported in the model.

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1. Introduction

This paper links three economic phenomena with a sovereign debt model. One of these phenomena is that international capital flows have not flowed to capital-scarce countries as much as we would expect under diminishing returns to capital. In fact, capital has flowed mostly to the rich, capital-abundant countries. Explanations for this phenomenon, or the "Lucas Paradox" (1990), include fundamental differences, such as differences in institutions, education, or other missing factors of production. Explanations also include international capital market imperfections, such as asymmetric information and sovereign risk. In this paper, I will assume that there is a combination of sovereign risk, increasing returns to scale, and collateral differences across sectors that help explain why most capital has not flowed to the poor countries.

Secondly, a phenomenon that has received a lot of attention is the negative relationship between natural resource abundance and economic growth. Many empirical studies have shown that the more natural resource abundant, the lower the growth rate of per capita GDP (for instance, see Sachs and Warner (1995) and Gylfason (2001)). Theoretically, the literature has given many reasons why natural resource abundance can be a curse (for a couple of the preliminary works, see Krugman (1987), and Matsyuama (1992)). In this paper, natural resources are not a curse, but instead the poor countries invest heavily in that sector because of the inability to access enough funds to invest in sectors that exhibit increasing returns to scale.

Third, this paper relates the first two phenomena to the existence of a bimodal or twin-peak world income distribution (for instance, see Quah (1996) and Jones (1997)). Since 1960, there seems to be a movement towards a cluster of countries at a low income and cluster of countries at a high income level. In this paper, I will show that poor countries can get stuck producing in less efficient sectors because of sovereign risk and collateral differences across sectors, while countries with sufficiently high income levels can take advantage of economies of scale.

I examine sovereign borrowing in a two-sector economy. The sovereign can invest in sectors such as natural resources (oil, minerals, timber, etc.), agriculture, and low-skilled manufacturing (textiles, toys, etc,), or the borrower can invest in sectors such as high-skilled manufacturing (engineering, information technology, etc) and other sectors that require investments in infrastructure (roads, bridges, utilities, etc.), education, and health. The first type of sectors require less initial investment and get higher returns at low levels of investment than the second type of sectors. The second type of sectors may yield higher returns than the first type of sectors, but only after a sufficient amount of capital is invested. The first type of sectors are characterized by diminishing returns to scale (DRS), while the second type of sectors are characterized by increasing returns to scale (IRS) at low levels of investment.

I find that countries below a threshold income level may only get enough loans to invest in the DRS sector, but countries above the threshold will invest mostly in the IRS sector. If the borrower could commit to repay, then it could invest in the IRS sector and be better off, but it cannot commit to repay under high levels of debt because of the lack of punishment from default. It is also possible that even if

the borrower would have an incentive to repay the loan if the funds were invested as specified in a contract, the borrower may invest differently and default on the loan. Under collateral differences across sectors, the lender may only loan enough so that the sovereign will have an incentive to only invest in the less-productive sector. However, countries with sufficiently high income can receive enough loans to invest mostly in the IRS sector until they become indifferent between repaying and defaulting.

What can be done to help the poor countries? The results from this paper suggest familiar policy implications, but depending on the income of the sovereign borrower, some policies may have a substantial effect, but those same policies may also have little or no effect in this model. For example, if countries are stuck producing in sectors with low productivity, there can be several things than can be done to push them over the threshold to produce in the sectors with high productivity. An obvious remedy would be foreign aid. However, in this model, some foreign aid may have little effect on a poor country because the lenders may reduce the loan amount in fear of the sovereign investing in the IRS sector and defaulting. A sufficiently large amount of aid may give disproportionally more help to the poor country by inviting more loans to take advantage of increasing returns to scale. A country whose income level is sufficiently high will benefit from aid more than a very poor country because they can credibly commit to repay when investing in sectors that have increasing returns to scale. As in other sovereign debt models, a sovereign borrower can benefit from increasing collateralization or the punishment for default. However, in this model an increase in punishment can also substantially change the composition of investment in the sovereign nation. If a country has an incentive to only invest in the DRS sector, and the DRS sector becomes more collateralized, it may actually lead to an equilibrium with most investment in the IRS sector. In other models, monitoring investment may also benefit the borrower. However, in this model, the poorest borrowers will not benefit from monitoring. Countries with a sufficiently high income may benefit from monitoring, but the monitoring may only provide limited help.

Though empirical evidence supports increasing returns to scale, theoretical literature incorporating such returns remains sparse. Investment in infrastructure and manufacturing has been shown to have increasing returns to scale. For instance, Rodriquez (2008) finds evidence of increasing returns to manufacturing across nations. The empirical work on infrastructure and IT in the U.S. by Duggal, Saltzman, and Klein (1999, 2007) supports initial increasing returns. Fingleton and McCombie (1998) find evidence of increasing returns in EU manufacturing. Oliveira, Jayme, Jr., and Lemos (2006) find evidence of increasing returns in manufacturing in Brazil, and Park and Kwon (1995) find support for economies of scale in Korean manufacturing. Two exceptions that do discuss increasing returns and sovereign debt are Karayalcin, McCollister, and Mitra (2002) and Spiegel (1995). Karayalcin, et al. show how including increasing returns in infrastructure can create a situation where a minimum amount of debt is necessary in order for repayment to be made. Spiegel shows that increasing returns could enhance long-term lending strategies.

In section 2, I will set up the model and solve the maximization problem of the borrower, given an income and debt level. The equilibrium debt level will be solved for in section 3, including the case with collateral differences across sectors. In section 4, the relationship between income and debt will be

examined, along with its implications regarding foreign aid and income distribution. Perfect monitoring of investment will be examined in section 5, and the discussion will conclude in section 6. An Appendix that includes simulation examples is covered in section 7.

2. The Model

I build a two-period, two-sector, small open-economy model of sovereign debt. The focus will be on the optimal actions of a social planner or government, and thereby I ignore any externality issues involved with increasing returns to scale. In the first period, the borrowing country receives an endowment income, Y_1. Output in the second period, Y_2, depends on investment. A borrowing country can invest in two sectors of the economy, one with diminishing returns, and one with increasing returns to capital. The lenders can observe the output of the borrowing country, but they cannot control how the borrower invests its loans (this assumption will be relaxed later). The borrowing country is sovereign, so they can default on their loan. As in Obstfeld and Rogoff (1996), I will assume if a country defaults, the lenders can extract some portion of the borrower's output.

Although this model uses direct punishment in case of default, in reality it may be difficult to validate it with many recent examples. Models based on reputation and fear of losing access to capital markets must also have to make assumptions that are difficult to justify.[1] The reality is that there may be many reasons why a borrower does or does not default. A borrower may be in fear of sanctions or interference with trade, political repercussions, losing future access to credit markets, or the borrower may just be morally obliged to repay. While this paper uses direct punishments, the result is that the rich country can be punished more in absolute terms than a poor country in case of default. This logic seems reasonable since a rich country typically has more collateral through more international entanglements than a poor country.

Depending on the particular investments, a country may be more or less susceptible to punishment in case of default. If a poor country invests the loan in natural resources and low-skilled manufacturing, sectors which poor countries tends to export, then defaulting on the loan can lead to costly sanctions or possibly a seizure of assets. If a poor country instead invests in a sector such as infrastructure or education, then defaulting on the loan may not result in much punishment. To compare and contrast with previous models, I will examine the problem under both identical and different punishments across sectors.

Although the borrower may have many considerations when deciding to repay a loan or default, including political or religious reasons, the focus in this paper will only be on financial incentives. Thus, the borrowing country chooses to repay only if default is too costly:

$$\text{Repayment} = \min\left[(1+r)D_2, n[F^N(K_2^N)+K_2^N)]+z[F^M(K_2^M)+K_2^M)]\right], \quad (1)$$

[1] Models based on reputation must typically assume that the borrower must continuously need more funds in the future, and the borrowers are infinitely lived.

where K_2^N is the capital devoted to the diminishing-returns sector, and K_2^M is the capital devoted to the increasing-returns sector. Capital does not depreciate. $F^N(K_2^N)$ is the output from the diminishing-returns sector (DRS), and $F^M(K_2^M)$ is the output from the increasing-returns sector (IRS). D_2 is the amount borrowed. The punishment for default is the fraction η of the DRS good and z of the IRS good.

Production in the IRS sector yields low returns for low levels of investment, but it exhibits increasing returns until the increasing returns are eventually exhausted and become diminishing returns after some investment level. Production in the DRS sector yields high returns for initial investment levels, but exhibits diminishing returns throughout. Graphically, the type of production functions which will be discussed are of the kind represented in Figure 1. Although, in reality, investment in IRS sectors may increase production in the DRS sectors, such as through infrastructure, I will assume for convenience that it does not. Similarly, I can assume that there is sufficient existing infrastructure (roads and bridges) to extract a lot of the natural resources, and adding such infrastructure will mainly benefit other businesses.

In previous models of sovereign debt, borrowers may have an incentive to default by consuming more in the first period and investing less in the second period. For example, without uncertainty, a loan that exceeds a debt threshold could lead to a discontinuous crash in investment in order to reduce anticipated creditor sanctions (Obstfeld and Rogoff 1996). That result would be the same in a two-sector model, making it even more difficult for a poor country to get enough loans to take advantage of increasing returns. In the two-sector set up, the borrower may not invest in the sectors as the lender may want, and this could also lead to default. To focus only on the investment composition decision and abstract from the effects of consumption smoothing or default stemming from first-period consumption, I will assume that the representative agent of the small country gets utility only from second-period consumption:

$$U(C^N, C^M) = C_2^N + C_2^M . \tag{2}$$

For ease of exposition, set prices equal to one for both goods, and those prices are taken and do not change. Since prices are taken by the sovereign, the borrower only cares about maximizing its output, which would then maximize its consumption and utility.

Investment in each sector is

$$\begin{aligned} &I_1^N = Y_1 + D_2 - I_1^M = K_2^N \\ &\text{and} \\ &I_1^M = Y_1 + D_2 - I_1^N = K_2^M . \end{aligned} \tag{3}$$

Consumption in the second period is

$$C_2 = F^N(K_2^N) + F^M(K_2^M) + K_2^N + K_2^M - \Re , \tag{4}$$

where $\Re$ is the repayment. Since the borrowing country cannot commit to repay, its repayment must never exceed the cost of sanctions. The lenders must make $(1+r)D_2$ in order to be willing to lend. Therefore, the sanctions under default must be at least as large as the risk-free return on the debt,

$$n[F^N(K_2^N)+K_2^N]+z[F^M(K_2^M)+K_2^M]\geq(1+r)D_2. \quad (5)$$

The borrowing country is therefore trying to solve:

$$\max_{K_2^N,K_2^M} F^N(K_2^N)+K_2^N+F^M(K_2^M)+K_2^M - \min[(1+r)D_2, n[F^N(K_2^N)+K_2^N]+z[F^M(K_2^M)+K_2^M]], \quad (6)$$

given $D_2=K_2^N+K_2^N-Y_1$. Under default, the maximization problem is

$$\max_{K_2^N,K_2^M} F^N(K_2^N)+K_2^N+F^M(K_2^M)+K_2^M - n[F^N(K_2^N)+K_2^N]-z[F^M(K_2^M)+K_2^M].$$

Solving, if an interior maximum exists, the maximum is where the net marginal rates of return are equalized:

$$(f'^N+1)(1-n)=(f'^M+1)(1-z).$$

Under repayment, the maximization problem is

$$\max_{K_2^M,K_2^N} F^M(K_2^M)+F^N(K_2^N)+K_2^N+K_2^M-(1+r)D_2$$
$$=\max_{K_2^M,K_2^N} F^M(K_2^M)+F^N(K_2^N)+K_2^N+K_2^M-(1+r)(K_2^N+K_2^M-Y_1).$$

Solving, if an interior maximum exists, then the maximum is achieved where the marginal rates of return are equalized, $f'^N=f'^M$. Given an income and debt level, there can be at least two points that set the marginal rates equal (one may be a minimum). As will be discussed, if the initial income of the borrower is too low, the maximum utility will be achieved when all the funds are invested in the decreasing-returns sector. A key assumption that is made here is that the sovereign borrower does not have enough funds to invest efficiently in both sectors, where $f'^N=f'^M=r$, nor will the sovereign nation be able to borrow enough to make the efficient investments because of the incentive to default.

In order to get explicit solutions, I would have to specify explicit production functions and parameters, which is done in the background to help illustrate the results (see Appendix), but they are not specified here since the parameters and numbers are arbitrary. However, I can implicitly describe the possible equilibrium under reasonable assumptions. We know that the lenders will not lend any amount which will lead to default, but they will lend as much as possible up to that amount if the lenders are competitive. The amount of the loan depends on how much the lenders expect the borrower to invest in each sector. To compare these results to previous models, let's first assume $n=z$, and then we can look at the implications of different punishments for different sectors. Under equal punishments across sectors, if there is an interior equilibrium, investments will be made until marginal returns are equalized

between the two sectors, $f'^{N}(K_2^N) = f'^{M}(K_2^M) \geq r$. In this case, regardless of whether the borrower has an incentive to default or repay, if an interior maximum exists, it is achieved when the marginal products are equalized. If the borrower is defaulting, then capital will flow to the sector with the highest net marginal return, $(f'^{N}+1)(1-n)$ or $(f'^{M}+1)(1-z)$, but if $n = z$, this reduces to $f'^{N}(K_2^N)$ or $f'^{M}(K_2^M)$, which is also the optimal allocation under repayment. Thus, if the borrower is being punished by the same fraction of output, whether output came from the DRS or IRS sector, if there is an interior maximum, it is achieved when the marginal products are equalized across sectors. Therefore, given an income and debt level, the optimal investment-composition decision of the borrower does not depend on whether it is repaying or defaulting if the collateral is the same across sectors.

Figure 2 illustrates the investment-composition decision of the borrower given its income and loan amount, while under equal punishment fractions across sectors. Given the utility function, the borrower will invest everything, so the only question is how much will be invested in each sector, and whether the borrower will repay or default. Figure 2 shows the possible equilibria if the borrower is given a credit constraint, $Y_1 + D_2 = \bar{I}$. Given an income and loan, the line $\bar{I}$, with a slope of negative one, represents all the different combinations of investment that the borrower can make in the two sectors. For instance, if all the income and loans were invested in the diminishing-returns sector (increasing-returns sector), the borrower would be at the vertical (horizontal) intercept on the $\bar{I}$ line in Figure 2.

Points **B** and **G** in Figure 2 represent the two possible interior equilibrium investment levels given an income and debt level. These points are the possible interior equilibria because all funds are invested and the marginal returns in each sector are equalized (although point **B** could be a minimum). The curved line represents where the marginal returns are equalized, $f'^{N}(K_2^N) = f'^{M}(K_2^M)$, and it is U-shaped because of the shape of the production functions. For instance, a large investment in the diminishing-returns sector will yield a similar marginal product as a small investment in the increasing-returns sector, such as the case at point **B**. As investment in the IRS sector increases, the marginal return increases in the IRS sector, and an equivalent marginal return in the DRS sector would be achieved with less investment. Once investment in the IRS sector is large enough such that the increasing returns are exhausted, then more investment in the IRS sector will achieve lower marginal returns, such as the marginal returns achieved with more investment in the DRS sector, and this explains the U-shaped curve.

The dotted line in Figure 2 represents the minimum amount of income and debt needed to make investment in the IRS sector possible. If $\bar{I}$ was inside the dotted line, then all the investment would be in the DRS sector since the marginal return of the IRS sector never reaches the marginal return from the DRS sector. For sufficiently low income levels, and consequently less loans, even if everything was invested in the IRS sector, the marginal return in the IRS sector may still be lower than what the marginal return would be if everything was invested in the DRS sector. Therefore, the borrower would never have an incentive to invest anything in the IRS sector if the credit constraint was inside the dotted line in Figure 2.

However, even if the credit constraint was to the right of the dotted line, the borrower may still not have an incentive to invest any in the DRS sector. Even if a higher marginal return can be achieved by investing all the funds the IRS sector as opposed to investing all in the DRS sector, the borrower may still get more output by investing only in the DRS sector. To get high marginal returns in the IRS sector, the borrower has to accept low marginal returns at low levels of investment, which may not be worth it unless the investment is large enough. Therefore, all capital will flow into the DRS sector if the income is too low and the loan is insufficient for the borrower to invest in both sectors. The amount of income necessary for investment in the DRS sector will be illustrated in the next section when the equilibrium debt level is solved for.

When punishment for default is the same fraction of output no matter which sector the output comes from, then the lender knows that the borrower will invest in a way that maximizes gross output. The question for the lenders is how large can the loan be until the borrowers have an incentive to default. If the borrower was punished differently according to the sector from which the output comes from, $n \neq z$, then Figure 2 would also have to include a curve which represents where the net marginal returns are equalized, $(f'^{N}+1)(1-n)$ or $(f'^{M}+1)(1-z)$. The borrower would then have to compare the value of its optimal decision under default, with an interior maximum achieved where the net marginal returns are equalized, with its optimal decision under repayment, where an interior maximum is achieved where the marginal returns are equalized. The lenders would have to solve the borrower's problem in order to know how much they can lend. This analysis is done in the next section.

3. The Equilibrium Debt Contract

The previous section solves the investment composition decision of the borrower given an income and debt level, but the decision of the borrower may affect the amount of the loan that the lenders would be willing to give in the first place. The difficulty is solving for the amount of the loan that the lenders would lend, given that the borrower can use the loan to invest in whatever sector it wants and may default depending on the composition of the investment. However, if we assume that the punishment for default is the same across sectors, $n = z$, then the lenders don't need to worry about the borrower's composition of investment, they only need to worry about loaning too much. This means that if the lenders give a loan that the borrower would repay if it invested according to the contract, the borrower would not have an incentive to invest differently and default. If we assume that punishment differs across sectors, $n \neq z$, then the lenders may have to decrease the loan because the borrower may not invest in a way that would create an incentive for repayment, even if the borrower would have an incentive to repay if it invested according to the contract. In either case, in order for the lenders to know how much they can loan to the borrower and still get repaid, they must know the highest amount of debt that would make the borrower indifferent between repaying and defaulting.

First, we know that if the borrower has a sufficiently low income, then at low levels of debt, the borrower will not have an incentive to invest anything in the IRS sector if we assume the IRS sector yields low marginal returns at low levels of investment and the DRS sector yields high marginal returns at low levels of investment. However, with enough income and debt, the borrower may be able to invest enough to take advantage of the increasing returns, with the optimal amount of investment being

where the net marginal products of capital are equalized across sectors. Therefore, the value of defaulting can be written as

$$V^d = \max[(1-n)[F^N(Y_1+D_2)+Y_1+D_2],(1-n)[F^N(\alpha(Y_1+D_2))+\alpha(Y_1+D_2)] + (1-z)[F^M((1-\alpha)(Y_1+D_2))+(1-\alpha)(Y_1+D_2)]], \quad (7)$$

where, $D_2 = K_2^N + K_2^M - Y_1$, $\alpha = \frac{\hat{K}_2^N}{\hat{K}_2^M + \hat{K}_2^N}$, and $\hat{K}_2^N, \hat{K}_2^M$ solves the first-order condition $(1-n)(f'^N+1) = (1-z)(f'^M+1)$ that corresponds to an interior maximum (not a minimum). The equation states that under default the borrower may get the most value from investing everything in the DRS sector and keeping the fraction $1-n$ of their output and investment; or the borrower may get the most value from keeping a fraction of their output and investment from both sectors, while investing in each sector so that the net marginal returns are equalized. As discussed above, at low levels of income and debt the borrower will get the most value from investing everything in the DRS sector, while at high levels of income and debt the borrower will get the most value by investing mostly in the IRS sector.

Analogously, the value of repayment can be written as

$$V^R = \max[F^N(Y_1+D_2)+Y_1+D_2-(1+r)D_2, F^N(\gamma(Y_1+D_2))+F^M((1-\gamma)(Y_1+D_2))+(Y_1+D_2)-(1+r)D_2, \quad (8)$$

where $\gamma = \frac{\tilde{K}_2^N}{\tilde{K}_2^M + \tilde{K}_2^N}$, and $\tilde{K}_2^N, \tilde{K}_2^M$ solves the first-order condition $f'^N = f'^M$ that corresponds to an interior maximum. Under repayment, the borrower may get the most net output from investing everything in the diminishing-returns sector; or the borrower may get the most net output from investing in both sectors in such a way that sets the marginal returns equal across sectors. The equilibrium debt is the amount that sets the utility from defaulting equal to the utility from repayment, $V^R = V^d$. Any debt that exceeds that amount will result in default, while the lenders have no reason to restrict debt below such an amount.

First, we can illustrate the outcomes under the assumption of equal default punishment fractions across sectors, $n = z$. A sufficiently low income will lead to a loan that will not be large enough in order for the borrowers to have an incentive to invest in the increasing-returns sector. This case is illustrated in Figures 3a and 3b. In Figure 3a, the value of repayment and the value of default are shown at each debt level, given an initial income level Y_1. Each of the curves have a kink, which represents the debt level at which investment goes from only the diminishing-returns sector to a combination of the DRS and IRS sector. As explained before, at low investment levels, where investment equals $Y_1 + D_2$, the borrower has an incentive to only invest in the DRS sector. At some higher investment level, the borrower will have an incentive to invest in both sectors at the point where the marginal returns are equalized. After the kink, the value of default and the value of repayment increases at a higher rate than before because

of increasing returns. In this example, however, the borrower is too poor to receive enough loans to have an incentive to invest any in the IRS sector. The value of default and the value of repayment are equalized at the debt level, D_2^*, which is not enough for the borrower to invest in the IRS sector.

As debt increases, both the value of repayment and the value of default increase; and under plausible parameters, the value of default will increase more than the value of repayment. To show this, I need to show that $\partial V^d / \partial D_2 > \partial V^R / \partial D_2$. For example, if all investment is in the DRS sector, the inequality is true if $1+r > n(1+ f'^N(K_2^N))$, where $f'^N(K_2^N)$ is the marginal return in the DRS sector. This condition is true with any reasonable parameter. In the case where investment is in both sectors, the assumption that default becomes more attractive as debt increases is also true under plausible parameters, although the necessary inequality doesn't simplify as neatly. Note that this implies that there is not a range of default, where higher debt makes repayment more attractive. Throughout this paper, it will be assumed that the value of default increases more than the value of repayment as debt increases, which will be the same as assuming that we will not have a combination of both extremely severe punishment for default and extremely high marginal returns.

Figure 3b illustrates the relationship between debt and the composition of investment. For low levels investment, all the funds, $Y_1 + D_2$, will go to the diminishing-returns sector, since the increasing-returns sector only yields sufficient returns at high levels of investment. In this example, the equilibrium debt level is not enough to entice the borrower to invest any funds in the IRS sector. As we can see in Figure 3b, there is a debt level, which is higher than the equilibrium debt level, where the amount of funds invested in the diminishing-returns sector falls, while the borrower invests most funds in the increasing-returns sector. Since all funds are invested, the difference between total investment, $Y_1 + D_2$, and investment in the DRS sector, K_2^N, is the investment in the IRS sector, K_2^M.

Once income is sufficiently large, the borrowers can get enough loans to make investment in the IRS sector worthwhile. When the punishment fractions for default are the same across sectors, there is a threshold income level that allows just enough debt so that the borrower is indifferent between investing only in the DRS sector and investing in both sectors such that the marginal products are equalized. As illustrated in Figures 4a and 4b, at such an income level, the lenders would be willing to give just enough loans such that the borrower is not only indifferent between repaying and defaulting when investing in the DRS sector, but the borrower is also indifferent between repaying and defaulting while investing optimally in both sectors. A lower income level will lead to less debt and will give the borrower an incentive to only invest in the DRS sector. A higher income level will lead to more debt and will give the borrower an incentive to mostly invest in the IRS sector.

It may not be obvious that there exists an income level that will allow the borrower just enough debt to make it indifferent between repaying and defaulting at two sets of investment levels, as illustrated in Figure 4b. For a given income level, the borrower will be given loans until it becomes indifferent between repaying and defaulting; and depending on the income level, the highest debt that sets $V^d = V^R$ may be where the borrower invests only in the DRS sector, or it may be large enough for the borrower to invest in the IRS sector. For the income level that yields enough debt to make the borrower

indifferent between repayment and default under DRS investment, if it gives the same utility for investing in both sectors and repaying, then it must also give the same utility from investing in both sectors and defaulting. If the value of repayment under only DRS investment is equal to the value of repayment when investment is made in both sectors, it must mean that the output is the same at each set of investments. If the value of default is equal to the value of repayment at one of those points, then it must be the case that the value of default is equal the value of repayment at both points. If the output is the same, and the punishment for default is the same fraction no matter which investments took place, then the net output will be the same at both points. Therefore, we have the situation in Figure 4b, where the value of default and the value of repayment are equal at two points at the threshold income level. We will see later that if we assume different punishments across sectors, then we will have a different threshold since the optimal investment composition depends on whether the borrower will default or repay (in such a case, it is not obvious that $\partial V^R / \partial Y_1 > \partial V^d / \partial Y_1$).

At a lower income level than illustrated in Figures 4a and 4b, the situation is that of Figures 3a and 3b, where the borrowers is only given enough debt to set the value of default equal to the value of repayment when investment is only in the DRS sector. At a higher income level than in Figures 4a and 4b, the highest debt that achieves $V^d = V^R$ will be where the borrower invests mostly in the IRS sector. Such an equilibrium debt and the composition of investment are illustrated in Figures 5a and 5b.

In Figure 5a, the borrower's income is high enough so that it can get enough debt to have an incentive to invest in the IRS sector. The value of default and the value of repayment intersect after the kinks in the curves, which is where the borrower has enough funds to take advantage of the increasing returns. In Figure 5b, we see that for the given income and debt level, the borrower will choose to invest most funds in the IRS sector (the difference between the two lines), where the marginal products are equalized across sectors. When we assume the same punishment fraction across sectors, the curves will never cross where the value of default and the value of repayment are on different sides of the kink in the curves in Figure 5a, where the borrower would be indifferent between investing only in DRS sector and repaying and investing in both sectors and defaulting. However, as we will see later, if we assume different punishments then we can have such a case.

3.2. Equilibrium under different default punishments for each sector, $n > z$

As described earlier, the diminishing-returns sector may be more collateralized than the increasing-returns sector. If a poor country invests in natural resources and low-skilled manufacturing, sectors from which poor countries tends to export, then defaulting on a loan may lead to costly sanctions or possibly a seize of assets. If a poor country instead invests in sectors such as infrastructure or education, then defaulting on an international loan may not result in much punishment. For instance, oil revenues may be easier to extract than a bridge or a school. Therefore, we should set $n > z$, and examine the effect that has on equilibrium debt and investment composition.

If a borrower is very poor, then the equilibrium under different sector punishments will look similar to Figures 3a and 3b. The borrower will not have enough income and debt to have an incentive to invest in the IRS sector. The lender will give a loan that will make the borrower indifferent between repaying and

defaulting, and at a sufficiently low income level, the borrower will be indifferent between repaying and defaulting after investing only in the DRS sector. At a higher but still sufficiently low income level, we get a result that differs from Figures 3a and 3b and from Figures 4a and 4b. The borrower may still not have enough income to get a large enough loan to invest in the IRS sector, but the lenders may give a loan that makes the borrower indifferent between investing in the DRS sector and repaying and investing mostly in the IRS sector and defaulting.

As illustrated in Figure 6a, the debt that makes the borrower indifferent between repaying and defaulting is after the kink in the V^d curve, but it is before the kink in the V^R curve. This means that at the equilibrium debt level, the highest value from defaulting comes from investing mostly in the IRS sector, but the highest value from repaying comes from investing everything in the DRS sector. When repaying, utility is maximized by either investing where the marginal returns are equalized, $f'^N(K_2^N) = f'^M(K_2^M)$, or where everything is invested in the DRS sector, whichever yields the most gross output. When defaulting, utility is maximized by either investing where the net marginal returns are equalized $(1-n)(f'^N+1) = (1-z)(f'^M+1)$, or where everything is invested in the DRS sector, whichever yields the most net output. If $n > z$, then an interior maximum under default is achieved when the gross marginal return in the DRS sector is higher than the gross marginal return in the IRS sector. Therefore, net output under default may be higher when investment is mostly in the IRS sector, even if gross output is greater when everything is invested in the DRS sector. At an income that is almost high enough, but still insufficient to yield enough debt to invest in the IRS sector, the borrower will be given enough loans such that they are indifferent between investing and repaying in the DRS sector and investing mostly in the IRS sector and defaulting. In this example, the lenders do not want the borrower to invest in the IRS sector because of the lack of punishment if the borrowers default after investment in that sector.

In Figure 6b, we see that the utility from repaying is maximized when all funds are invested in the DRS sector, but utility from defaulting is maximized when most funds are invested in the IRS sector and K_2^{ND} is invested in the DRS sector, which is where the net marginal returns are equalized. For very low income and debt levels, the utility from repaying and the utility from defaulting are both maximized when investment is only in the DRS sector, such as in Figures 3a and 3b. At high levels of income, both the utility from repaying and the utility from defaulting is maximized when most investment is made in the IRS sector. Therefore, there exists some threshold income level that yields just enough debt such that the borrower is indifferent between investing only in the DRS sector and repaying, investing mostly in the IRS sector and repaying, and investing some greater proportion in the IRS sector and defaulting. Such an income level is illustrated in Figures 7a and 7b.

As illustrated in Figure 7a, the initial income is just large enough such that the equilibrium debt level is at the kink in the V^R curve, which means the borrower is indifferent between investing everything in the DRS sector and repaying and investing mostly in the IRS sector and repaying, and investing mostly in the IRS sector and defaulting. There is no kink in the default curve in this example, which means the borrower has enough initial income that it would always want to invest in the IRS sector if it was to

default. When output from the DRS sector is more collateralized than output from the IRS sector, then at the threshold income level the value from defaulting will be maximized when most of the funds are invested in the IRS sector. This occurs because at a given output level the borrower will get more net output from investing mostly in the IRS sector under default since the IRS sector has less collateral. Therefore, while the value of repayment from investing only in the DRS sector equals the value of repayment from investing mostly in the DRS sector, the value of default from investing only in the DRS sector is less than the value of default from investing mostly in the IRS sector.

Figure 7b shows the investment composition at a threshold income level under different punishments across sectors. As we can see from the vertical dotted line, three sets of investment levels achieve the same utility. The borrower can invest everything in the DRS sector (set $Y_1 + D_2 = K_2^N$ and $K_2^M = 0$) and repay, or the borrower can invest mostly in the IRS sector where the marginal products are equalized and repay, or the borrower can invest a greater proportion in the IRS sector where the net marginal products are equalized (where $(1-n)(f'^N+1) = (1-z)(f'^M+1)$) and default. At that given income level, any more debt and the borrower will have an incentive to invest mostly in the IRS sector and default (under plausible parameters, $\partial V^d / \partial D_2 > \partial V^R / \partial D_2$).

If income was higher than the level illustrated in Figures 7a and 7b, then the borrower would be allowed enough debt to invest mostly in the IRS sector. In general, however, when the IRS sector is less collateralized than the DRS sector, more income doesn't mean more debt, as will be illustrated later. However, once the borrower's income crosses the threshold level shown in Figures 7a and 7b, more income will allow for more debt and investment in the IRS sector, assuming plausible parameters. At a higher income level than portrayed in Figure 7a, the V^R curve will increase more than the V^d curve, which will allow for more debt. It can be shown that $\partial V^R / \partial Y_1 > \partial V^d / \partial Y_1$ for any given reasonable parameters when comparing the interior maximums or when comparing the case where everything is invested in the DRS sector. However, when looking at the case where the borrower is getting just enough income to have an incentive to invest in the IRS sector under default, then an income may not allow for more debt, as will be shown later.

3.3. A change in collateral

With equal punishment fractions across sectors, $n = z$, it may be obvious that an increase in punishment may allow a borrower to take advantage of increasing returns and invest mostly in the IRS sector. For example, under equal punishments across sectors, if the lenders could increase the punishment for default, the borrowers will be willing to repay a larger amount of debt, which will allow for more loans and may give the borrower an incentive to invest mostly in the IRS sector. If the borrower was near the threshold as in Figure 4a, then increasing the punishment will shift the V^d curve down and to the right in Figure 4a, which will increase the equilibrium debt. The result would then look like Figures 5a and 5b, where the borrower has an incentive to invest mostly in the IRS sector.

What may not be obvious, however, is that if the punishment is greater in the DRS sector, $n > z$, then increasing punishment in the DRS sector may also allow the borrower to contract for enough funds to

invest mostly in the IRS sector, even if it means that the borrower is investing less in the DRS sector than with the lower punishment. Also, increasing *n* may actually decrease the total amount of collateral since the borrowers may invest mostly in the IRS sector, of which a smaller fraction is collateral. For instance, suppose that the income and punishment fractions gave the borrower an incentive to invest only in the DRS sector, where the punishment for defaulting is $n(F^N(Y_1+D_2)+Y_1+D_2)$. Suppose at the equilibrium, as in Figures 6a and 6b, the borrower is given just enough debt to be indifferent between investing only in the DRS sector and repaying and investing in both sectors and defaulting, which can be expressed as

$$\begin{aligned}&\max V^R = F^N(Y_1+D_2)+Y_1+D_2-(1+r)D_2 = \\ &\max V^d = (1-n)[F^N(\alpha(Y_1+D_2))+\alpha(Y_1+D_2)]+(1-z)[F^M((1-\alpha)(Y_1+D_2))+(1-\alpha)(Y_1+D_2)],\end{aligned} \tag{9}$$

where α is the fraction of the funds that are invested in the DRS sector when the net marginal returns are equalized across sectors. When n is increased, the maximum utility from defaulting decreases since the borrower is receiving a smaller fraction from the DRS sector output under default, and at the original debt level the maximum that can be achieved from repaying exceeds the maximum that can be achieved from defaulting, $\max V^R > \max V^d$. If the utility from repayment is higher than the utility from defaulting, the lenders can give more loans until the borrower becomes indifferent between repaying and defaulting. As the amount of loans increase, both the value of default and the value of repayment increases, with the value of defaulting increasing at a higher rate than the value of repaying (again assuming $\partial \max V^R / \partial Y_1 > \partial \max V^d / \partial Y_1$, which is true under any plausible parameters). A new equilibrium debt level may be achieved where the borrower may have an incentive to invest in the IRS sector and repay.

An increase in n is illustrated in Figure 8. The dotted line represents the value of default after an increase in punishment on output from the decreasing-returns sector. At the old equilibrium debt level, D_2^*, the maximum value from repaying exceeds the maximum value from defaulting after the increase in punishment for the decreasing-returns sector output under default. At the new equilibrium debt level, $D_2^{*'}$ in Figure 8, the borrower invests in both sectors such that the marginal returns are equalized, $f'^N(K_2^N) = f'^M(K_2^M)$, and repays the debt. The borrower would also get the same utility from investing in both sectors such that the net marginal returns are equalized, $(1-n)(f'^N+1) = (1-z)(f'^M+1)$, and defaulting. At the new equilibrium debt level, the borrower would have defaulted at the old punishment fraction of the DRS sector output. However, when n increases, the collateral for investment in both sectors is now high enough to entice the borrower's to repay, with the total punishment under default equaling

$$n[F^N(\alpha(Y_1+D_2))+\alpha(Y_1+D_2)]+z[F^M((1-\alpha)(Y_1+D_2))+(1-\alpha)(Y_1+D_2)]. \tag{10}$$

Before the increase in n, if the borrower invested everything in the DRS sector and did not repay, then the actual collateral taken by the lenders may be more than what they could have taken if the borrower defaulted after the increase in n. The output from the borrower will be more, but the fraction of

output extracted by the lender under default may be less than before since the IRS sector is not highly collateralized. Thus, an increase in collateral in the DRS sector may actually decrease overall collateral, yet it still may make it possible to sustain a contract for more funds, and possibly enough to take advantage of increasing returns to scale. Of course, having less collateral after an increase n is a little misleading. If the borrower defaults, the investment would be different than if it repaid, and the punishment would be higher after the increase in n.

For example, suppose a contract added a clause that allowed the lending country to seize oil export revenues in case of default on the loan. Adding this collateral would increase the amount of the loan for any given contract in which the borrower would produce (and export) oil. Further, suppose the borrower has been too poor to invest in industries with increasing returns to scale, such as the case illustrated in Figures 6a and 6b. Before the added clause, lenders would not loan enough money for borrower to invest in both sectors where the marginal products are equalized, since the borrower has less collateral in the IRS sector and would have an incentive to default. However, with the added punishment for default with oil output, the borrower may now have an incentive to repay a loan that allows the borrower to invest in both the IRS and DRS (oil) sector. This is possible since the portion that is invested in oil production, when marginal products are equalized, is punished at a higher fraction than before the clause. Before the clause, if the borrower invested in both sectors, it would default, but adding more punishment in the DRS sector could bring down the utility from default just enough to make repayment optimal. The result from including the possibility of expropriating oil exports may be less oil produced, since a lot of funds are now invested in other sectors, but the borrower will still get a larger loan and achieve higher overall output. The total collateral that can be seized by the lenders under default may actually decrease because the output from the IRS sectors may have less collateral than the DRS sector. However, if the borrower would have defaulted before the clause, it would have invested in both sectors, but after the clause the punishment for such an investment increases. In previous models, increasing the punishment for default benefits the borrower, but in this model it can also shift investment to more productive sectors, and the increase in punishment can have a more substantial impact because of increasing returns to scale.

4. Income and debt

After analyzing the possible contracts under different initial income levels, we can now describe a general relationship between initial income and debt. In Figure 9, the relationship between initial income and debt is drawn, given that the DRS sector is more collateralized than the IRS sector, $n > z$. A borrower with an initial income of zero can still contract for some funds, given that the initial returns are very high in the DRS sector, and subsequently the punishment for default is high since the lender will take a fraction of the output. For higher levels of initial income, up to Y_1^1, the amount of collateral increases, and the returns are high, so the amount of loans increase. In this range, the borrowers are investing only in the DRS sector since they do not have enough funds to take advantage of increasing returns. Because of diminishing marginal returns, the curve is concave, since the punishment will increase with more income and investment, but at a diminishing rate.

Once initial income surpasses the level Y_1^1, the amount of debt will decrease until it reaches the level Y_1^2. When a borrowing country has an initial income of Y_1^1, it is indifferent between investing everything in the DRS sector and repaying, investing everything in the DRS sector and defaulting, and investing in both sectors (where $(1-n)(f'^N+1)=(1-z)(f'^M+1)$) and defaulting. To put in another way, the income is just high enough such that the highest debt that the borrower can be trusted to repay allows for just enough funds to make the borrower indifferent between investing in just the DRS sector and investing mostly in the IRS sector and defaulting. Note that at this point, the value of repaying when investing in both sectors will be lower than when defaulting because at a given output level the punishment will be lower when funds are invested in the IRS sector. As initial incomes increase above Y_1^1, the borrowers needs less debt to have an incentive to invest in the IRS sector and default. While the borrower initially has more income to be extracted under default, if the borrower has an incentive to invest in the IRS sector, the fraction of the output that will be extracted will be less than the fraction that would be extracted when all investment is in the DRS sector (in this case we are comparing two different sectors, and it is possible to have $\partial V^R/\partial Y_1 < \partial V^d/\partial Y_1$). Between initial incomes Y_1^1 and Y_1^2, the lender does not want the borrower to invest anything in the IRS sector because the borrower will not have an incentive to repay. Therefore, as incomes increase in this range, debt will decrease.

After the income level Y_1^2, initial income is positively related to debt. For a borrowing country who has an initial income of Y_1^2, it receives loans such that it is indifferent between investing all funds in the DRS sector and repaying, investing most funds in the IRS sector and repaying, and investing even more funds in the IRS sector and defaulting. Such an income and debt level was illustrated in Figures 7a and 7b. As income increases from Y_1^2, utility from investing in both sectors will exceed utility from investing only in the DRS sector, regardless as to whether the borrower defaults or repays. Given that investment is in both sectors, when income increases, the value of repayment increases more than the value of default (under plausible parameters), and more debt will be allowed to the borrowers. As diminishing marginal returns set in, the punishment for default increases at a diminishing rate, so the equilibrium debt will increase at a diminishing rate.

Note that this relationship between initial income and debt supports a twin-peak world income distribution. The poor countries that can only get enough debt to invest in the sectors that exhibit diminishing returns will converge because the debt decreases between Y_1^1 and Y_1^2. Growth would then take off for those incomes above Y_1^2 because debt and income will be positively related. Divergence can then exist between a country with an initial income between $Y_1^1$and Y_1^2, and a country with an initial income above Y_1^2. Thus, initial income and sovereign debt can lead to twin peaks in world income if there is increasing returns with low punishment for default.

4.2. Foreign aid and lending: substitutes or complements?

From the relationship between initial income and debt, we can analogously examine the effectiveness of foreign aid. In some cases, foreign aid can decrease international lending, but in other cases, foreign aid can lead to more lending, as we can observe from the relationship between income and debt in Figure 9. For incomes lower than Y_1^1, more foreign aid leads to more debt, until the sum of the foreign aid and initial income equals Y_1^1. For initial incomes above Y_1^1, the equilibrium debt is just enough to make the borrower indifferent between investing in the DRS sector and repaying and investing in both sectors and defaulting. Since the increasing-returns sector becomes more attractive as the amount of funds increase, for a given equilibrium debt level, a small amount of aid will give the borrower an incentive to invest in the IRS sector and default, instead of investing everything in the DRS sector and repaying. Therefore, for initial incomes between Y_1^1 and Y_1^2 in Figure 9, the amount of debt will decrease if aid is increased and if the sum of the aid and initial income does not surpass Y_1^2. In fact, as illustrated in Figure 9, if the initial income is between Y_1^1 and Y_1^2, aid will only increase debt if the aid is so large that it brings the income level above Y_1^2 to the point that it yields more loans.

For initial incomes greater than Y_1^2, an increase in foreign aid will increase the available funds for the borrower. At Y_1^2, the borrower receives just enough loans so that it is indifferent between investing all funds in the DRS sector and repaying, investing most funds in the IRS sector and repaying, and investing even more funds in the IRS sector and defaulting. Any foreign aid will give the borrower an incentive to invest in both sectors and repay, assuming $\partial \max V^R / \partial Y_1 > \partial \max V^d / \partial Y_1$, which again is true under any plausible parameters. Therefore, if the initial income for the borrowing country is equal to or greater than Y_1^2, any amount of aid will increase the loans that the borrower can be trusted to repay.

In summary, foreign aid and foreign debt will be complements for very low income levels, between zero and Y_1^1, if the aid is small or if the aid is so large that it surpasses Y_1^2 enough to allow for more debt. In the same income range, foreign aid and foreign debt can be substitutes if the aid is enough (but not too much) to give the borrower an incentive to invest in the IRS sector and default. For example, as can be seen in Figure 9, if the initial income is just below Y_1^1, then some aid, such as an amount that brings income to just below Y_1^2, will lead to less debt. For initial incomes between Y_1^1 and Y_1^2, foreign aid and foreign debt will be substitutes when the aid is small, but complements if the aid is large enough. For incomes above Y_1^2, foreign debt and foreign aid will be complements.

5. Sector-controlled loans

How would things change if the foreign lenders could control how their loans get invested? In many cases, we can imagine that lenders do have some control as to where their funds get invested. For instance, we can think of the lenders as the ones directly purchasing the physical capital, for which the borrowers then have to repay (or not), such as the case when someone is financing the purchase of a car. The borrower, therefore, cannot easily shift those funds to an alternative investment. In this model, with no capital flight or secret consumption, a sufficiently poor country with income below Y_1^1 will not gain if the lender can control where the sovereign invests the borrowed funds. A sufficiently

poor country will get enough debt such that it is indifferent between repaying and defaulting after investing all funds in the DRS sector. It cannot get enough debt to have an incentive to invest some funds in the IRS sector, since the IRS sector has low initial returns and the value of defaulting will exceed the value of repayment if the borrower was given enough debt to overcome the initial low returns. The borrower could invest its initial income how it sees fit, but it would not get any higher utility from investing any of its own funds in the IRS sector. Therefore, for a very poor country, controlling which sector the funds go to will not help.

If the funds are monitored for the borrowers who have an initial income between Y_1^1 and Y_1^2 in Figure 9, the borrowers may or may not be better off than if none of the funds are monitored. Whether or not the borrower becomes better off, and how much it becomes better off, depends on the assumptions about the production functions and the assumptions about the way the loan is bargained. Again, when a borrowing country has an initial income of Y_1^1, it is indifferent between investing everything in the DRS sector and repaying, investing everything in the DRS sector and defaulting, and investing in both sectors (where $(1-n)(f'^N+1)=(1-z)(f'^M+1)$) and defaulting. For a borrower whose initial income level lies just above Y_1^1, if the funds were not monitored, the borrower would get a loan amount that would make it indifferent between investing everything in the DRS sector and repaying and investing in both sectors and defaulting, such as the case illustrated in Figures 6a and 6b. If the loans were monitored, the result would only be different if the lender could give more loans and prevent the borrower from investing in both sectors and defaulting. The lender could do this if the debt is higher than the amount of funds that the borrower would have invested in the DRS sector under default. If the lender could force more funds in the DRS sector than the amount that the borrower would have invested under default, then the borrower may have an incentive to repay.

For example, in Figure 10, the borrower will be allowed enough debt until the amount of debt equals the amount of investment in the DRS sector that the borrower would have invested if it defaulted and received the same utility as repayment under all investment in the DRS sector. In the figure, as we go from left to right, the amount of capital invested in the IRS sector increases from zero to Y_1+D_2. As we go from right to left, capital invested in the DRS sector increases from zero to Y_1+D_2. On the vertical axis, utility from defaulting and repaying is measured. The box is for a given income and debt level. We can see from the horizontal dotted line in the figure that if $K_2^{N^*}=D_2$ the utility from defaulting at that point equals the utility from repaying when all funds are invested in the DRS sector. If the debt was smaller than $K_2^{N^*}$ in Figure 10 (keeping total investment fixed), the borrower could invest less in the DRS sector, more in the IRS sector, and achieve a higher utility from defaulting. If the debt was larger than $K_2^{N^*}$ in Figure 10, the lender can allow even more funds since it could force those funds to be invested in the DRS sector, and the borrower would then have an incentive to repay. As debt increases, the curves shift up (and lengthen), with the value of default increasing more than the value of repayment at each given debt level. Therefore, at some debt level, the value of repayment will equal the value of default when all the funds are invested in the DRS sector. Once debt reaches such a

threshold, controlling the sector in which the loans are invested will not help since default will be more attractive at each investment set that includes investment in the IRS sector.

For income levels above Y_1^2 in Figure 9, just as for lower income levels, controlling which sectors the loan is invested in will be beneficial only if the loan is a large enough portion of the total funds invested when the IRS sector becomes attractive to invest in. As we will also see, the degree at which the borrower benefits will depend on how the loan is negotiated in the first place. Again, at Y_1^2, with no monitoring, the borrower receives just enough loans so that it is indifferent between investing all funds in the DRS sector and repaying, investing most funds in the IRS sector and repaying, and investing even more funds in the IRS sector and defaulting, as illustrated in Figures 7a and 7b. For higher income levels than Y_1^2, without monitoring, the borrower receives just enough loans so that it is indifferent between investing most funds in the IRS sector and repaying and investing even more of its funds in the IRS sector and defaulting. Any more debt than equilibrium, without monitoring, will give the borrower an incentive to invest most funds in the IRS sector and default. If the loans are monitored, the lender may be able to prevent the borrower from investing too much in the IRS, and in doing so, allow even more loans to the borrower than if the loans were not monitored.

Suppose that the borrower has an initial income a little above Y_1^2, so that it is given enough debt to be indifferent between investing most funds in the IRS sector and repaying and investing even more funds in the IRS sector and defaulting (imagine a debt a little higher than shown in Figure 7b). If the lender could control more than the amount that the borrower was going to invest in the DRS sector without monitoring, then the lender can give more loans that will be repaid. It may turn out however, as will be shown, given that the DRS sector is more collateralized than the IRS sector, the lenders may want a bigger debt level than the borrower would like to obtain. The equilibrium debt level would then depend on how the loan is negotiated.

Take for example, Figures 11a and 11b. In Figure 11a, the borrower is given some loans, some are forced to be invested in the DRS sector, and the borrower invests the rest in the IRS sector. The horizontal line is where the value of repayment equals the value of repayment, and the amount of debt must be at least as large as the DRS investment in order for that point to be a possible contract. As we can see, if funds were not monitored, the borrower would invest more in the IRS sector and default. However, if the loans in Figure 11a exceeded the amount of DRS investment where the value of default equaled the value of repayment, the lenders can give more loans and force more of it in the DRS sector, as in Figure 11b. Even though the borrowers can invest more from the additional loans, if it means that the lenders can make the investment go in the DRS sector, the borrowers may actually be worse off. The horizontal line shows the utility received when the debt is at the lower level in Figure 11a, which is higher than when the debt is higher in Figure 11b. If the lender could control a large amount of the funds, they may want to give more loans than the borrowers would prefer. The amount of loans, therefore, depends on how the loans are negotiated. If the lender could give the borrower an ultimatum, then the borrower would take the loans since it is better than nothing. If the borrower could negotiate, then the borrower may want to accept a lower amount of debt, be able to invest more of it in the more productive sector, and achieve a higher net output than with a larger loan. With a larger loan,

the borrower will be forced to invest most of it in the less productive sector, which can be worse for the borrower than a smaller loan that is invested mostly in the IRS sector.

6. Conclusion

The objective of this paper has been to analyze the effects of increasing returns and collateral differences on a sovereign borrower. The purpose of this analysis is to help explain why poor countries may invest in less productive sectors than rich countries, give a reason why the world income distribution can have twin peaks, and explain the effectiveness of alternative policies to help increase investment and output in the poor countries. Many other papers have also used a two-sector economy to explain differences in economic growth, including the "Dutch disease" literature. This paper differs from those in that the mechanism which inhibits growth is initial income, sovereign risk, and collateral differences across sectors. Poor countries don't suffer from a resource curse in this model, but instead they heavily invest in natural resources because they do not have enough initial income and they cannot attain enough debt to invest in the sectors that have increasing returns. This paper argues that a main reason holding the borrower from investing in the more productive sector could be the lack of collateral in such sectors. Investments in infrastructure, research and development, education, and health may pay off handsomely if a large amount of funds are allocated to those sectors. However, international lenders may not be interested in loaning funds for such investments since a sovereign borrower may not be held accountable if they default after investing in those sectors. For instance, a lender may have a difficult time extracting a road or a school. However, if loans go towards investing in natural resources, a sector that a poor sovereign borrower may heavily export, the lender may find it easier to punish the borrower under default. Sectors such as natural resources or agriculture may have high initial returns, but those returns are diminishing as the resources become scarce.

The results from this paper suggest different policy implications for different income levels. For example, the poorest of the poor would benefit significantly from foreign aid, as it would also increase foreign lending because of the increase in collateral. However, the poorest countries would not benefit from perfect monitoring of investment in this model, since the borrower will only have an incentive to invest in the DRS sector regardless of whether or not the lenders can control where the funds go. For poor countries that have a little more income, some foreign aid may not significantly help because it may lead to fewer loans, since the borrower may have an incentive to invest less in the collateralized sector and more in the less-collateralized sector and default. At these income levels, policies that will improve monitoring of investment may help increase investment and output since the lenders may be able to prevent the borrower from investing in sectors that are less collateralized, and as a result the lender will give more loans.

For the countries that have enough income and debt to invest in the IRS sector and repay, they will be significantly helped by foreign aid because it will increase collateral and allow them to take advantage of increasing returns. Improving monitoring of investment could increase investment, but it could also lead to excess lending. For instance, if the lenders had control over which sectors the funds were invested in, they may require all loans go in the DRS sector, while the borrower may be better off if they received less loans but could invest a large portion of the funds in the IRS sector.

At all income levels, as in previous studies, increasing punishment for default will help the borrower. However, the significant difference in this paper is that it shows how an increase in punishment for default from output in the low productivity sector may actually decrease investment and output in the that sector, while increasing investment and output in the high productivity sector. Since the borrower will always want to invest at least some funds in the DRS sector, increasing punishment for default in that sector will increase the collateral for any investment portfolio that includes investment in the DRS sector. Before an increase in punishment in the diminishing-returns sector, a borrower may have not been allowed enough debt to have an incentive to invest in a portfolio that included mostly investment in the IRS sector, with little investment in the DRS sector. However, an increase in punishment in the DRS sector may give the borrower an incentive to invest in mostly in the IRS sector and repay since the collateral from the investment in the DRS sector is high enough.

In summary, this paper helps explain how some poor countries can be inherently stuck investing in less productive sectors because of sovereign debt problems. The paper also discusses the effects of possible policies that can be taken to alleviate the problems with sovereign debt and increase investment and output in the poor countries. However, the results of this paper suggest that countries at different income levels will be affected differently from the same policies.

7. Appendix

To illustrate the results from the figures, the following production functions were used:

$$F(K_2^N) = 6(K_2^N)^{1/3},$$

and

$$F(K_2^M) = \frac{25(K_2^M)^{13/6}}{(K_2^M)^2 + 500}.$$

Other similar functions can be used as well. I used an interest rate of 0.10. To illustrate the results under collateral differences across sectors, I set $z = 0.07$ and $n = 0.27$ (for the initial results, I set both parameters equal).

The variable of interest is the initial income, Y_1, and how it affects the equilibrium debt and output. The value of default is given in equation (7) and the value of repayment is given in equation (8). To solve for the equilibrium debt level, and to construct Figure 9, I set the value of default equal to the value of repayment at different initial income levels. To construct Figures 10 and 11, initial income and debt is given, so insert ($Y_1 + D_2 - K_2^M$) in the place of K_2^N in the value functions.

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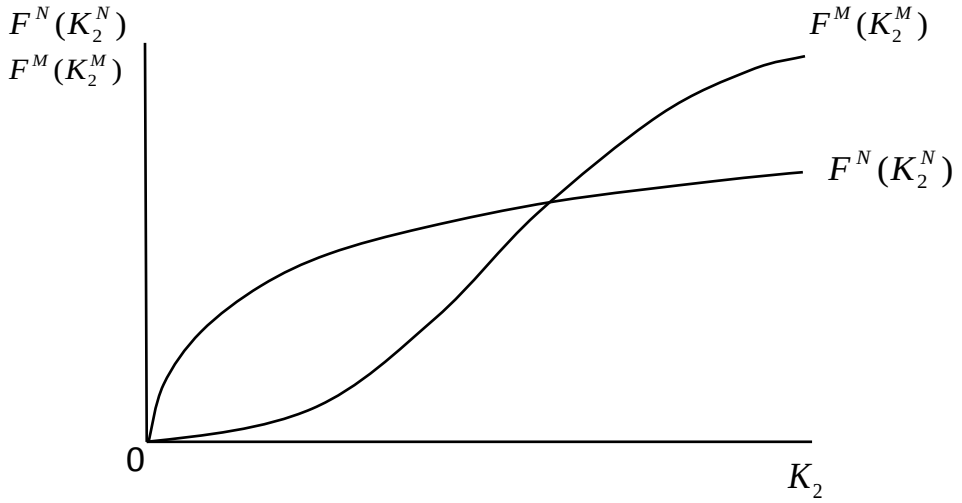
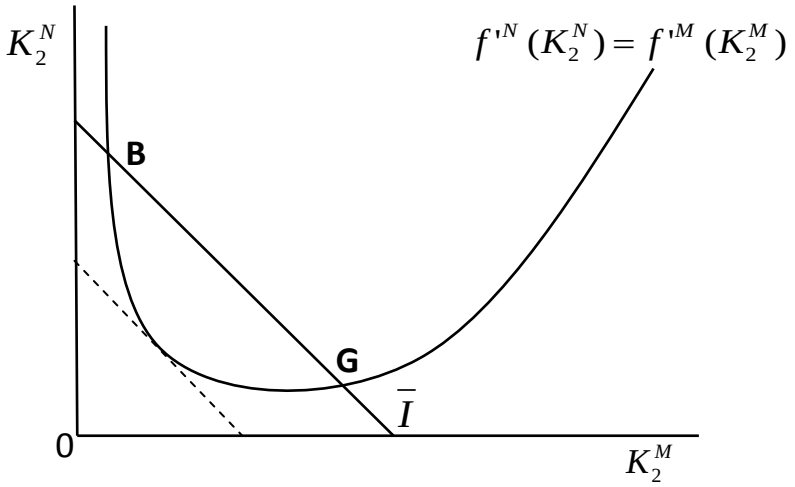
Figure 1: Production function of the IRS and the DRS sector



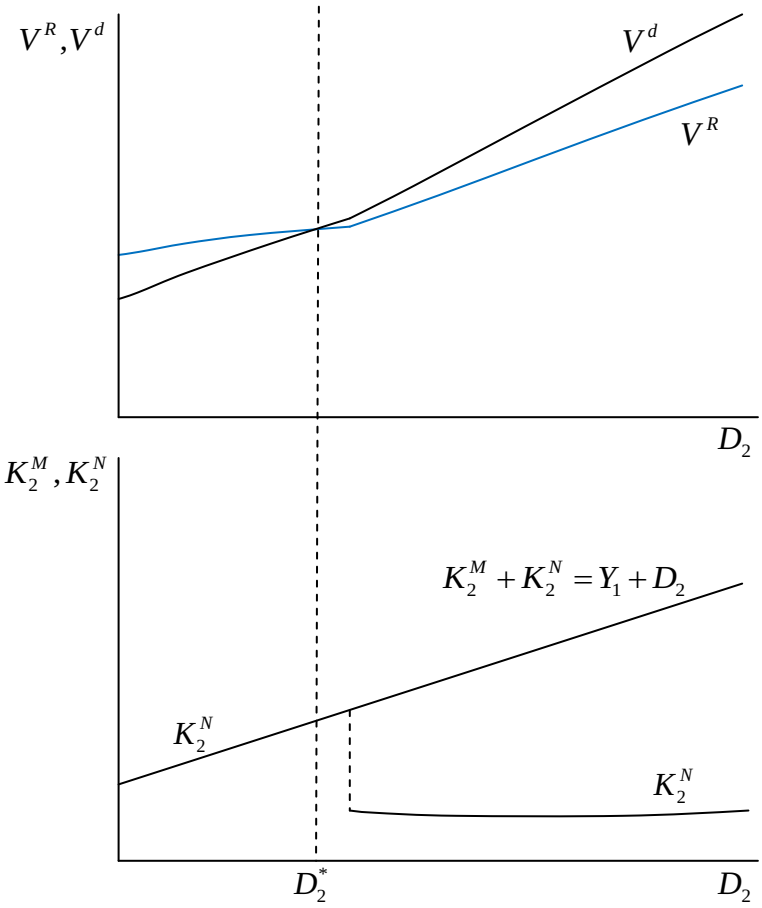
Notes: Output in the diminishing-returns sector (DRS), $F^N(K_2^N)$, or natural resource sector, increases as investment in that sector increases, but at a diminishing rate. The initial marginal product is high, but it goes down as the resources are extracted. Output in the increasing-returns sector (IRS), $F^M(K_2^M)$, or manufacturing sector, has a low marginal product at low levels of investment, but output initially increases at an increasing rate. At high levels of investment, the IRS sector is more productive than the DRS sector.

Figure 2: Equilibrium Investment composition given an income and debt level



Notes: Given an initial income and a debt level, the borrower can invest their funds in the IRS sector, K_2^M, or DRS sector, K_2^N, and the possible combination is represented by $\bar{I}$. If the fraction of income extracted under default is the same across sectors, and the borrower has enough income and debt, the optimal investment will equalize the marginal products across sectors, $f'^N(K_2^N) = f'^M(K_2^M)$. Points B and G are where all funds are invested and the marginal products are equalized.

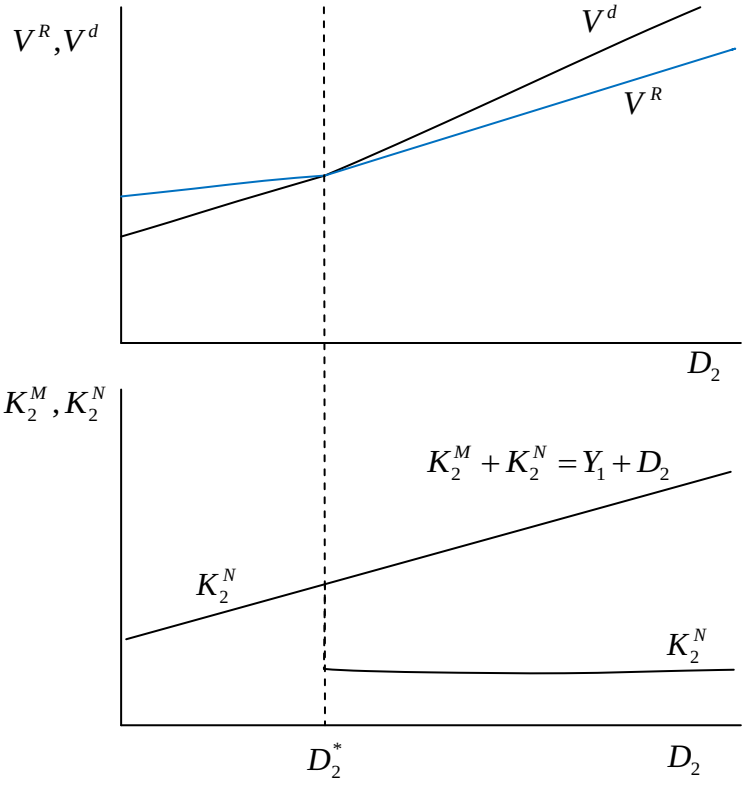
Figure 3a and 3b: Equilibrium debt and investment composition levels for a sufficiently poor country



Notes: In Figure 3a, the maximum value of default and the maximum value of repayment are given as a function of debt for a given initial income level. The kink in the curves represents the debt level that the borrower starts to invest some capital in the increasing-returns sector (where $f'^N(K_2^N) = f'^M(K_2^M)$). In this case, the borrower is not given enough debt to have an incentive to invest in the increasing-returns sector.

Figure 3b illustrates the relationship between debt and the composition of investment. For low levels investment, all the funds, $Y_1 + D_2$, will go to the diminishing returns sector, since the increasing-returns sector only yields sufficient returns at high levels of investment. In this example, the equilibrium debt level is not enough to entice the borrower to invest any funds in the IRS sector. As we can see in the graph, at a higher debt level than the equilibrium debt level the amount of funds invested in the diminishing-returns sector falls, while the borrower invests most funds in the increasing-returns sector (the difference between the two lines).

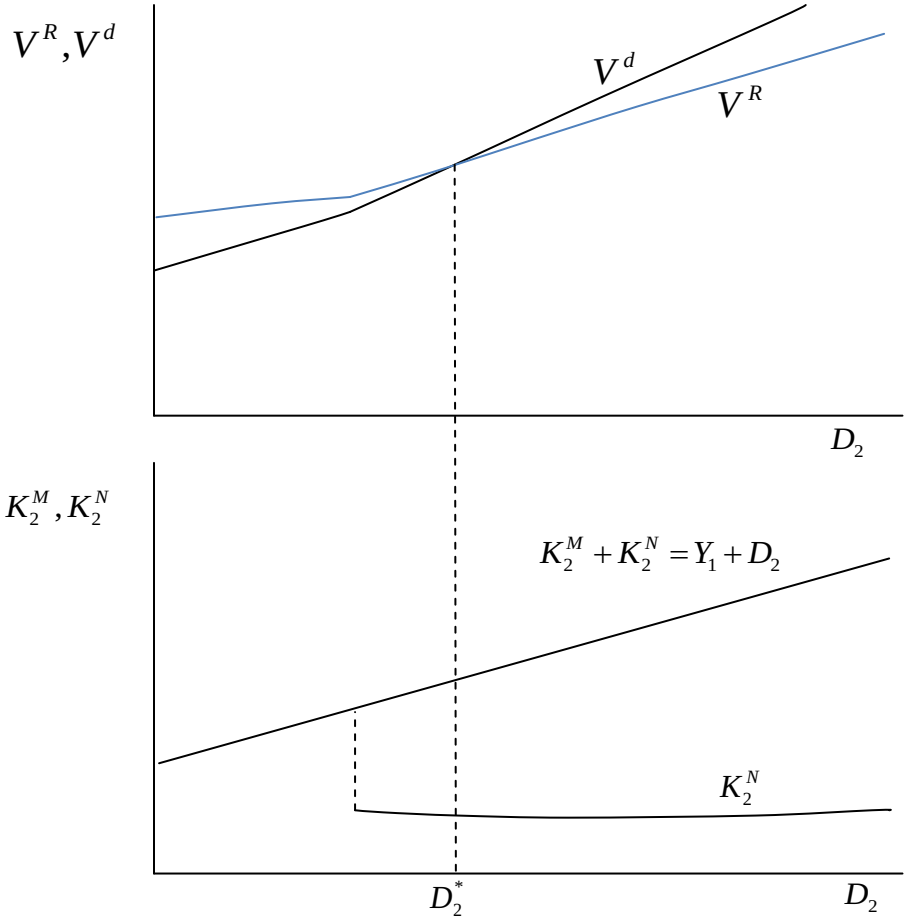
Figure 4a and 4b: Equilibrium debt and investment composition levels for a threshold income level.



Notes: In Figure 4a, the maximum value of default and the maximum value of repayment are given as a function of debt for a given initial income level. The kink in the curves represents the debt level that the borrower starts to invest some capital in the increasing-returns sector (where $f'^N(K_2^N) = f'^M(K_2^M)$). In this case, the borrower is given just enough debt to be indifferent between investing all the funds in the DRS sector and investing most of the funds in the IRS sector.

Figure 4b illustrates the relationship between debt and the composition of investment. For low levels investment, all the funds, $Y_1 + D_2$, will go to the diminishing returns sector, since the increasing-returns sector only yields sufficient returns at high levels of investment. In this example, the equilibrium debt level is just enough to make the borrower indifferent between investing all the funds in the DRS sector and investing most of the funds in the IRS sector.

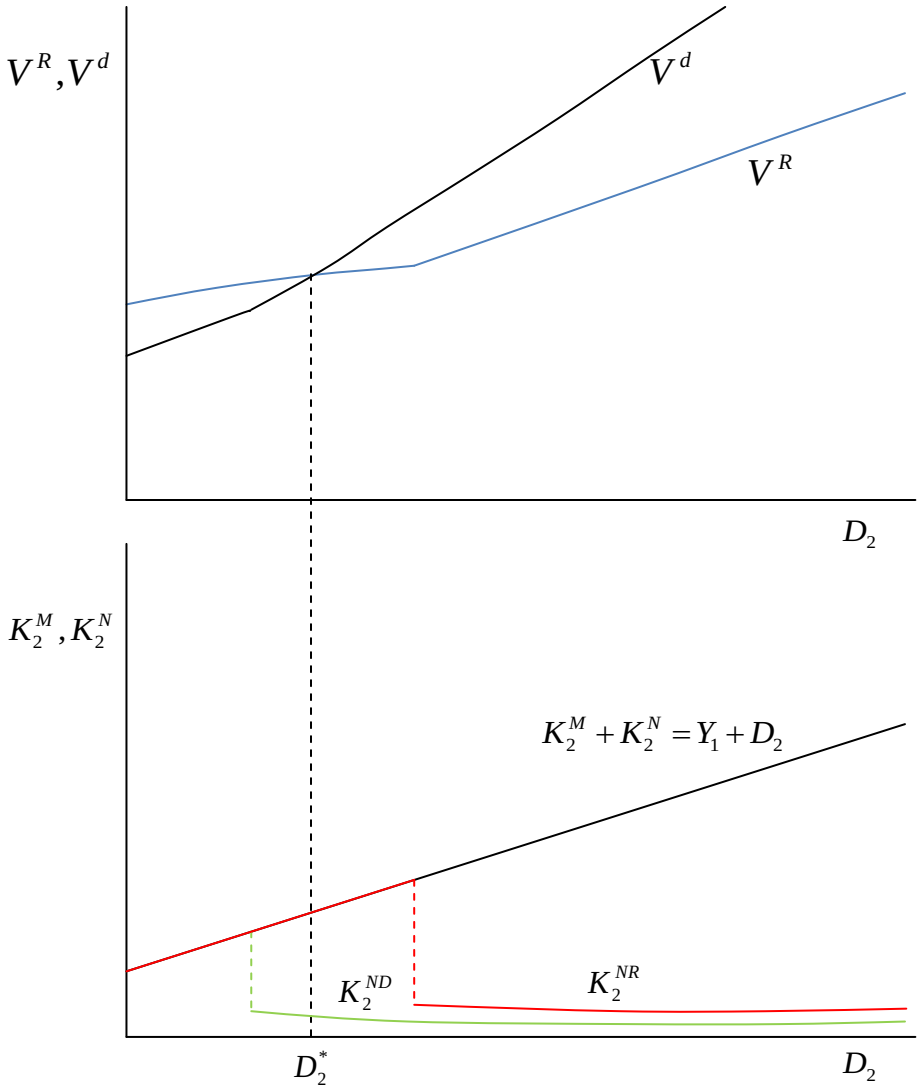
Figures 5a and 5b: Equilibrium debt and investment composition levels for a sufficiently high income level.



Notes: In Figure 4a, the maximum value of default and the maximum value of repayment are given as a function of debt for a given initial income level. The kink in the curves represents the debt level that the borrower starts to invest some capital in the increasing-returns sector (where $f'^N(K_2^N) = f'^M(K_2^M)$). In this case, the borrower is given enough debt to invest some funds in the IRS sector.

Figure 4b illustrates the relationship between debt and the composition of investment. For low levels investment, all the funds, $Y_1 + D_2$, will go to the diminishing returns sector, since the increasing-returns sector only yields sufficient returns at high levels of investment. In this example, however, the equilibrium debt level is enough to entice the borrower to invest funds in the IRS sector, which is the difference between the two curves.

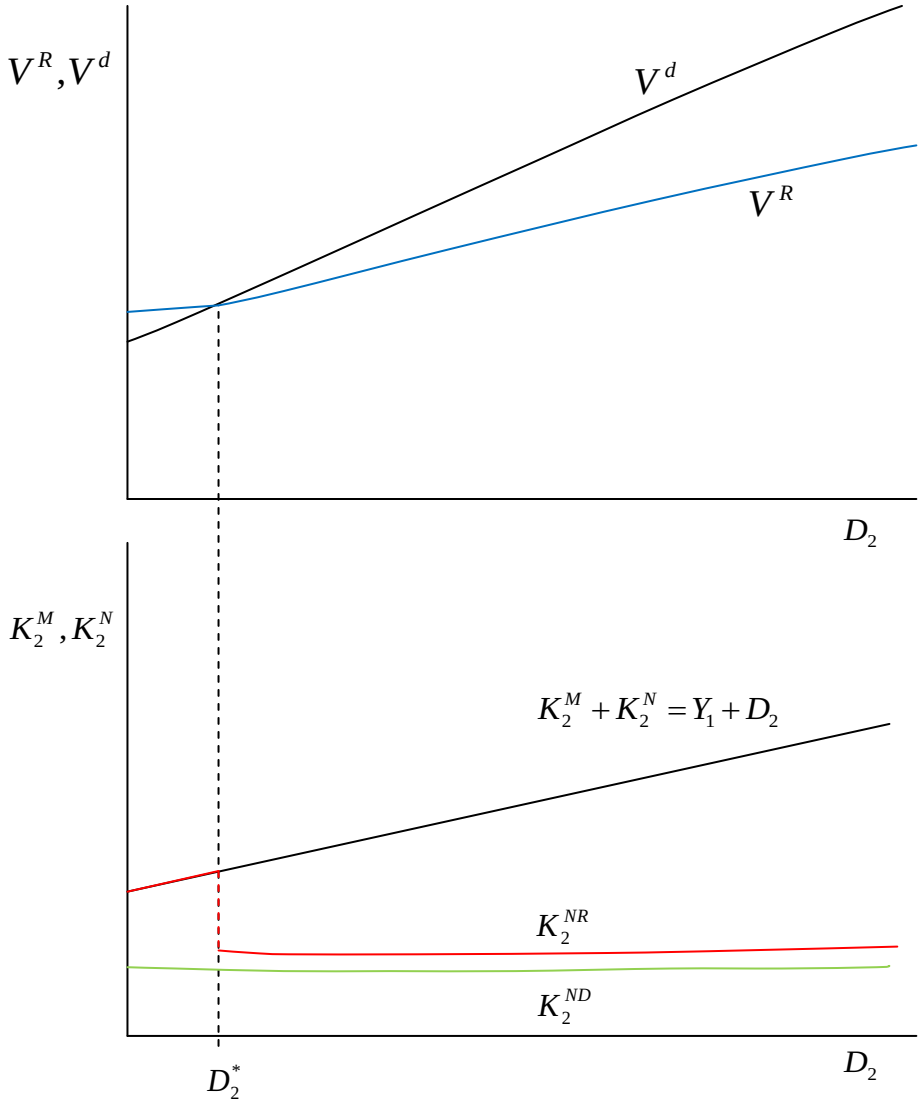
Figures 6a and 6b: Equilibrium debt and investment composition levels for a poor country under collateral differences across sectors.



Notes: In Figure 6a, the maximum value of default and the maximum value of repayment are given as a function of debt for a given initial income level. The kink in the curves represents the debt level that the borrower starts to invest some capital in the increasing-returns sector. In this case, the borrower is indifferent between investing all funds in the DRS sector and repaying or investing mostly in the IRS sector and defaulting.

Figure 6b illustrates the relationship between debt and the composition of investment. For very low levels investment, all the funds, $Y_1 + D_2$, will go to the diminishing returns sector. However, since the IRS sector is less collateralized, the borrower may choose to invest in the IRS sector under default, but not under repayment.

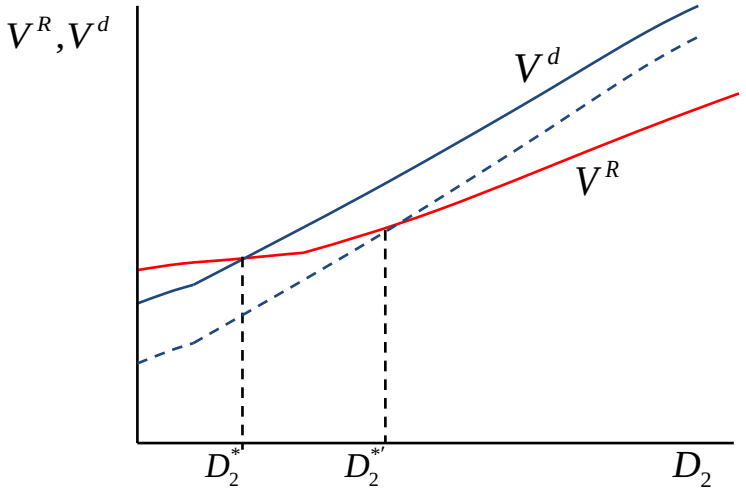
Figures 7a and 7b: Equilibrium debt and investment composition levels for a threshold country under collateral differences across sectors.



Notes: In Figure 7a, the maximum value of default and the maximum value of repayment are given as a function of debt for a given initial income level. The kink in the curves represents the debt level that the borrower starts to invest some capital in the increasing-returns sector. In this case, the borrower is at a threshold and is indifferent between investing all funds in the DRS sector and repaying or investing mostly in the IRS sector and defaulting. Any more income and the borrower will invest mostly in the IRS sector.

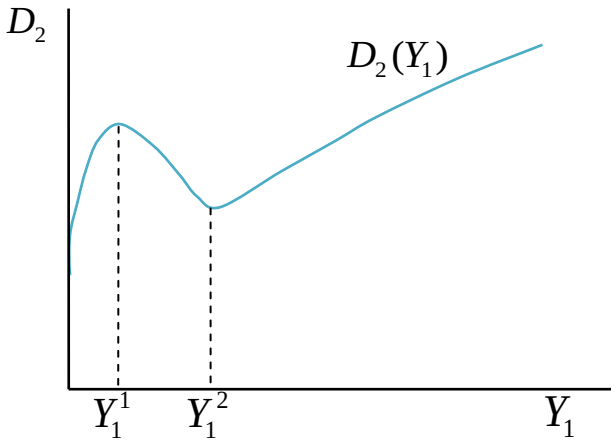
Figure 7b illustrates the relationship between debt and the composition of investment. The borrower is at a threshold, where any more income will allow more debt and the borrower could invest in the IRS sector and repay.

Figure 8: Equilibrium debt after an increase in punishment in the decreasing-returns sector.



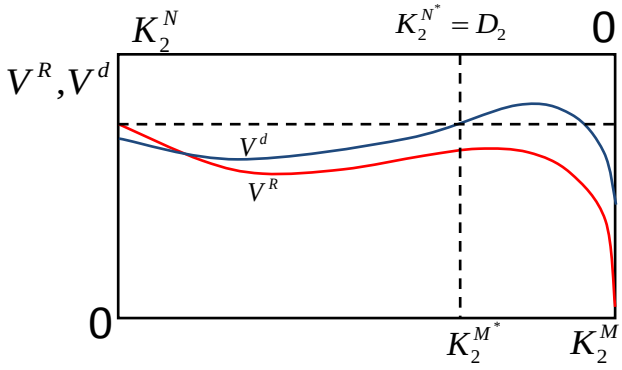
Notes: In Figure 8, the value of defaulting decreases because of the increase in punishment from DRS-sector output. Since an optimal investment portfolio always includes some DRS investment (because of the initial high returns), every possible investment by the borrower becomes can be punished more severely. More debt is allowed, and the borrower in this case now has an incentive to invest in both sectors.

Figure 9: Income and debt when decreasing-returns sector is more collateralized



Notes: Figure 9 shows the relationship between income and debt when the diminishing-returns sector is more collateralized than the increasing-returns sector. As income increases from zero to Y_1^1, debt increases because of the increase in the collateral. As income surpasses Y_1^1, debt decreases because the lender does not want the borrower to invest in the less–collateralized IRS sector and default. As income surpasses Y_1^2, debt increases as the borrower is given debt to invest in the IRS, since the high returns increases punishment enough to support loans.

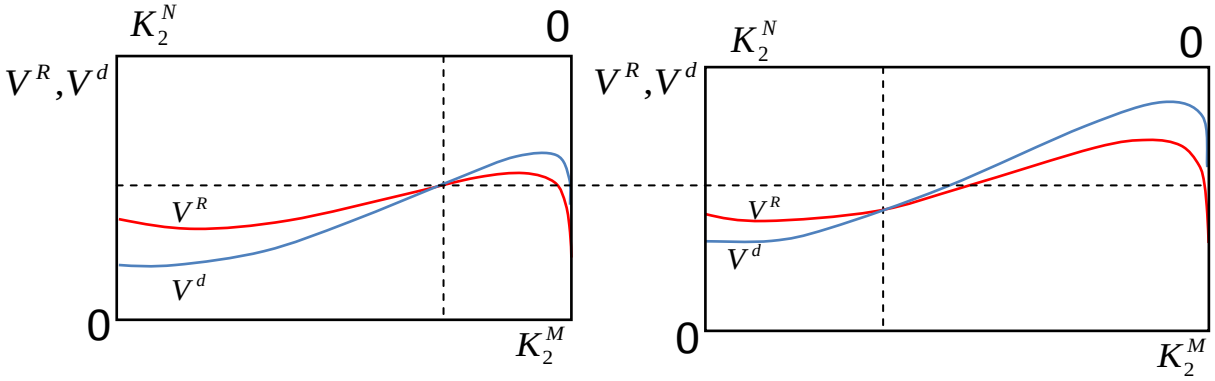
Figure 10: Investment composition with the equilibrium debt level under sector-controlled loans



Notes: Figure 10 shows the value from default and repayment given an initial income and sector-controlled loans. Looking at the figure, we can see that the borrower would prefer to invest more than $K_2^{M^*}$ in the IRS sector and default. However, if the lender could force $K_2^{N^*}$ of the funds to be in the DRS sector, then the borrower will not have an incentive to default. If $K_2^{N^*} = D_2$, then we are at equilibrium. If the lender gave more loans, the borrower would default because in Figure 10, the value from defaulting will increase more than the value from repayment, under reasonable parameters (and the figure will expand). If debt was less, then the lenders are forgoing profit.

Figure 11a: Optimal debt for the borrower when the lenders can control the sector where the loans are invested

Figure 11b: Optimal investment for the lenders when they can control the sector where the loans are invested



Notes: Given the same parameters, Figure 11a is an example of what an optimal debt contract is for a borrower if the lender can control where the loans are invested, while Figure 11b is an example of what an optimal debt contract is for a lender. In Figure 11a, the borrower is given funds such that they are allowed to invest heavily in the IRS sector, where the value of default equals the value of repayment. However, in Figure 11a, if the loans exceed the amount of capital invested in the DRS sector, the lender may give more loans that will be repaid (see Figure 10 for explanation). As the debt increases, both the value of default and the value of repayment increases, with the value of default increasing more at each investment set. Since the DRS sector is more collateralized, as debt increases, the value of repayment will exceed the value of default only if most of the investment is in the DRS sector. If the debt equals the amount invested in the DRS sector where the curves intersect in Figure 11b, the lenders have maxed out the amount of loans they can give that will be repaid. In this example, the borrower was better off in Figure 11a, where they received less loans than in Figure 11b, because in Figure 11a they were allowed to invest more heavily in the IRS sector than in Figure 11b.