The radial variation of wind in tropical cyclones is commonly, though not particularly accurately, represented using the Holland (1980) analytical profile. This “parametric” model forms the basis of storm surge, sea state and windstorm underwriting catastrophe (CAT) simulations. It starts with the following specification of the radial pressure profile:

\[ p(r) = P_c + (P_\infty - P_c) \exp\left\{-\left(\frac{A}{r}\right)^B\right\}. \]

Here \( P_c \) is the central surface pressure; \( P_\infty \) is the surface pressure far from the center; \( A \) is a size parameter; and \( B \) is the shape parameter. Calculation of the gradient wind, \( v \), derives from the radial momentum equation, neglecting asymmetric and vertical acceleration and assuming slow evolution of the axially symmetric wind and pressure fields:

\[ \frac{v^2}{r} + fv = \frac{1}{\rho} \frac{\partial p}{\partial r}, \]

where \( f \) is the Coriolis parameter \( = 2\Omega \sin \varphi \), and \( \rho \) is the density of air \( \sim 1.2 \text{ kg m}^{-3} \). If the first term on the left is much larger than the second, the wind is said to be in “cyclostrophic” balance. If the second term is larger, the wind is in “geostrophic” balance. Show that in the former case \( Ro \gg 1 \) and in the latter \( Ro \ll 1 \), where \( \text{Ro} = v / \Omega r \) is the Rossby number. Differentiate \( p(r) \) with respect to \( r \) and obtain an expression for the cyclostrophic wind, \( \frac{v_c^2}{r} = \rho^{-1}(\frac{\partial p}{\partial r}) \). Since it can be shown that the maximum cyclostrophic wind is found at \( r = A \), obtain an expression for \( V_{\text{max}} \). In MatLab, plot \( p(r) \) and \( v_c(r) \) from \( r = 2 \text{ km} \) to \( r = 200 \text{ km} \) for \( A = 20 \text{ km} \), \( B = 1.5 \), and \( V_{\text{max}} = 50 \text{ m s}^{-1} \).

Extra credit: Rewrite the radial momentum equation in terms of the cyclostrophic wind, \( v^2 + frv - v_c^2 = 0 \). Solve for the gradient wind and manipulate the equation so that it is cast in terms of the cyclostrophic wind and the cyclostrophic Rossby number \( \text{Ro}_c = \frac{v_c}{fr} \). Add the gradient wind computed at latitude \( 20^\circ \), where the Coriolis parameter \( f = 5 \times 10^{-5} \), to your plot of the cyclostrophic wind. Is it stronger or weaker than the cyclostrophic wind?