

1. Show that for any $k > 2$, 2^{3k-1} divides $(2^k + 1)(2^k + 3)\dots(2^{k+1} - 1) - (2^k - 1)(2^k - 3)\dots(2^k - 2^k + 1)$ but 2^{3k} does not

2. Let $p > 2$ be a prime number and $\frac{m}{n}$ is the irreducible fraction $\frac{m}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{p-1}$. Show that p divides m .