Barra on Campus
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Welcome to Barra on Campus, a software donation program sponsored by MSCI Barra, Inc.

Barra on Campus enables professors of finance and business administration at the MBA or diploma level in universities throughout the world to share Barra’s extensive expertise in the classroom setting. We invite students to learn the concepts of portfolio theory using software products used by thousands of investment professionals worldwide.

The Barra on Campus program introduces you to Barra’s core applications, Aegis for equity and Cosmos Global Risk Manager for fixed income assets.

The Barra on Campus contact at your school receives a welcome letter explaining where to download the applications and data required. Installation instructions and pdf copies of the sections of this handbook are also posted on MSCI Barra’s password-protected website.

Note: For analysis purposes, the Barra on Campus program includes equity and fixed-income data from 2006. Data updates are not provided as part of the program.
Handbook Structure

Theory

Chapters 1 through 4 explore the basic concepts that underlie Barra products. A fundamentals chapter provides an overview of risk theory and multiple-factors models (MFM). The Equity chapter explains Barra’s USE3L model. A Fixed Income chapter explains the basics about interest rate models and risk. Finally, a chapter on optimization briefly outlines some of the mathematical concepts behind optimization theory and gives an overview of the optimization constraints that can be used in Barra’s applications.

Reference

Appendices include definitions of the USE3L model descriptors, sectors, and industry classifications, as well as detailed analytics for equity asset selection risk. A glossary and index are also provided.

The Barra on Campus Handbook also includes surveys for both students and professors. Please complete the appropriate survey at the end of the term and return it to us. Your feedback is important.

The Barra on Campus software is a complete set of equity, fixed income, and asset allocation programs. A comprehensive online help function is included with each program to answer your application questions. If you still have questions, please have your primary contact call the Help Desk toll free at (888) 588-4567.
About You

The Barra on Campus Handbook is written with the needs of students in mind. Students should be familiar with basic statistics and finance concepts in order to maximize the usefulness of the material. Examples of basic statistics concepts are correlation, covariance, mean, standard deviation, and normal distribution. Examples of basic finance concepts are CAPM, return and risk, duration and convexity, and the use of alphas and betas.

About MSCI Barra

Barra enables investment professionals around the world to make strategic investment decisions with confidence. Our comprehensive risk management solutions have set the industry standard for over 25 years. Barra’s strength in both equity and fixed income risk modeling is the result of almost three decades of focused research into the risk factors in markets around the world. Clients trust our products and services to support their business-critical portfolio and enterprise-wide risk-management needs.

In 2004, Barra, Inc. (“Barra”) was acquired by MSCI. MSCI develops and maintains equity, fixed income, multi-asset class, REIT and hedge fund indices that serve as benchmarks for an estimated USD 3 trillion on a worldwide basis. MSCI Barra is headquartered in New York, with research and commercial offices around the world. Morgan Stanley, a global financial services firm and a market leader in securities, asset management, and credit services, is the majority shareholder of MSCI, and Capital Group International, Inc. is the minority shareholder. MSCI Barra is a service mark of Morgan Stanley Capital International Inc. (“MSCI” or “MSCI Barra”).
Chapter 1

Risk and Model Fundamentals

- Introduction
- Risk Measures
- Measuring Performance Vs. a Benchmark
- Portfolio Management Strategies and Risk
- Multiple-Factor Models
Introduction

Superior investment performance is the product of careful attention to four elements:

• Forming reasonable return expectations.
• Controlling risk so that the pursuit of opportunities remains tempered by prudence.
• Controlling costs so that investment profits are not dissipated in excessive or inefficient trading.
• Controlling and monitoring the total investment process to maintain a consistent investment program.

These four elements are present in any investment management problem, be it a strategic asset allocation decision, an actively managed portfolio, or an index fund — managed bottom-up or top-down, via traditional or quantitative methods.

Figure 1: The Performance Pyramid
Superior investment results depend on understanding and controlling risk. Barra models and software permit you to minimize risk within your constraints, and to balance your pursuit of return with acceptable risk. In this Handbook we discuss how Barra models are put together, how risk is optimally controlled, and how a risk model can contribute to a deeper understanding of investment performance.

Risk Measures

A definition of risk that meets the criteria of being universal, symmetric, and flexible is the standard deviation of return.¹

Standard Deviation

A portfolio’s future return is described by a probability distribution. Every return has a probability of occurring, with one expected return having the largest chance of being realized. The width of this probability distribution represents risk to an investor. This is the uncertainty associated with the portfolio’s future value.

The standard deviation (\(\sigma\)) can be used to quantify the width of a probability distribution and describe the uncertainty, variability, or risk associated with the return of an asset or a portfolio.

¹ An economist would call the standard deviation a measure of uncertainty rather than risk.
If $R_p$ is a portfolio's total return, then the portfolio's standard deviation of return is denoted by

$$\sigma_p \equiv \text{Std}[R_p]$$

A portfolio's excess return ($r_p$) differs from the total return ($R_p$) by a constant $R_F$, so the risk of the excess return is equal to the risk of the total return. We typically quote this risk, or standard deviation of return, on an annualized basis. We also occasionally refer to this quantity as volatility.$^2$

The rough interpretation of standard deviation is that an asset return will be within one standard deviation of its expected value two-thirds of the time and within two standard deviations 19 times out of 20. Figure 3 on page 5 illustrates this idea.

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For example, suppose that we have determined that the standard deviation of an asset’s return is 3% and that its expected return is 2%. Using the normal distribution, we can determine that there is roughly a two-third probability that the asset will return between -1% and 5% in the next year; a 95% likelihood that it will return between -4% and 8%; and a 5% chance that it will return less than -4% or greater than 8%.

There is considerable evidence that financial returns are not normal. In particular, extreme values occur more often than predicted by a normal distribution. Still, at the levels of one or two standard deviations, financial returns can be treated as approximately normal. Assuming a normal distribution of portfolio or asset returns enables us to assign explicit probabilities to most reasonable returns.

**Variance**

One disadvantage of the standard deviation is that it is not additive, that is, the standard deviation of a portfolio is not simply the weighted average of the standard deviations of the individual assets in the portfolio.
Fortunately, the square of standard deviation, variance ($\sigma^2$), can be used to add sources of risk together, as in the case of adding the risks of individual assets to determine the risk of a portfolio.

The formulae are

$$\sigma = \text{Std}[r] = \sqrt{\text{Var}[r]} \quad \text{Eq. 1}$$

$$\sigma^2 = \text{Var}[r] = \mathbb{E}[(r - \bar{r})^2] \quad \text{Eq. 2}$$

where

$r$ = return

$\bar{r}$ = expected or mean return

$\text{Std}[x]$ = standard deviation of $x$

$\text{Var}[x]$ = variance of $x$

$\mathbb{E}[x]$ = expected value of $x$

The standard deviation is the more common risk indicator since it is measured in the same units as return. Of course, if the standard deviation is known, the variance can be easily computed and vice versa.

**Other Measures of Risk**

*Value-at-risk* (VaR) characterizes the potential loss in a portfolio over a given time period for a chosen probability level. A typical statement of VaR would be “with 2.5% probability, losses will exceed 4%.”

*Beta* ($\beta$) captures the sensitivity of a portfolio to market movements. Typically, higher beta portfolios or assets tend to exhibit greater volatility.
The coefficient of determination, $R^2$, estimates the amount of variability in a portfolio that is explained by the market’s volatility. It is directly related to beta.

For fixed-income assets, duration measures the price responsiveness of an interest-sensitive asset to changes in interest rates. Duration is a reasonably good predictor as long as the change in interest rates is small and affects the entire yield curve. Macaulay duration describes the weighted average time to receipt of a series of cash flows, where the weights are the proportions of the present values of the cash flows to the present value of the entire series of cash flows.

As we will see in Chapter 3, “Fixed Income Multiple-Factor Modeling”, Barra employs principal component analysis to provide accurate measures for fixed-income risk.

**Multiple-Factor Models**

Multiple-factor models (MFMs) are formal statements about the relationships among asset returns in a portfolio. The basic premise of MFMs is that similar assets display similar returns. Assets can be similar in terms of fundamental company data (such as industry, capitalization, and indebtedness) or other attributes such as liquidity or duration.

MFMs identify common factors and determine return sensitivity to these factors. The resulting risk model describes a return as the weighted sum of common factor return and specific return, or that part of the return not attributed to model factors. In Barra’s MFMs, the risk profile responds immediately to changes in fundamental information. For more information, see “Multiple-Factor Models” later in this chapter.
Measuring Performance Vs. a Benchmark

Portfolio performance is most often measured with respect to a benchmark for investment performance and risk analysis. The benchmark is also known as the normal portfolio—that is, the asset basket a manager would hold in the absence of any judgmental information. It may reflect the manager’s particular style and biases.

While total risk describes the uncertainty of investment outcomes for an entire portfolio, active risk describes the risk that arises from the manager’s effort to outperform the benchmark. While outperforming the benchmark is the ultimate goal of an active manager, it is certainly not always achieved. Active risk (or tracking error) measures the uncertainty surrounding the portfolio’s performance with respect to its benchmark.

You may associate tracking error with passive management strategies and active risk with active management strategies. For the purposes of Barra’s analysis, these measures are considered the same.
Tracking Error and Active Risk

Active risk describes the expected volatility of the difference between the managed portfolio return and the benchmark return. They are both measured as the annualized standard deviation of active returns.

![Graph showing portfolio and benchmark returns with active return and active risk](image)

*Figure 4: Portfolio and Benchmark Returns*

The difference between the portfolio’s and the benchmark’s returns is active return. The volatility of that return is active risk.

Portfolio Management Styles

Passive Management

In its broadest sense, passive management refers to any management strategy that does not rely on the possession of superior information. More specifically, disclosure of a passive investment strategy offers no competitive information that would undermine the strategy’s validity.

One type of passive management is *indexing* or tracking the performance of a particular index. An example is the “buy-and-hold” philosophy which exposes the portfolio only to benchmark risk. The second form of passive management is constructing a portfolio to match pre-specified attributes or constraints. The portfolio may be yield-biased with a selected beta or match an index within certain parameters. This is often called *enhanced indexing.*
Passive management procedures are distinguished by the following attributes:

- They exclude any transactions in response to judgments about security valuations and the market as a whole.
- They contain relatively minimal residual risk with respect to the benchmark or index.
- They often involve weighting by industry or sector.

Active Management

Active management refers to investment strategies designed to increase return by using superior information. In other words, the active manager seeks to profit from information that would lose its value if all market participants interpreted it in the same way. If, for example, an investment manager observed that securities with certain characteristics performed better (or worse) than expected, the manager could hold a larger (or smaller) proportion of that security to increase the subsequent value of the portfolio.

By following active management strategies, investors can add value to their portfolio if they predict returns better than the consensus expectations of the market. Information is obtained through ongoing research to forecast such things as yield curve changes, factor and industry returns, and macro-economic directions. At any given time, portfolio construction should reflect the tradeoff between risk and return—that is, any contribution to risk should be offset by the contribution to reward.

For a discussion of active investment strategies, see “Portfolio Management Strategies and Risk” on page 11.
Alphas

*Alpha* ($\alpha$) generally refers to the expected exceptional return of a portfolio, factor, or individual asset. It measures how much an asset or portfolio has moved up or down during a given year. A positive alpha is the extra return that arises from taking specific (non-market) risk rather than passively taking market return. For example, an asset with an alpha of 1.40 means that, if the market and asset beta is 0, its price will rise by 40% in a year.

The use of alphas is a distinction of active management. It indicates that a manager believes a portion of expected return is attributable to particular factors, such as the rate of growth in earnings per share.

**Portfolio Management Strategies and Risk**

**Equity Portfolio Strategies**

For equity portfolios, the strategies include market timing, sectoral emphasis, and stock selection.

*Market timing* is the process of altering market risk exposure based on short-term forecasts in order to earn superior returns. The manager seeks to sell before the market goes down and buy before the market goes up. However, this strategy increases the variability in the portfolio beta, inducing increased systematic risk through time. Barra MFMs assist market timing by giving the investor a robust beta estimate for any portfolio and indicating the most efficient way to take on or reduce market risk, including the use of futures.
The second type of active management is *sectoral emphasis*. Sectoral emphasis can be thought of as a combination of the other active strategies. It is both factor timing and a broad version of stock selection. The manager attempts to increase residual return through manipulating common factor exposures. For example, the manager can bet on an industry of high-yield stocks. Because several sectors can be emphasized at any given time, diversification is possible. Barra MFMs possess detailed industry and risk index exposure information that can be utilized for any combination of sectoral tilts.

Finally, *stock selection* is a portfolio allocation strategy based on picking mispriced stocks. It uses security alphas to identify overvalued and undervalued stocks. The manager can then adjust the asset proportions in the portfolio to maximize specific return (discussed below, on page 18). These active holdings, in both positive and negative directions, increase residual risk and portfolio alpha. The primary objective of this strategy is to manage asset holdings so that any change in incremental risk is compensated by a comparable change in return. Barra MFMs facilitate stock selection by extending the risk model down to the individual equity level.

*Figure 5 on page 13* illustrates the typical prevalence of these various types of risk in a single stock, a small portfolio, and a multiple-portfolio situation. In each case, the manager’s goal is to earn a superior return with minimum risk. The use of a multiple-factor model permits the manager to pursue these active management strategies with maximum precision.

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3 A stock with a price that is low relative to its alpha is deemed undervalued.
Fixed-Income Portfolio Strategies

For fixed income portfolios, the strategies include market timing, sectoral rotation, and bond selection.

As with equities, market timing is the process of altering market risk exposure based on short-term forecasts in order to earn superior returns. Asset allocation is based on predictions on which way interest rates are going. The manager makes the portfolio more or less sensitive to the interest rates by altering the maturity structure. Allocating more capital to long-term holdings makes the portfolio more sensitive to interest rate changes; allocating more to short-term holdings makes it less sensitive. Demographic shifts, economic trends, and socio-political changes are closely watched to anticipate changes in the yield curve. But this strategy is much more difficult to implement with fixed income instruments than with equities. Barra MFMs assist market timing by forecasting forward rate curves for any market and for any yield curve shape and by providing a detailed analysis of exposure to interest rates for any portfolio.
The second type of active management, *sectoral rotation* attempts to outperform the benchmark by making anticipatory spread bets. The manager predicts changes in market price relationships between different types of bonds and allocates funds accordingly. For example, if the spread between a retailer-issued bonds and treasury bonds is anticipated to widen because of weakening consumer demand, the manager would reduce holdings of retailer-issued bonds. On the other hand, if the spread is anticipated to tighten, the manager would accumulate more retailer-issued bonds. Barra MFMs facilitate allocation decisions by forecasting the volatility of spreads and the risk rate of credit migration.

Lastly, *bond selection* is a portfolio allocation strategy based on picking undervalued bonds. Mispriced bonds are found by credit and fundamental analyses. The manager accumulates bonds whose ratings she predicts will improve because the issuer’s fundamentals are strengthening, the market has not correctly calculated the underlying cashflow of the issue, or macroeconomic directions favor the issuer. Alternatively, the manager picks bonds that have higher yields than their peers. For example, a manager who is comparing the historical price and yield of two similar treasury bonds would select the relatively cheaper one. Barra MFMs facilitate bond selection by extending the risk model down to the individual security level.

*Figure 6 on page 15* illustrates the typical prevalence of these various types of risk in a single bond, a small portfolio, and a multiple-portfolio situation. In each case, the manager’s goal is to earn a superior return with minimum risk. The use of a multiple-factor model permits the manager to pursue these active management strategies with maximum precision.
Barra and the Investment Process

In order to implement a desired strategy, an investment portfolio manager routinely participates in activities ranging from asset risk analysis to scenario testing to portfolio optimization. Optimization refers to the process of establishing an efficient frontier, or a set of portfolios with maximum expected net return for a given level of risk, based on defined parameter constraints. For more information about optimization, see Chapter 4, “Optimization,” starting on page 87.

In order to automate routine tasks, importing and maintaining asset-level and portfolio data is of course critical for the use of the software. Barra facilitates this process by providing wide coverage to access global data and indices from a variety of institutional investment firms. Asset selection and trades will be governed by the investment firm’s philosophy and management style and influenced by the strategies discussed above.

*Figure 6: Diversification and Risk—Bonds*

Term structure risk grows as a proportion of total risk as portfolio size increases.
Multiple-Factor Models

Portfolio risk and return can be decomposed along two dimensions: that which is due to factors which are prevalent throughout the market and that which is due to the idiosyncratic nature of the securities in the portfolio. A multiple-factor model is a powerful tool to shed light on these sources of risk and return within a portfolio.

Single Factor

For single-factor models, the following equation describes the excess rate of return:

\[ r_j = X_j f + u_j \]  \hspace{1cm} \text{Eq. 3}

where

- \( r_j \) = total excess return over the risk-free rate of security \( j \)
- \( X_j \) = sensitivity of security \( j \) to the factor
- \( f \) = rate of return on the factor
- \( u_j \) = non-factor or specific return of security \( j \)
Multiple Factors

We can expand this model to include $K$ factors. The total excess return equation for a multiple-factor model becomes

$$r_j = \sum_{k=1}^{K} X_{j,k} f_k + \epsilon_j$$

Eq. 4

where

$X_{j,k}$ = sensitivity of security $j$ to factor $k$

$f_k$ = return of factor $k$

$\epsilon_j$ = non-factor or specific return of security $j$

If we unfurl Equation 4, it takes the form of Figure 5 on page 21. The asset’s return is broken out into the return due to individual factors and a portion unique to the asset and not due to the common factors. In addition, each factor’s contribution is a function of the portfolio’s exposure or weight in the factor and the return of that factor.

Exposures ($X_{j,k}$)

By observing patterns over time, common factors can be identified and exposures to these factors can be determined. An exposure represents the sensitivity of an asset to a model factor. Model factors are based around market or fundamental data which is readily available for most securities, and can be computed fairly easily at the end of any month. Factors have different possible exposure values. In the Cosmos fixed-income risk model, for example, each asset has a currency risk factor exposure of either 0 or 1: the asset will be exposed to the currency factor or not exposed. Fixed-income assets may have a swap spread factor exposure equal to the spread duration.
Because these factors are based on fundamental or market information, the Barra model’s profile of a security will respond immediately to any changes in the company’s structure or the market’s behavior. Most Barra models update security exposures on a monthly basis using the last trading day’s information to compute exposures for the coming month. In the case of interest rate exposures for fixed-income assets, the Cosmos application calculates these on the fly within the application (for a discussion of the shift, twist, and butterfly exposures, see “Term Structure Risk” on page 50).

**Factor Return ($f_k$)**

We define factor returns as pure measures of the factor’s actual performance in isolation of any other effects. It is critical that this return be an unbiased estimate of how that factor actually behaves. This enhances risk analysis, portfolio construction, and performance analysis.

Since factor returns are not readily observable, we must derive their behavior. Recall that asset exposures are computed at the end of each month. Using our multiple-factor model framework and the observed asset returns over the next month, we can solve for what the factors must have returned during the course of the month. This is done with a cross-sectional regression over the returns and exposure of assets used in model estimation — known as the estimation universe — for that month.

**Specific Return ($\varepsilon_j$)**

Specific return is the difference between the actual return and the predicted model return on an asset. Since the model explains whatever can be attributed to common factors, specific return is that part of the return that is not associated with common factors. The use of the term “specific” refers to that aspect of the issue that is specific to the issue and unrelated to all other issues.

Specific risk — referred to in Barra’s Aegis product as asset selection risk — represents the uncertainty in asset or portfolio return that arises from the unpredictable component of the specific return.
Decomposing Risk

Barra’s multiple-factor models (MFMs) isolate components of risk based on correlations across assets sharing similar characteristics.

Active risk has two main sources: common factors, which are prevalent throughout the market, and security specific risk, which reflects the idiosyncratic nature of each asset. Both total risk and active risk can be decomposed into common-factor and asset-selection components.

Risk decomposition enables us to identify and isolate the contribution of each of these sources to active risk:

\[
\text{var(common factor risk)} + \text{var(active specific risk)} = \text{var(total active risk)}
\]

Chapter 2, “Equity Multiple-Factor Modeling” and Chapter 3, “Fixed Income Multiple-Factor Modeling” in this Handbook describe the common factors and specific risk methodology used in Barra’s models.
Advantages of MFMss

Multiple-factor models offer several advantages for security and portfolio analysis:

- A more thorough breakdown of risk and, therefore, a more complete analysis of risk exposure than other methods such as single-factor approaches.
- Because economic logic is used in their development, MFMss are not limited by purely historical analysis.
- MFMss are robust investment tools that can withstand outliers.
- As the economy and characteristics of individual issuers change, MFMss adapt to reflect changing asset characteristics.
- MFMss isolate the impact of individual factors, providing segmented analysis for better informed investment decisions.
- From an applications viewpoint, MFMss are realistic, tractable, and understandable to investors.
- MFMss are flexible models allowing for a wide range of investor preferences and judgment.

Of course, MFMss have their limitations. They predict much, but not all, of portfolio risk. In addition, they predict risk, not return; investors must choose the investment strategies themselves.
Mathematics

In constructing a multiple-factor risk model, Barra uses historical returns to create a framework for forecasting future risk. Each month, we use a representative universe of assets to estimate returns to the common factors. The analysis begins with the decomposition of asset return into a common factor piece and a specific piece, shown mathematically in Equation 5.

Multiple-Factor Framework for Return Decomposition

\[ r_i = x_{i,1}f_1 + x_{i,2}f_2 + \ldots + x_{i,k}f_k + \epsilon_i \]  

Eq. 5

where

- \( r_i \) = excess return of asset \( i \)
- \( x_{i,k} \) = exposures of asset \( i \) to factor \( k \)
- \( f_k \) = return to factor \( k \)
- \( i \) = specific return of asset \( i \)

We can represent the monthly returns for the asset universe as a single matrix equation of \( n \) assets and \( m \) factors. Each row represents one of the assets in a portfolio or bond universe.

Return Decomposition (Matrix Format)

\[
\begin{bmatrix}
  r_1 \\
  r_2 \\
  \vdots \\
  r_n
\end{bmatrix} =
\begin{bmatrix}
  x_{1,1} & x_{1,2} & \ldots & x_{1,m} \\
  x_{2,1} & x_{2,2} & \ldots & x_{2,m} \\
  \vdots & \vdots & \ddots & \vdots \\
  x_{n,1} & x_{n,2} & \ldots & x_{n,m}
\end{bmatrix}
\begin{bmatrix}
  f_1 \\
  f_2 \\
  \vdots \\
  f_m
\end{bmatrix}
+ \begin{bmatrix}
  \epsilon_1 \\
  \epsilon_2 \\
  \vdots \\
  \epsilon_n
\end{bmatrix}
\]
At the end of a month, we know the return of each asset as well as its factor exposures at the beginning of the month. We estimate the factor returns via regression. The factor returns are the values that best explain the asset returns.

**Calculating Risk**

The covariance matrix is used in conjunction with a portfolio’s weight in each asset and the factor exposures of those assets to calculate portfolio risk. **Equation 6 on page 22** is the underlying form of our risk calculations:

**Portfolio Volatility or Risk**

\[
\sigma_p = \sqrt{\mathbf{h}_p (\mathbf{XFX}^T + \mathbf{S}) \mathbf{h}_p^T}
\]

Eq. 6

where

- \( \sigma_p \) = volatility of portfolio returns
- \( \mathbf{h}_p \) = vector of portfolio weights for \( n \) assets: \[
\begin{bmatrix}
  h_1 \\
  \vdots \\
  h_n
\end{bmatrix}
\]
- \( \mathbf{X} \) = matrix of factor exposures of \( n \) assets to \( m \) factors:

\[
\begin{bmatrix}
  x_{1,1} & x_{1,2} & \cdots & x_{1,m} \\
  x_{2,1} & x_{2,2} & \cdots & x_{2,m} \\
  \vdots & \vdots & \ddots & \vdots \\
  x_{n,1} & x_{n,2} & \cdots & x_{n,m}
\end{bmatrix}
\]
Covariances, Volatilities, and Correlations

An alternative representation of variances and covariances is volatilities and correlations. The latter representation is easier to interpret. Recall that the diagonal of the covariance matrix contains factor variances so that the factor volatility (or standard deviation) is equal to

\[
\sigma_j = \sqrt{\text{var}(f_j)}
\]

The correlation between two factors is equal to

\[
\rho_{i,j} = \frac{\text{cov}(f_i, f_j)}{\sigma_i \sigma_j}
\]

Note that the covariance matrix can be reconstructed from the factor volatilities and correlations.

Risk Prediction with MFMs

To estimate a portfolio’s risk, we must consider not only the security or portfolio’s exposures to the factors, but also each factor’s risk and the covariance or interaction between factors.

\[
F = \text{factor returns variance-covariance matrix for } m \text{ factors:}
\begin{bmatrix}
\text{Var}(f_1) & \text{Cov}(f_1, f_2) & \cdots & \text{Cov}(f_1, f_m) \\
\text{Cov}(f_2, f_1) & \text{Var}(f_2) & \cdots & \text{Cov}(f_2, f_m) \\
\vdots & \vdots & \ddots & \vdots \\
\text{Cov}(f_m, f_1) & \text{Cov}(f_m, f_2) & \cdots & \text{Var}(f_m)
\end{bmatrix}
\]

\[
S = \text{specific risk covariance matrix}
\]
To calculate the variance of a portfolio, you need to calculate the covariances of all the constituent components.

Without the framework of a multiple-factor model, estimating the covariance of each asset with every other asset is computationally burdensome and subject to significant estimation errors. For example, using an estimation universe of 1,400 assets, there are 980,700 covariances and variances to calculate.

\[
V(i,j) = \text{Cov}[r(i),r(j)]
\]

where

\[
V(i,j) = \text{asset covariance matrix}
\]

\[
i,j = \text{individual assets}
\]

\[
V = \begin{bmatrix}
V(1,1) & V(1,2) & \cdots & V(1,N) \\
V(2,1) & V(2,2) & \cdots & V(2,N) \\
V(3,1) & V(3,2) & \cdots & V(3,N) \\
\vdots & \vdots & \ddots & \vdots \\
V(N,1) & V(N,2) & \cdots & V(N,N)
\end{bmatrix}
\]

Figure 8: Asset Covariance Matrix
For \(N=1,400\) factors, there are 980,700 covariances and variances to estimate.

A multiple-factor model simplifies these calculations dramatically. This results from replacing individual asset profiles with categories defined by common characteristics (factors). Since the asset selection risk is assumed to be uncorrelated among the assets, only the factor variances and covariances need to be calculated during model estimation.\(^4\)
For example, in the U.S. Equity Model (USE3L), 68 factors capture the risk characteristics of equities. This reduces the number of covariance and variance calculations to 2,346. Moreover, since there are fewer parameters to determine, they can be estimated with greater precision.

---

4 Barra’s models allow asset selection risk to be correlated between some assets, such as two securities from the same issuer.
Chapter 2

Equity Multiple-Factor Modeling

A Barra equity risk model is the product of a thorough and exacting model estimation process. It is an extensive, detailed process of determining the factors that describe asset returns. In this chapter we provide a brief overview of this process.

- Overview
- Data Acquisition and Cleaning
- Risk Index Formation
- Industry Allocation
- Factor Return Estimation
- Covariance Matrix Calculation
- Updating the Model
- Risk Decomposition
Overview

Many methods exist for forecasting a stock’s future volatility. One method is to examine its historical behavior and conclude that it will behave similarly in the future. An obvious problem with this technique is that results depend on the length of history used and the type of history used. The security might have changed over time due to mergers, acquisitions, divestitures, or other corporate actions. The past contains information that may be no longer relevant to the present. Yet, this is the approach most commonly used for measuring beta.

A more informative approach uses insights into the characteristics and behavior of the stock and market as a whole, as well as the interactions between them.

Common Factors

Common factors shared by stocks — such as country membership (and trends in that market), industry membership (and trends in that industry), recent trends in the stock’s price and the market’s returns, current fundamental ratios, and trading activity—not only help explain performance, but also anticipate future volatility. Industry membership factors may significantly impact assets within an industry group, while country membership and currency factors often dominate global portfolios.

Barra’s equity risk models capture these various components of risk and provide a multifaceted, quantitative measure of risk exposure. As described in this chapter, Barra combines fundamental and market data to create risk indices. Together with industry groups, risk indices provide a comprehensive partition of risk.
Model Estimation Flow Chart

The creation of a comprehensive equity risk model is an extensive, detailed process of determining the factors that describe asset returns. Model estimation involves a series of intricate steps that is summarized in Figure 9 below, and explained in more detail in this chapter.

**Phase I: Factor Exposures**

![Diagram showing data flow and manipulations for Phase I]

**Phase II: Factor Return Estimation**

![Diagram showing monthly cross-sectional weighted regression for Phase II]

**Phase III: Risk Estimation**

![Diagram showing risk estimation process]

*Figure 9: Data Flow for Model Estimation Process*

Ovals denote data flow; rectangles represent manipulations of and additions to data.
Data Acquisition and Cleaning

Model estimation starts with the acquisition and standardization of data. Barra uses both market information (such as price, dividend yield, or capitalization) and fundamental data (such as earnings, sales, or assets). We pay special attention to capital restructurings and other atypical events to provide for consistent cross-period comparisons.

Market data and fundamental data for each equity risk model are gleaned, verified, and compiled from more than 100 data feeds supplied by 56 data vendors. Market information is collected daily. Fundamental company data is derived from quarterly and annual financial statements.

After data is collected, it is scrutinized for inconsistencies, such as jumps in market capitalization, missing dividends, and unexplained discrepancies between the day's data and the previous day's data. Special attention is paid to capital restructurings and other atypical events to provide for consistent cross-period comparisons. Information then is compared across different data sources to verify accuracy.

Our robust system of checks and our data collection infrastructure, which has been continuously refined for more than 25 years, ensure that Barra's risk models utilize the best available data.
Risk Index Formation

Descriptor Selection and Testing

Descriptor candidates are drawn from several sources. This step involves choosing and standardizing variables that best capture the risk characteristics of the assets. To determine which descriptors partition risk in the most effective and efficient way, Barra tests the descriptors for statistical significance. Useful descriptors often significantly explain cross-sectional returns.

For some descriptors, market and fundamental information is combined. An example is the earnings-to-price ratio, which measures the relationship between the market’s valuation of a firm and the firm’s earnings.

Descriptor selection is a largely qualitative process that is subjected to rigorous quantitative testing. First, preliminary descriptors are identified. Good descriptor candidates are individually meaningful; that is, they are based on generally accepted and well-understood asset attributes. Furthermore, they divide the market into well-defined categories, providing full characterization of the portfolio’s important risk features. Barra has more than two decades of experience identifying important descriptors in equity markets worldwide. This experience informs every new model we build.

Selected descriptors must have a sound theoretical justification for inclusion in the model. They must be useful in predicting risk and based on timely, accurate, and available data. In other words, each descriptor must add value to the model. If the testing process shows that they do not add predictive power, they are rejected.
Standardization

The descriptors are normalized; that is, they are standardized with respect to the estimation universe. The normalization process involves setting random variables to a uniform scale. A constant (usually the mean) is subtracted from each number to shift all numbers uniformly. Then each number is divided by another constant (usually the standard deviation) to shift the variance.

The normalization process is summarized by the following relation:

\[
\text{normalized descriptor} = \frac{\text{raw descriptor} - \text{mean}}{\text{standard deviation}}
\]

Barra regresses asset returns against industries and descriptors, one descriptor at a time, after the normalization step.

Combining Descriptors

Risk index formulation and assignment to securities is the next step. This process involves collecting descriptors into their most meaningful combinations. Though judgment plays a major role, Barra uses a variety of techniques to evaluate different possibilities. For example, we employ statistical cluster analysis to assign descriptors to risk indices.

Descriptors for the model are selected and assigned to risk indices using proprietary formulas. Risk indices are composed of descriptors designed to capture all the relevant risk characteristics of a company.

Risk index formulation is an iterative process. After the most significant descriptors are added to the model, remaining descriptors are subjected to stricter testing. At each stage of model construction, a new descriptor is added only if it adds explanatory power to the model beyond that of industry factors and already assigned descriptors.
Industry Allocation

Along with risk index exposures, industry allocations are determined for each security. Industry allocation is defined according to what is appropriate to the local country environment. Each security is classified into an industry by its operations. Barra either uses a data vendor’s allocation scheme or creates one that categorizes the assets in the estimation universe better.

Most equity models allocate companies to single industries. For the U.S. and Japan, however, sufficient data exist to allocate to multiple industries. For these markets, industry exposures are allocated using industry segment data (such as operating earnings, assets, and sales). The model incorporates the relative importance of each variable in different industries. For example, the most important variable for oil companies would be assets; for retail store chains, sales; and for stable manufacturing companies, earnings. For any given multi-industry allocation, the weights will add up to 100%.

Multiple industry allocation provides more accurate risk prediction and better describes market conditions and company activity. Barra’s multiple-industry model captures changes in a company’s risk profile as soon as new business activity is reported to shareholders. Alternative approaches can require 60 months or more of data to recognize changes that result from market prices.
Factor Return Estimation

The previous steps have defined the exposures of each asset to the factors at the beginning of every period in the estimation window. The factor excess returns over the period are then obtained via a cross-sectional regression of asset excess returns on their associated factor exposures. (For a review of how factor return estimation works, see “Multiple-Factor Models” starting on page 16.) The resulting factor covariance matrix is used in the remaining model estimation steps.

Covariance Matrix Calculation

Factor returns are used to estimate covariances between factors, generating the covariance matrix used to forecast risk. The simplest way to estimate the factor covariance matrix is to compute the sample covariances among the entire set of estimated factor returns. Implicit in this process is the assumption that we are modeling a stable process and, therefore, each point in time contains equally relevant information.

A stable process implies a stable variance for a well-diversified portfolio with relatively stable exposures to the factors. However, considerable evidence shows that correlations among factor returns change. In some markets periods of high volatility are often followed by periods of high volatility. The changing correlations among factor returns and the changing volatility of market portfolios belie the stability assumption underlying a simple covariance matrix.
For certain models we relax the assumption of stability in two ways. First, in computing the covariance among the factor returns, we assign more weight to recent observations relative to observations in the distant past. Second, we estimate a model for the volatility of a market index portfolio—for example, the S&P 500 in the U.S. and the TSE1 in Japan—and scale the factor covariance matrix so that it produces the same volatility forecast for the market portfolio as the model of market volatility (described in “Scaling to Match a Market Forecast” on page 36).

**Scaling the Covariance Matrix**

**Exponential Weighting**

Suppose that we think that observations that occurred 60 months ago should receive half the weight of the current observation. Denote by $T$ the current period, and by $t$ any period in the past. $t=1,2,3,\ldots, T-1$, and let $\delta = 5^{1/60}$. If we assign a weight of $\delta^{T-1}$ to observation $t$, then an observation that occurred 60 months ago would get half the weight of the current observation, and one that occurred 120 months ago would get one-quarter the weight of the current observation. Thus, our weighting scheme would give exponentially declining weights to observations as they recede in the past.

Our choice of 60 months was arbitrary in the above example. More generally, we give an observation that is half-life months ago one-half the weight of the current observation. Then we let

$$\delta = 5^{1/\text{HALFLIFE}}$$  

**Eq. 7**
and assign a weight of

\[ w(t) = \delta^{T-t} \]  \hspace{1cm} \text{Eq. 8}

The length of the half-life controls how quickly the factor covariance matrix responds to recent changes in the market relationships between factors. Equal weighting of all observations corresponds to half-life = \( \infty \). Too short a half-life effectively throws away data at the beginning of the series. If the process is perfectly stable, this decreases the precision of the estimates. Our tests show that the best choice of half-life varies from country to country. Hence, we use different values of half-life for different single country models.

**Scaling to Match a Market Forecast**

In some markets, market volatility changes in a predictable manner. We find that returns that are large in absolute value cluster in time, or that volatility persists. Moreover, periods of above normal returns are, on average, followed by lower volatility, relative to periods of below-normal returns. Finally, we find that actual asset return distributions exhibit a higher likelihood of extreme outcomes than is predicted by a normal distribution with a constant volatility.

Barra applies variants of these systematic scalings as appropriate to its single country models.\(^1\) Before scaling is applied to any model, a scaling model is tested and validated.\(^2\) If Barra can satisfactorily fit a scaling model to the volatility of a market proxy portfolio, the model is used to scale the factor covariance matrix so that the matrix gives the same risk forecast for the market portfolio as the scaling model. Only the systematic part of the factor covariance matrix is scaled.

---

1 Some markets, such as the emerging markets, are not scaled.

2 The scaling models are Generalized Auto-Regressive Conditional Heteroskedasticity (GARCH) techniques or Daily Exponentially Weighted Index Volatility (DEWIV).
The Extended GARCH Model

Barra models use different approaches to forecast future volatility levels. The use of a GARCH submodel is one. The following discussion lays out the general theory of extended GARCH modeling.

Denote by $r_t$ the market return at time $t$, and decompose it into its expected component, $E(r_t)$, and a surprise, $e_t$, thus:

$$ r_t = E(r_t) + e_t $$

Eq. 9

The observed persistence in realized volatility indicates that the variance of the market return at $t$, $Var(r_m)$, can be modeled as:

$$ Var(r_m) = \omega + \alpha e_{t-1}^2 + \beta Var(r_m)_{t-1} $$

Eq. 10

where:

- $(r_m)_t$ = market return at time $t$
- $e_{t-1}^2$ = recent realized volatility
- $Var(r_m)_{t-1}$ = recent forecast of market volatility

The coefficients $\omega$ (forecasted volatility of the market), $\alpha$ (size of recent realized volatility), and $\beta$ (size of volatility forecasts) are estimated through modeling.

---

3 The form of the variance forecasting function distinguishes GARCH models from one another.
This equation, which is referred to as a GARCH(1,1) model, says that current market volatility depends on recent realized volatility via $\varepsilon_{t-1}^2$, and on recent forecasts of volatility via $\text{Var}(\tilde{r}_m)_{t-1}$.

If $\alpha$ and $\beta$ are positive, then this period’s volatility increases with recent realized and forecast volatility. GARCH(1,1) models have been found to fit many financial time series. Nevertheless, they fail to capture relatively higher volatility following periods of below-normal returns. We can readily extend the GARCH(1,1) model to remedy this shortcoming by modeling market volatility as:

$$\text{Var}(\tilde{r}_m) = \omega + \alpha \varepsilon_{t-1}^2 + \beta \text{Var}(\tilde{r}_m)_{t-1} + \theta \varepsilon_{t-1}$$  \hspace{1cm} \text{Eq. 11}

If $\theta$ is negative, then returns that are larger than expected are followed by periods of lower volatility, whereas returns that are smaller than expected are followed by higher volatility.

### Specific Risk Modeling

During factor returns estimation, Barra separates out specific returns and forecasts specific risk. This portion of total risk is related solely to a particular stock and cannot be accounted for by common factors. The greater an asset’s specific risk, the larger the proportion of return attributable to idiosyncratic, rather than common, factors.

Referring to the basic factor model:

$$\tilde{r}_i = \sum_k \chi_{ik} \tilde{f}_k + \tilde{\epsilon}_k$$  \hspace{1cm} \text{Eq. 12}
The specific risk of asset $i$ is the standard deviation of its specific return, $\text{Std}(\tilde{e}_i)$. The simplest way to estimate the specific risk matrix is to compute the historical variances of the specific returns. This, however, assumes that the specific return distribution is stable over time. Rather than use historical estimates, we model specific risk to capture fluctuations in the general level of specific risk and the relationship between specific risk and asset fundamental characteristics.

An asset’s specific risk is the product of the average level of specific risk that month across assets, and each asset’s specific risk relative to the average level of specific risk. Moreover, our research has shown that the relative specific risk of an asset is related to the asset’s fundamentals. Thus, developing an accurate specific risk model involves a model of the average level of specific risk across assets, and a model that relates each asset’s relative specific risk to the asset’s fundamental characteristics.

Methodology

Denote by $S_t$ the average level of specific risk across assets at time $t$, and by $V_{it}$ (asset $i$’s specific risk relative to the average). In equation form,

$$\text{Std}(\tilde{u}_i) = S_t (1 + V_{it})$$  \hspace{1cm} Eq. 13

where

- $\text{Std}(\tilde{u}_i)$ = asset specific risk
- $S_t$ = average level of specific risk at time $t$
- $V_{it}$ = asset $i$’s specific risk relative to the average
We estimate a model for the *average level of specific risk* via time-series analysis, in which the average level of realized specific risk is related to its lagged values and to lagged market returns. Similarly, we estimate a model of relative specific risk by regressing realized relative specific risks of all firms, across all periods, on the firm fundamentals, which include the Barra risk factors.

For the formulae for modeling the average and relative levels of specific risk, estimating the scaling coefficients, and obtaining the final specific risk forecast, see Appendix C, “Equity Asset Selection Risk Modeling,” starting on page 123.

### Updating the Model

Last, the model undergoes final testing and updating. Tests include comparing risk forecasts against results obtained from alternative models, and comparing *ex ante* forecasts with *ex post* realizations of beta, specific risk, and active risk. For each upcoming month’s risk forecast, Barra incorporates new information from company fundamental reports and market data, and recalculates the covariance matrix.

The latest data are collected and cleaned. Descriptor values for each company in the database are computed, along with risk index exposures and industry allocations. Next, a cross-sectional regression is run on the asset returns for the previous month. This generates factor returns which are used to update the covariance matrix. Finally, this updated information is distributed to users of Barra’s applications software.
Risk Decomposition

Barra’s Aegis Portfolio Manager application enables you to decompose risk in the manner shown in Figure 10. For a brief description of each risk category, see Table 1 on page 42.

![Figure 10: Risk Decomposition for Equity Models](image)

Note: For Barra’s Aegis clients who subscribe to multiple-country models, the risk composition will also include country and currency risk.
### Table 1: Equity Model Risk Categories

<table>
<thead>
<tr>
<th>Risk Category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Timing</td>
<td>The risk due to exposure to the market, equal to the beta times the market risk. In variance terms, it is equal to the beta squared times market variance.</td>
</tr>
<tr>
<td>Common Factors</td>
<td></td>
</tr>
<tr>
<td>- Risk Indices (Style)</td>
<td>For a listing of risk indices and constituent descriptors used in the USE3L model, see Appendix A, “USE3L Descriptors”.</td>
</tr>
<tr>
<td>- Sectors and Industries</td>
<td>For a listing of sectors and industries used in the USE3L model, see Appendix B, “USE3L Sector and Industry Definitions”.</td>
</tr>
<tr>
<td>- Covariances</td>
<td>A statistical measure of the similarity of return outcomes between risk indices and sectors.</td>
</tr>
<tr>
<td>Asset Selection</td>
<td>Risk arising from the specific or idiosyncratic behavior of assets. Asset selection risk is independent of common factor risk categories, and is uncorrelated (or negligibly correlated) with the specific risk of other companies.</td>
</tr>
</tbody>
</table>
Chapter 3

Fixed Income
Multiple-Factor Modeling

This chapter provides an overview of the fixed-income model used by Barra’s Cosmos Global Risk Manager product.
Overview

Constructing the Cosmos fixed-income risk model is something like putting a puzzle together. The pieces must fit with each other and show the entire picture of risk, as shown in Figure 11.

Term structure and yield spread estimation are processes that involve asset valuation. Barra’s Cosmos risk model defines five common factor risk categories: currency, emerging market, term structure, swap spread, and credit risk. Factor exposures determine the sensitivity of portfolio return to market changes. The exposure of an asset to a risk factor is the sensitivity of its excess return to a change in the factor level. For example, the exposure of a bond to a change in the average level of interest rates is its effective duration.

Figure 11: Piecing Together the Cosmos Fixed-Income Risk Model
The component of return not explained by the common factors can be quite volatile. This return is called *specific return*, and it is idiosyncratic to the issue or to the issuer. The risk due to variation in specific return is captured by the Cosmos *specific risk* model.

**Risk Decomposition**

Barra’s approach to fixed-income risk analysis starts with a decomposition of excess return into its common factor piece and a specific piece. The Cosmos model groups common factors into *global factors*, which arise in situations where currencies are mixed or when the local currency has only a small effect on the risk of an issue, and *local factors*, which affect assets within a particular market.

In a refinement of the basic risk breakdown shown in Figure 7 on page 19, Figure 12 on page 46 shows how Cosmos individually calibrates specific risk to each local market and combines it with local market common factor risk to determine total local market risk. Specific risk is discussed later in this chapter.

---


2. Fixed-income assets with lower credit quality tend to have more volatile specific return.

3. The specific risk of some corporate bonds is captured with a rating migration model that depends on the issuer, not on the issue. For more information, see “Modeling Event Risk” on page 74.
Cosmos determines a local market by currency. For example, bonds denominated in the Japanese yen may be thought of as a single market. In the yen market, the main sources of risk are changes in Japan interest rates and credit spreads. However, the structure of a local market can be more complex. For example, the European Monetary Union local market contains several sovereign governments issuing a common currency.

In each local market, the common factors are changes in interest rates and changes in spreads. The common factor local market risk of a bond is determined by the volatilities of the term structure and spread risk factors in that market, the correlations between factors, and the bond exposures to the risk factors.

Figure 12: Refined Risk Decomposition.
Factors can be prevalent only in a local market or throughout the global market. Total local market risk includes both common factor and specific risk.
Except for the euro block, each local market has three interest rate risk factors. These are the first three principal components of a key rate covariance matrix: shift, twist, and butterfly (STB). The euro market has three term structure factors for each country and three that describe average changes in rates across the euro zone. Every local market has a swap spread factor. The U.S. dollar, sterling, and euro markets have detailed credit spreads (to swap) as well. For a detailed discussion of these risk factors, see “Local Market Factors” on page 49.

Bonds in any local market may be exposed to global factors. Barra has two kinds of global risk factors: currency factors and emerging market spread factors. Both are quite volatile relative to local market risk factors and therefore constitute a significant component of portfolio risk. Barra has factors for 65 currencies and 26 emerging market issuers. For a detailed discussion of global risk factors, see “Global Factors” on page 67.

One of the most important parts of the Cosmos risk model is its factor covariance matrix (for a discussion of the mathematics, see “Calculating Risk” on page 22). This matrix contains the volatilities and correlations of the common factors. Factors fall naturally into subsets determined by the local market to which they belong or to the type of risk they describe (currency, for example). These subsets impose a block structure on the covariance matrix. Each diagonal block contains covariances of a factor subset. Off-diagonal blocks contain covariances between factors in distinct subsets Figure 13 on page 48 provides a schematic look at the block structure of the covariance matrix.

The time series of local market factor returns are used to generate factor variances and covariances in the Cosmos covariance matrix. For global factors, time series of returns are based on changes in exchange rates (for currency factors) and index spread levels (for emerging market factors). Details about time series for each common factor grouping are presented later in this chapter.

---

4 A covariance matrix literally contains variances and covariances. However, the information in a covariance matrix can be equally well represented as correlations and volatilities. These latter quantities are easier to interpret.
Figure 13: Graphic View of Common Factor Blocks in Covariance Matrix
Local Market Factors

Barra’s local market factors break down into subsets corresponding to individual local markets, which are distinguished by currency. Each subset contains interest rate and spread risk factors. In Cosmos, every bond is assigned to a local market according to its principal currency. The common factor piece of a bond’s local market risk is estimated from the volatilities of the factors in its local market block, the correlations among them, and the exposures of the bond to the factors.

Most Cosmos local markets contain four factors. Three of these model risk due to a change in interest rates (see “Term Structure Risk” on page 50). The fourth is a spread risk factor based on monthly changes of the spread between the swap and sovereign curves (see “Swap Spread Risk” on page 62). The only instruments not exposed to swap spread are sovereign issues denominated in the local currency.

In addition, Cosmos has detailed sector-by-rating spread factors in the United States, the United Kingdom, and euro zone (see “Credit Spread Risk” on page 64). All credit spreads are measured to the swap curve. Credit instruments denominated in the U.S. dollar, the British pound, or the euro may be exposed to one of these credit spreads.

The euro zone presents a more complicated picture due to the presence of more than one sovereign issuer. A single set of interest rate factors is not sufficient to capture the disparate credit qualities of the EMU sovereigns. Consequently, Barra provides interest rate factors for each sovereign issuer as well as factors for changes in average euro zone rates. Thus, euro-denominated sovereign bonds from EMU member countries will be exposed to the term structure factors of the country’s local market. All other euro-denominated bonds will be exposed to the general EMU term structure factors.

5 In some markets, such as Norway, there are an insufficient number of sovereign bonds to support three interest rate factors, and fewer factors are used.
**Term Structure Risk**

A term structure of interest rates is a curve that maps each maturity or term to the interest rate at which money can be borrowed for that maturity. Typically, it is more expensive to borrow for longer periods of time so term structures tend to slope upward. However, this is not always the case.

Since interest rates are neither bought nor sold, a term structure is a derived quantity. Each day, Barra estimates term structures for more than 25 sovereign issuers and swap curves in 16 local markets. In most cases, including all the sovereigns, these term structures are estimated using index bond prices from the previous market day’s close.

Since both rates and rate volatilities vary by term, a fixed-income risk model must take account of the term structure of interest rates.

In this section, we examine three models of term structure risk. The first has a single factor given by average changes in rates. This model is often called a *duration model* because the exposure of a portfolio to the single factor is effective duration. The second is a multiple-factor *key rate model* whose factors are changes in rates at key maturities. Finally, we look at a *shift-twist-butterfly model* that is based on the way term structures actually move. This model has fewer factors than a key rate model yet has virtually the same explanatory power. We compare these models below and conclude that the shift-twist-butterfly model is best.

---

6 Based on monthly changes in a yield curve estimated from the pool of actively traded EMU sovereign issues. Bonds are weighted by the GDP of the issuer. See Lisa Goldberg and Anton Honikman, “The Euro Yield Curve: Projecting the Future,” *Euro* (December 1998).

7 In the EMU local market, Barra uses the same swap curve for all member countries.

8 Barra calculates swap spot curves from swap par yields and LIBOR rates.
Historical Perspective: Duration Models

Experience shows that in developed markets, rates tend to move up and down roughly in parallel. This observation led early risk analysts to manage money with only a single factor: average change in interest rates. **Effective duration** is the exposure of a bond or portfolio to the average change in rates. In mathematical terms, effective duration is given as follows:

**Effective Duration**

\[
D_{\text{eff}} = -\frac{1}{P} \left( \frac{\partial P}{\partial r} \right)
\]

where

\[
D_{\text{eff}} = \text{effective duration} \quad \frac{\partial P}{\partial r} = \text{change in price with respect to change in level of rates}
\]

To a first approximation, portfolio return is the product of effective duration and the average change in rates,

\[
\frac{\Delta P}{P} \approx \Delta r D_{\text{eff}}
\]

where \(\Delta P/P\) is the bond’s return. It immediately follows that the bond return variance is approximately equal to the factor variance times the squared effective duration. In mathematical terms, this is expressed as

\[
\sigma^2 \left( \frac{\Delta P}{P} \right) \approx \sigma^2 (\Delta r) D_{\text{eff}}^2
\]

The disadvantage of the duration model is that it does not account for slope change and reshaping of the term structure. As we will see below (page 53), the use of a new set of factors will capture these term structure movements.
Key-Rate Model

We now move from the single-factor duration model to the opposite extreme: a key-rate model. Cosmos term structures are based on “key rates” at a distinguished set of maturities. A key-rate risk model is one whose defined factors are changes in key rates. At the core of the model is the covariance matrix of key rate changes. Since the full maturity dependence is represented, this model completely describes term structure variation.

Table 2 shows the key rate correlations and volatilities of the Australia market. A close investigation of the table immediately reveals the main shortcoming of a key rate model. Neighboring key rates are typically 90% correlated. Even long and short rates have roughly 60% correlation. This means that the key rate model has more factors than necessary.

Table 2: Monthly Correlation Matrix for the Australia Market (January 1988 to January 2001)

<table>
<thead>
<tr>
<th></th>
<th>1 yr</th>
<th>2 yrs</th>
<th>3 yrs</th>
<th>4 yrs</th>
<th>5 yrs</th>
<th>7 yrs</th>
<th>10 yrs</th>
<th>20 yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>1.00</td>
<td>0.91</td>
<td>0.86</td>
<td>0.83</td>
<td>0.81</td>
<td>0.72</td>
<td>0.63</td>
<td>0.56</td>
</tr>
<tr>
<td>2 yrs</td>
<td>1.00</td>
<td>0.97</td>
<td>0.95</td>
<td>0.93</td>
<td>0.84</td>
<td>0.75</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>3 yrs</td>
<td>1.00</td>
<td>0.99</td>
<td>0.98</td>
<td>0.92</td>
<td>0.84</td>
<td>0.77</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 yrs</td>
<td>1.00</td>
<td>0.99</td>
<td>0.94</td>
<td>0.88</td>
<td>0.81</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 yrs</td>
<td>1.00</td>
<td>0.97</td>
<td>0.91</td>
<td>0.84</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 yrs</td>
<td>1.00</td>
<td>0.97</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 yrs</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 yrs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

Source: Barra
Shift-Twist-Butterfly Model

The high degree of correlation among changes in key rates is just a quantitative restatement of the intuition underlying the duration model: rates move roughly in parallel. However, there is more to term structure risk than level change.

In 1991 Barra introduced a three-factor interest rate risk model. The factors reflect the way term structures actually move: shift, where rates change roughly in parallel; twist, where the short and long end of the curves move in opposite directions, steepening or flattening; and butterfly, where the curvature of the term structure changes or flexes. Together, these three principal components capture more than 95% of interest rate variation in most developed markets. The shift-twist-butterfly model (STB) remains the model of choice for interest rate forecasts.

How do we turn these intuitive term structure movements into risk factors? At Barra, we apply a principal components analysis\(^9\) to a key rate covariance matrix estimated from a history of term structure changes. This analysis amounts to a rotation of the covariance matrix to a diagonal matrix. The new factors underlying the rotated matrix are orthogonal, weighted combinations of changes in interest rates. The diagonal elements are the in-sample variances of the factors.

The characteristic shapes of shift, twist, and butterfly persist across markets and time periods. Figure 14 displays the shift, twist, and butterfly factors in the Germany market. Like most markets, the shift factor tends to be slightly humped at the short end because short rates tend to be more volatile than long rates.

---

\(^9\) A principal components analysis of a covariance matrix gives rise to a set of orthonormal factors and associated volatilities that can be used to reconstruct the original matrix. For more details see, for example, Richard A. Johnson and Dean W. Wichern, *Applied Multivariate Statistical Techniques*, 5th ed., Pearson Education, 2001.
As an example of how shift, twist, and butterfly account for term structure movement, consider the change in the U.S. term structure following a Federal Reserve System attempt to stimulate the U.S. economy by decreasing interest rates. This can be seen in a comparison of the spot rate curve at the time of the decrease and one month after.

Figure 14: Germany Shift, Twist, and Butterfly Shapes
The key information about the Fed's actions is conveyed by the downward movement of most of the spot rates. In other words, the underlying information about each spot rate movement is the same: rates decreased. This information is conveyed with a different magnitude, however, at each vertex.

The movement of the curve can clearly be decomposed into a large shift movement (the level of interest rates went down) and a large twist movement (the slope of the curve went up).

Like any model, the STB approach has shortcomings. The most common concern is that the STB model requires frequent re-estimation. The model depends on many choices, including the time period and weighting scheme used in the estimation of the underlying key rate matrix. How are these choices made? How often do the factors need to be updated?

There are no set answers to these questions. Fortunately, overwhelming empirical evidence shows that the factors are quite stable over time. Most reasonable methodologies for building shift, twist, and butterfly factors give rise to models that accurately forecast risk. Factors only need to be updated when the market structure changes.
In January 2001, we re-estimated the term-structure risk factors for all Cosmos markets. At that time we experimented with different weighting schemes and determined that the exponential weight had little effect on the factor shapes or on their explanatory power. Further, we discovered that in most markets, the estimation period had surprisingly little effect. Figure 16 shows the shift, twist, and butterfly factors for the U.K. market estimated in 1995 and in 2000. The shapes are very close indeed. The U.K. market is not unusual in this regard; similar results were obtained in most markets.

To take the point further, consider Table 3. It draws a comparison between the percentage of variance explained in sample by the new factors and the percentage of variance explained by the old factors out of sample over the period 1995–2000. In most cases the difference is negligible.
The large incremental change in the percentage of variance explained by the Spain factors merits attention. In particular, the introduction of long bonds in February 1998 changed the length of the market from 15 to 30 years. This supports the conclusion that when re-estimation is required it is because of a structural change in the market.

### Table 3: Percentage of Variance Explained by Shift-Twist-Butterfly Model Before and After Re-Estimation

<table>
<thead>
<tr>
<th>Market</th>
<th>New Factors (%)</th>
<th>Old Factors (%)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>98.9</td>
<td>98.5</td>
<td>0.4</td>
</tr>
<tr>
<td>EMU</td>
<td>98.0</td>
<td>96.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Japan</td>
<td>98.5</td>
<td>98.1</td>
<td>0.4</td>
</tr>
<tr>
<td>Spain</td>
<td>95.1</td>
<td>90.1</td>
<td>5.1</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>97.7</td>
<td>96.1</td>
<td>1.7</td>
</tr>
<tr>
<td>United States</td>
<td>98.8</td>
<td>98.0</td>
<td>0.7</td>
</tr>
</tbody>
</table>

The large incremental change in the percentage of variance explained by the Spain factors merits attention. In particular, the introduction of long bonds in February 1998 changed the length of the market from 15 to 30 years. This supports the conclusion that when re-estimation is required it is because of a structural change in the market.

### Shift-Twist-Butterfly Factor Returns

Each month, Barra calculates shift-twist-butterfly factor returns in each market by a cross-sectional regression of bond excess returns onto factor exposures. The regression universe is the set of bonds that are in an established market index at the start and end of the month. Barra uses the time series of STB returns along with returns to the other factors to generate the Cosmos covariance matrix.

Table 4 on page 58 shows volatilities for a sample 3x3 sub-block generated by shift, twist, and butterfly returns in the United Kingdom. Volatilities are reported in basis points per year. The annualized numbers are obtained from the monthly time series by multiplying the monthly estimate by $\sqrt{12}$. In a developed market, the typical shift volatility is 65–100 basis points per year (roughly 20–30 basis points per month).
Term Structure Factor Exposures and Normalization

By definition, the exposure of a bond or portfolio to a risk factor is the sensitivity of its return to changes in the factor level. For example, effective duration is the sensitivity of return to change in average level of rates.

As we have seen, the shift factor is roughly parallel, and for non-callable bonds, the exposure to shift will be a number comparable to effective duration. Principal component analysis determines the relative weights of the factors on each of the key maturities, but not the absolute size of the factors. After calculating factor shapes, Barra normalizes the base shift factors, as well as normalizes the twist, and butterfly factors to have the same magnitude as the shift.

Term structure factor exposures are computed by numerical differentiation. The term structure is shocked up and down by a small scalar multiple of the factor and the bond is revalued. The difference between “up” and “down” values is divided by twice the size of the shift. The mathematical formula is given as:
Many fixed-income investors are accustomed to using key rate durations (KRD) as hedging tools. Key rate durations measure the sensitivity of portfolio return to the change in the level of a particular interest rate. The relationship between the Cosmos term structure factor exposures and key rate durations is given in the next formula.

In particular, if the factor is the average change in rates, all the weights are equal to 1, and the exposure is effective duration. We thus obtain the well-known formula

$$D_{\text{eff}} = \sum D_i$$
Spread Risk

Historically, international bond managers focused largely on government bond issues. Recently, managers have increased the exposure of their global fixed-income portfolios to corporate and agency bonds, foreign sovereigns, supranationals, and credit derivatives. These bonds are subject to spread risk.10

Spread risk in bond portfolios arises for two reasons: market-wide spread risks and credit event risks. Market-wide spread risk arises from changes in the general spread level of a market segment. For example, the spreads of BBB-rated telecom bonds might widen. Credit event risk arises when an individual issuer suffers an event that affects it alone. It is the risk associated with changed in company fundamentals. For example, Ford Motor Company’s car sales might fall relative to other automobile makers due to a product recall and bad publicity.

In each market, a single factor accounts for changes in the difference between the swap and sovereign curves. For markets with detailed credit models (such as the United States, United Kingdom, Japan, and euro zone), Barra decomposes credit spread risk into two components that are modelled separately.

Over the period of January 1993 to September 2002, yield spreads on BBB bonds ranged from a low of 70 basis points to a high of 240 basis points. Figure 17 on page 61 shows a time series of monthly changes in BBB spreads over the U.S. treasury curve. The figure shows a spike in volatility beginning with the currency crisis in autumn of 1998 followed by a period of persistently high volatility. Risk of this type is modeled with spread factors, described in detail below.

10 The market perceives varying levels of creditworthiness among EMU sovereigns, giving rise to spreads between EMU sovereign issuers, so we estimate a term structure for each EMU sovereign.
A Layered Approach to Spreads

There is no universally accepted choice of benchmark against which to measure credit spread. In the United States, Japan, and United Kingdom, credit spreads have historically been measured against the local treasury curve, which has been interpreted as the curve of default-free rates. However, sovereign budget surpluses and other changes in monetary policy\(^{11}\) have corrupted treasury curves with risk premia. Further, there is no single sovereign curve that can be used to price treasuries across the euro zone. The result of these conditions is that many investors have begun to look to the swap curve as a benchmark against which to measure credit spreads.

\(^{11}\) For example, after October 2001 the U.S. government stopped issuing 30-year treasury bonds.
These considerations have led Barra to take a layered approach to modeling spread risk. In each Cosmos local market, a single factor models fluctuations between the swap and sovereign curves. Further, in the United States, United Kingdom, and euro zone, a block of sector-by-rating factors capture the risk of fluctuations in credit spreads over the swap curve. In these markets, spread risk is decomposed into swap spread risk and risk due to credit spreads over the swap curve. In markets where there are not enough data to support sector-by-rating models, we account for variable credit quality by scaling the bond exposures to swap spread risk (as described in the next section). The same scaling approach also applies to U.S. dollar-, British pound-, and euro-denominated securities that are subject to default risk but cannot be mapped to one of the credit spreads.12

**Swap Spread Risk**

In each Cosmos local market, swap spread risk is based on a single factor: the monthly change in the average spread between the swap and treasury curve. Figure 18 on page 63 shows a time series of monthly changes in the Australian dollar swap spread. This series and others of the same type are fed into the Cosmos covariance matrix estimation.

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12 For example, a rating may be missing or the issuing firm may be unrated.
For bonds that are exposed to a credit factor or an emerging market factor in addition to the swap spread factor, the swap spread exposure is equal to the sensitivity of a bond’s return to change in the swap spread level. For most bonds in this category, this exposure is equal to the effective duration.13

Bonds that do not have additional factors to explain their credit risk use the swap spread factor to forecast risk. Since bonds of lower credit quality tend to be more volatile, we scale their exposure to the swap spread by their (higher) option-adjusted spread (OAS), which is typically less than 10 basis points.

For such bonds, the sensitivity to a change in the swap spread level (typically spread duration) is scaled by the ratio of the bond’s OAS to the swap spread level so that bonds with higher OAS will have correspondingly higher volatility forecasts.

13 For floating rate notes and mortgage-backed securities, Barra computes the exposure to swap spread differently, as the spread duration is not equal to the effective duration.
Credit Spread Risk

The Cosmos local market risk model includes credit risk factors in three of the most active markets: U.S. dollar, sterling, and euro. Empirical evidence indicates that credit risk is currency-dependent. As a result, Barra's model is based on currency-specific credit risk factors.

In each market, most factors are determined by combinations of sector and rating. The one exception is the CCC factor in the U.S. block, which does not take account of sector. Table 5 on page 65 shows the factors by market for the U.K., EMU, and U.S. Also note that the Canadian sector in the U.S. block applies to a U.S. dollar-denominated bond issued either by the Canadian government or by an issuer domiciled in Canada.

Monthly credit spread factor return series are generated by taking the average change in spreads for bonds present in a particular sector-by-rating category at both the start and end of each period.14 When fewer than six bonds are available in a category, the return for that sector-by-rating bucket is generated by all bonds with the same rating, independent of sector.

---

14 The average is duration weighted. The resulting factor return is equal to the spread return to a portfolio consisting of all bonds in the estimation weighted equally by value.
Figure 19 on page 66 shows a comparison of volatility estimates for factors common to the U.S. dollar, sterling, and euro blocks. U.S. dollar volatilities are higher than euro volatilities, sometimes by a factor of three. The volatility of the sterling sectors consistently lies between the two extremes. This is part of the motivation to use currency-dependent factors.

Figure 19 also shows a comparison of term structure shift, swap, and several credit spread volatilities for the U.S. dollar, sterling, and euro markets.
The exposure of a credit instrument to the factor with matching currency, sector, and rating is spread duration. Mathematically, spread duration is given by the following formula:

Credit Spread Exposure (Spread Duration)

\[
D_{spr} = - \frac{1}{P_{durry}} \left( \frac{\partial P_{durry}}{\partial S_{spr}} \right)
\]

where

- \( P_{durry} \) = dirty price of bond
- \( S_{spr} \) = parallel spread shift

Figure 19: Cross-Market Volatility Comparison.
U.S. dollar factor return volatilities generally exceed euro and sterling by a factor of two to three. Estimates as of May 31, 2001, based on monthly data weighted exponentially with a 24-month half-life.
Global Factors

A bond may be exposed to a global risk factor regardless of which local market it belongs to. In Cosmos 3.0, global factors are used to model two types of risk: currency risk and risk due to changes in emerging market spreads.

Currency factors must be global since currency risk is a matter of perspective. Consider a Norway-based investor with a portfolio of U.S. treasuries. U.S. treasuries belong to the U.S. dollar local market and their interest rate risk comes from changes in U.S. treasury rates. However, our Norwegian investor is also subject to the risk that comes from changes in the U.S. dollar/Norwegian kroner exchange rate.

It may be somewhat surprising to find that emerging market spread risk is handled through global factors since corporate spread risk is part of local market risk. The reason for this is economic and can be seen at a glance in Figure 21 on page 73. Emerging market risk can be enormous compared to other sources of risk and is largely dictated by the level of distress of the emerging market. Hence, the currency in which a security is denominated will not have a material effect on the risk forecast.

Because currency and emerging market risk are time sensitive, variances and covariances of global factors are estimated separately using high-frequency data and models with short memories. In addition, Barra estimates correlations between global and local market factors. These correlations are derived from monthly data weighted exponentially with a 24-month half-life.

15 In a distressed market, the risk forecast can exceed 2000 basis points per year.

16 The Cosmos risk model forecasts risk for a monthly horizon using monthly changes in level as the basis for local market factor volatility and correlation forecasts. Covariances between factors estimated from data histories with different frequencies will necessarily be constrained by the coarser time resolution.
Barra’s risk model combines the local market covariance matrix and global risk blocks with the correlations between local market and global factors to form the common factor risk matrix.

**Currency Risk**

The fluctuation of currency exchange rates is a significant source of risk faced by international investors. Often more than half the variation of a well-diversified portfolio of global equities or investment-grade bonds can be attributed to currencies. Consequently, for an international investor making currency bets or trying to hedge currency risk entirely, it is essential to have an accurate, reliable risk forecasting model that includes currency risk.

To capture the dynamics of risk in currency markets, the Barra currency risk factor block is effectively a covariance matrix combining two models: a volatility model and a correlation model.

Currency volatility levels behave like many other economic time series, varying dramatically across currencies and over time. Changes in volatility range from gradual to abrupt. General auto-regressive conditional heteroskedasticity (GARCH) models are standard tools used to forecast risk for variable volatility time series. These models posit that, conditional on today’s information, the next period return is normal with variance that is a function of previous returns and forecasts.

The daily currency forecasts are then time-scaled to conform with other Barra risk forecasts, which are based on a monthly horizon.

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17 For related discussions, see “Currency Dependence in Global Credit Markets” and “Forecasting Currency Return Volatility,” articles by Barra Research available at www.barra.com.
Numeraires and Currency Factor Exposures

The currency matrix is estimated from a U.S. dollar perspective. However, Cosmos 3.0 can linearly transform the numeraire, or base currency, to be any of the 65 currencies covered in the model.

The exposure of a bond to a currency is 1 if the bond is denominated in that currency, otherwise it is 0. Bonds with principal in one currency and interest in another have their exposures split according to their present value weights. Derivatives have no currency of denomination but rather depend on instruments denominated in a particular currency. In this case the currency exposures are the derivative deltas.

Currency Correlations

The correlation matrix is estimated using weekly currency return data relative to the U.S. dollar, exponentially weighted with a half-life of 17 weeks. This half-life is fit with a maximum likelihood estimation.

Emerging Market Risk

Much of the debt issued by emerging market sovereigns and companies domiciled in emerging markets is denominated in U.S. dollar, sterling, euro, and Japanese yen. This debt is subject to risk due to interest rates in the market indicated by the principal currency. In addition, it is subject to risk due to the creditworthiness of the issuer. Since the credit risk tends to be much larger than the interest rate risk, the factors used to model this risk are currency independent. In other words, they are global factors.
As with all other Cosmos credit spreads, emerging market spreads are measured to the swap curve. Thus, an emerging market instrument denominated in a developed market currency is exposed to the interest rate and swap spread factors in the local market determined by its currency as well as the emerging market spread indicated by its issuer. As an example, consider a bond issued by the government of Nigeria in British sterling. The bond will be exposed to the U.K. shift, twist, butterfly, swap, and currency factors as well as the Nigeria emerging market spread.

**Emerging Market Spread Risk**

The emerging market factor block forecasts risk for bonds issued in an external currency by an emerging market sovereign or by a company domiciled in an emerging market country. The returns for risk factors are estimated from changes in stripped spreads of country subindices in the J.P. Morgan Emerging Market Bond Index (EMBI) Global\(^\text{18}\) composed of 26 factors, one for each market.

Several markets, including Poland, South Africa, and Korea, appear in Cosmos both as developed local markets and as emerging markets. As an example, a bond will be mapped to the Korea local market factor block if it is issued by the Korean government (or a company domiciled in Korea) in Korean won. If the same bond is issued in a currency other than the Korean won, it will be mapped to the local market determined by the currency of the issue and to the Korea emerging market spread.

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\(^{18}\) The J.P. Morgan EMBI Global is an index of dollar-denominated government debt from 26 markets. The index includes both collateralized restructured (Brady) debt and conventional noncollateralized bonds. For more information on J.P. Morgan’s spread estimation methodology, see *Introducing the J.P. Morgan Emerging Markets Bond Index Global* (August 3, 1999), available from J.P. Morgan.
Spread Volatility and Risk Estimation

As in the case of currency factors, emerging market spreads can be quite volatile and tend to exhibit variable volatility over time. Barra estimates the emerging market block separately from other factors and relies on high-frequency data. Variance and covariance estimates are based on weekly data, weighted exponentially with an eight-week half-life. The emerging market block is time-scaled to a one-month horizon and subsequently integrated into the Cosmos risk model.

Figure 20 on page 72 shows a time series of average volatility forecasts across the various market regions. We see that realized volatility was fairly stable from June 1999 to December 2000 in Africa and Asia, declined by roughly a factor of two in Latin America (ex Ecuador), and declined by a factor of four in Eastern Europe and Russia. The eight-week half-life is considerably shorter than the time scale of volatility decrease in these markets, so the risk forecasts have only slightly lagged the changes in the markets.

Figure 21 on page 73 shows volatility in emerging markets on two dates one year apart. During this period, the Ecuadorian sucre was dollarized and macroeconomic conditions improved. The resulting large decrease in volatility is obvious. In contrast, we can see the increased volatility in Argentina, following continuing political and social pressures.

Figure 20  Annualized Volatility Forecasts (Averaged) for Regions of the EMBI Global. The volatility of the swap spread is shown for comparison.
Emerging Market Factor Exposures

An emerging market bond issued in an external currency is exposed to the spread indicated by the issuer. The exposure is the bond spread duration.\(^\text{20}\)

**Credit Spread Exposure (Spread Duration)**

\[
D_{spr} = -\frac{1}{P_{\text{dirty}}} \left( \frac{\partial P_{\text{dirty}}}{\partial S_{\text{spr}}} \right)
\]

where

- \( P_{\text{dirty}} \) = dirty price of bond
- \( S_{\text{spr}} \) = parallel spread shift

\(^{20}\) Spread duration is also the exposure of a bond to credit factors.
Specific Risk

Overview

Common factors do not completely explain asset return. Return not explained by the shift-twist-butterfly and spread factors is called specific return or asset-specific return. The risk due to the uncertainty of the specific return is called asset-specific risk. This risk tends to be quite small for government bonds and investment-grade corporates. By assumption, bond-specific returns are uncorrelated with one another as well as with common factor return. Consequently, portfolio specific risk diversifies away quickly as the number of bonds in the portfolio increases. Our general model for asset-specific risk is described below in this section.

Included under the heading of specific risk is corporate bond event risk. This is the risk that a company's debt may be repriced due to a change in perception of the company's business fundamentals. Credit rating agencies monitor these business fundamentals in relation to a firm's debt payment obligations, and assign debt ratings based on their estimate of the likelihood of repayment. The details of how this model is put together are explained the next section.

Modeling Event Risk

The Cosmos event risk model is based on rating migration and covers corporate issues denominated in U.S. dollar, sterling, and euro. The ingredients to the calculation are the historical rating migration matrix and average yield spreads estimated for bonds bucketed by currency and rating.
Rating Migration Matrix

Credit rating agencies update debt ratings as a firm’s condition changes. That is, credit events can trigger rating migration. For example, concerns about the debt level of the American company, Gap Inc.,\(^{21}\) caused Moody’s to lower their credit rating three times during a nine-month period in 2001 and 2002.\(^{22}\)

Credit rating information is consolidated in an annualized rating migration matrix. The matrix contains historical estimates of the likelihood of debt rating migrations over the horizon of one year. Before we include the matrix in our model, we first remove any firms that transition to “non-rated” and rebalance so that once again each column sums to 100%. Next, we eliminate mathematical inconsistencies present in the observed data.

Rating Spreads and Returns

In order to forecast the risk associated with rating migration, we need to estimate the size of the impact that a rating change has on spread. We do this by calculating the spread levels for each rating category.

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22 Changes in business fundamentals are not always accompanied by rating migration. For example, in March 2002, a money manager in a large investment company questioned the high level of short-term debt held by General Electric, a highly rated company in the United States. Spreads of GE Capital’s debt increased 8 to 15 basis points, but their credit rating remained strong. Investigating when an event will affect debt rating is an area of further Barra research. [David Feldheim, “GECC’S Credit Standing Seen Undented, Despite Criticism,” Morningstar–Dow Jones, March 22, 2002, news.morningstar.com (March 26, 2002).]
On each analysis date, bonds are grouped by rating and currency. The average spread over swap of each group is calculated. Figure 22 shows a time series of rating spread levels for U.S. dollar-denominated bonds. Note that the average spread for a bond below investment grade dwarfs the investment-grade spreads. Figure 23 on page 77 magnifies the investment-grade spreads to show more clearly their levels and relationships.
Table 6 shows a sample of U.S. dollar rating spread levels. As expected, spreads increase as credit quality diminishes.

Table 6: U.S. Dollar Rating Spread Levels, March 31, 2001 (in basis points)

<table>
<thead>
<tr>
<th>Rating</th>
<th>Spread Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>23.4</td>
</tr>
<tr>
<td>AA</td>
<td>45.02</td>
</tr>
<tr>
<td>A</td>
<td>79.43</td>
</tr>
<tr>
<td>BBB</td>
<td>146.09</td>
</tr>
<tr>
<td>BB</td>
<td>313.52</td>
</tr>
<tr>
<td>B</td>
<td>687.69</td>
</tr>
<tr>
<td>CCC</td>
<td>1919.43</td>
</tr>
</tbody>
</table>
For each initial and final rating pair, the spread change is the difference between the spread of the final level and the spread of the initial level. Note that the spread changes are zero when the initial and final ratings agree and they are negative if the initial rating is higher than the final rating. The returns generated by the spreads in Table 6 are displayed graphically in Figure 24.

23 The common factor returns we have encountered earlier in this chapter were changes to a factor level over the course of a month. By contrast, the specific returns we consider here are differences in rating spread levels on a particular date.

Figure 24: U.S. Dollar Spread Returns, March 31, 2001

Issuer Migration Risk Forecasting Model

We now explain how to combine the rating migration probabilities and the level spread returns (as seen in Figure 24) to generate issuer migration risk forecasts. For any bond, this forecast is the standard deviation of price return due to change in spread level.

---

23 The common factor returns we have encountered earlier in this chapter were changes to a factor level over the course of a month. By contrast, the specific returns we consider here are differences in rating spread levels on a particular date.
In order to calculate this, we first note the current rating of the bond and compute the price return to the bond that arises from each possible rating migration. The formula that converts spread return to price return is given in the equation below:

**Converting Spread Returns to Price Returns**

\[ r_{price} \approx r_{spr} D_{spr} \]

where

- \( r_{price} \) = price return variance due to spread change
- \( r_{spr} \) = spread return
- \( D_{spr} \) = spread duration

As an example, consider a five-year duration AA bond. On March 31, 2001, the AA spread level was 45 basis points and the BBB spread level was 146 basis points. Hence the price return for this bond due to change in level to BBB is

\[(45 - 146) \cdot 5 = -505 \text{ basis points}\]

The case when the end state is default requires special explanation. Here, the price return is simply the recovery rate. The recovery rate is uncertain and situation dependent. However, studies have shown that a typical recovery rate for secured senior debt is roughly 50%. Hence, we use this value in our model.\(^{24}\) In fact, the recovery value has only a small impact on the risk forecast for investment-grade bonds.\(^{25}\)

---

\(^{24}\) Barra is currently undertaking a study of recovery given default.

\(^{25}\) However, the recovery model may have a pronounced impact on bonds that are below investment grade.
Once we have calculated the price return for each possible end state, we compute the weighted standard deviation using the probabilities from the monthly market data. The formula for the rating migration risk forecast for a bond with initial rating $i$ and spread duration $D_{spr}$ is

**Forecasting Rating Migration Risk**

\[
\sigma^2 = \sum_j w_{ij} \left( D_{spr} \left( r_{spr,i} - r_{spr,j} \right) - r_{mean} \right)^2 + w_{i,def} \left( R - r_{mean} \right)^2
\]

where

- $\sigma^2$ = price return variance due to spread change or default
- $w_{ij}$ = probability that rating $i$ bond will migrate to rating $j$ over next month
- $D_{spr}$ = spread duration
- $r_{spr,i} - r_{spr,j}$ = difference between average spread for rating $i$ and rating $j$ (spread return)
- $r_{mean}$ = mean price return due to spread change or default
- $w_{i,def}$ = probability that rating $i$ bond will default over next month
- $R$ = recovery rate in default
Figure 25 below and Figure 26 on page 82 display time series of credit risk forecasts for the U.S. dollar spreads. Credit risk can be more conveniently expressed in terms of spread risk by dividing out by the spread duration. Sharp jumps in risk forecasts for all rating classes are apparent in the credit crash of mid-1998, due to the significant widening of spreads. However, forecasts have subsequently fallen back to their pre-July 1998 levels.

Table 7 on page 83 summarizes the model forecasts as of January 31, 2000, expressed as annualized standard deviation of interest rates or spreads. For the rating-based credit risk forecasts, a spread duration of five years is assumed. The first column shows average factor volatility forecasts for factors in different groups. The second column shows the specific or credit risk forecast for a single security.
An interesting observation from this table is that the classification of issuers into investment and speculative grade (BBB and above, and below BBB) neatly corresponds to the split between bonds whose common factor and credit risk are each less than their interest rate risk, and those for which they are greater. That is, the common factor and credit risks for ratings of BBB and above are all less than the 82 basis points of risk due to spot rate volatility, while those of lower-grade bonds are above this level. We can also see that interest rate risk is dominant for investment-grade bonds and credit risk is dominant for high-yield bonds.

Figure 26: Credit Risk Forecasts for a U.S. Dollar-Denominated Five-Year Duration Bond, by Rating, Based on the Transition Matrix (BB to CCC).
Proxies for Missing Rating Spreads

The calculation of rating migration risk for a bond requires estimates for all the rating spread levels. However, in some markets there are not enough data to estimate spreads below investment grade. In these cases we extrapolate from investment-grade spreads using rating spreads from a market for which data exists (notably the U.S.). We have shown empirically that spreads between the two markets for a below-investment-grade rating are proportional to those for an investment-grade rating.

Table 7: Common Factor and Specific Risk Forecasts for a U.S. Dollar-Denominated Bond as of January 31, 2000

<table>
<thead>
<tr>
<th>Type</th>
<th>Common Factor (basis points)</th>
<th>Specific/Credit (basis points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treasury/Spot</td>
<td>82</td>
<td>8</td>
</tr>
<tr>
<td>Agency</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>AA</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>A</td>
<td>22</td>
<td>33</td>
</tr>
<tr>
<td>BBB</td>
<td>34</td>
<td>79</td>
</tr>
<tr>
<td>BB</td>
<td>92</td>
<td>155</td>
</tr>
<tr>
<td>B</td>
<td>164</td>
<td>287</td>
</tr>
<tr>
<td>CCC</td>
<td>296</td>
<td>582</td>
</tr>
</tbody>
</table>
Modeling Issue-Specific Risk

Underlying the model for specific risk of domestic government bonds is the assumption that the risk of spread return is constant. In units of price return, this gives

*Monthly Specific Risk of Domestic Government Bonds*

\[
\sigma = D_{spr} \cdot b
\]

where

- \(\sigma\) = monthly specific risk for a domestic government bond
- \(D_{spr}\) = spread duration
- \(b\) = constant risk of spread return for a government bond

The constant \(b\) is calibrated to each market. Figure 27 on page 85 shows specific risk forecasts for five-year duration government domestic issues. The forecasts are made for a monthly horizon and reported in annualized terms.
For defaultable bonds in markets not covered by the credit migration model, Barra uses an extension of the issue-specific risk model.

Figure 27: Monthly Specific Risk Forecasts for Five-year Duration Government Bonds
Chapter 4

Optimization

Barra’s optimizer module in Cosmos Global Risk Manager and Aegis Portfolio Manager offers a powerful portfolio management tool that helps investment managers quickly construct optimal portfolios. It does this by automatically trading off return, risk, and transaction costs based on client-specified parameters. You can impose realistic policy or regulatory constraints and specify your parameters on an optimization, while accounting for transaction costs, short positions, penalties, and hedges to create portfolios that reflect your investment goals.
Optimizer Features

An optimization using Cosmos Global Risk Manager or Aegis Portfolio Manager can be useful regardless of your investment strategy. For example, you can

- Construct a portfolio to track an index, benchmark, or liability, maximizing active return for a given level of tracking error.
- Optimally weight your universe by incorporating estimates of relative attractiveness.
- Specify constraints and penalties on local market, currency, interest rate, sector, and credit rating exposures.
- Implement tilts towards factors that you expect to do well by assigning expected returns to industries, risk indices, and specific assets.
- Control holding size, turnover, number of assets, and transaction costs.
- Implement full or optimal currency hedging.
- Use scenario-constrained optimization to generate an “efficient frontier” or portfolios with different risk-return tradeoffs.
- Rebalance an existing portfolio to optimally match your risk and return profile.
How Optimization Works

Optimization creates an optimal portfolio by trading assets found in user-specified initial and universe portfolios. The objective is to maximize utility while taking into account any specified constraints.

The optimizer feature in Barra Aegis and Cosmos products calculates the optimization objective function as

\[
\text{objective function} = \text{utility} - \text{transaction costs} - \text{penalties}
\]

where utility = return – (risk aversion \times portfolio variance)

By maximizing the optimization objective function, you are trying to balance several competing objectives:

- Maximize expected returns.
- Minimize total risk or risk relative to a benchmark.
- Minimize transaction costs.
- Minimize penalties.

It is sometimes convenient to look at the objective function calculation as a tradeoff between expected net return (expected returns minus transaction costs minus penalties) and risk. The set of portfolios with maximum expected net return is known as the efficient frontier. The risk aversion parameter, which you adjust in the optimizer, characterizes your willingness to accept additional risk to achieve a higher level of expected net return. Therefore, each point on the frontier corresponds to a different risk aversion level, \( \lambda \).

Portfolio managers have different goals, therefore optimizations in Cosmos Global Risk Manager or Aegis Portfolio Manager allow managers to customize the parameters of the optimization to best meet their needs. One manager might have a target risk level for the portfolio and would like to find the corresponding maximum-return portfolio on the frontier. Another might have a target expected return level and the need to find the minimum
risk portfolio providing that target return. Another manager might choose an optimal portfolio by balancing the risk (as defined by the risk aversion parameter) and expected return components of the portfolio. If expected returns, transaction costs, or penalties are not included, the optimizer tries to minimize risk.

In addition to the goal of maximizing the objective function, the optimizer must meet any indicated constraints, such as the upper or lower bounds on the weights of assets, cash contributions or withdrawals, or transaction-type limitations.

Mean-Variance Optimization

To create an optimal portfolio, portfolio sponsors strive for high expected returns \( (r_p) \), and low expected risk \( (\sigma^2) \). Typically risk and return increase together, so the goal is to achieve the best compromise.

Mean-Variance optimization provides the best compromise by choosing the portfolio to maximize the utility, defined as

\[
U = r_p - \lambda \sigma^2
\]  
Eq. 14

where

\[
\lambda = \text{risk aversion parameter}
\]
When managing against a benchmark portfolio with excess return \( r_B \), the benchmark can be removed from the managed portfolio’s returns

\[
\theta \equiv r_p - \frac{\text{cov}[r_p, r_B]}{\text{cov}[r_B, r_B]} r_B \equiv r_p - \beta_B r_B
\]

to arrive at the residual return.

The residual return’s expected value is called alpha,

\[
\alpha \equiv \widetilde{\theta}
\]

and its variance is the residual risk,

\[
\omega^2 \equiv \text{cov}[\theta, \theta]
\]

The value-added utility,

\[
U_A \equiv \alpha - \lambda \omega^2
\]

is suitable for optimizing performance against a benchmark portfolio.
Two common performance metrics, Sharpe ratio (SR) and the information ratio (IR),\(^1\) are maximized by the optimization:

\[
SR \equiv \frac{r_p}{\sigma}
\]

\[
IR \equiv \frac{\alpha}{\omega}
\]

**Constraints**

Cosmos Global Risk Manager and Aegis Portfolio Manager enable clients to set the following optimization constraint types:

- upper and lower bounds for asset holdings
- transaction cost constraints
- trading constraints
- those listed in Table 8 on page 93 (Cosmos) and Table 9 on page 94 (Aegis).

---

### Table 8: Cosmos Optimizer Constraints

<table>
<thead>
<tr>
<th>Constraint Type and Constraints</th>
<th>Global</th>
<th>Local</th>
<th>Currency</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aggregate Characteristics:</strong> OA Duration, Mod OA Duration, Nominal Duration, Convexity, OA Yield, OA Spread, Average Coupon, Current Yield, Vertex Weight, Vertex Contribution to Duration, Average Maturity</td>
<td>✔️</td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td><strong>Barra Risk Measures:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beta</td>
<td>✔️</td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>Shift, Twist, Butterfly, and Swap Exposures and Weight-Exposures</td>
<td>✔️</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit Rating Exposures and Weight-Exposures</td>
<td>✔️</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Local Market:</strong> % Value or % Effective Value</td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td><strong>Market Category:</strong> Domestic, Eurobond, Foreign, Derivatives, Cash</td>
<td>✔️</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Sector:</strong> Government, Financial, Industrial, Utility, Supranational, Sovereign, Local Provincial, Agency, Cash, Pfandbriefe, Jumbo Pfandbriefe</td>
<td>✔️</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Quality Moody:</strong> Moody credit rating buckets Aaa to NR</td>
<td>✔️</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Quality S&amp;P:</strong> S&amp;P credit rating buckets AAA to NR</td>
<td>✔️</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Quality Model:</strong> Model credit rating buckets AAA to NR</td>
<td>✔️</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Asset Class:</strong> Bond (implicit exposure) and Cash Instruments (explicit exposure)</td>
<td>✔️</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Country of Issue:</strong> % Weight for each country and for Multinational; Barra Emerging Markets (Total)</td>
<td>✔️</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Currency:</strong> Total Currency, Implicit, Explicit</td>
<td>✔️</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constraint Type</td>
<td>Specific Constraints</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------------</td>
<td>----------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Beta</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industries</td>
<td>For a list of industries used in the USE3L model, see Appendix B, “USE3L Sector and Industry Definitions”.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk Indices</td>
<td>Currency sensitivity, earnings variation, earnings yield, growth, leverage, momentum, non-estimated universe, size, size non-linearity, trading activity, value, volatility, yield</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sectors</td>
<td>Basic materials, consumer (cyclical), consumer (non-cyclical), consumer services, commercial services, energy, financial, health care, industrials, technology, telecommunications, transport, utility</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Additionally, user data and formulas can be used as Aegis constraints.
Appendix A

USE3L Descriptors

This Appendix provides detailed definitions of the important descriptors that underlie the risk indices in USE3L. The method of combining these descriptors into risk indices is proprietary to Barra.
Volatility

Beta Times Sigma

Beta times sigma is computed as follows:

\[ \sqrt{\beta \sigma_e} \]

where \( \beta \) is the historical beta, and \( \sigma_e \) is the historical residual standard deviation. If \( \beta \) is negative, then the descriptor is set equal to zero.

Daily Standard Deviation

The daily standard deviation is computed as follows:

\[ \sqrt{N_{days} \left[ \sum_{t=1}^{T} w_t r_t^2 \right]} \]

where \( r_t \) is the return over day \( t \), \( w_t \) is the weight for day \( t \), \( T \) is the number of days of historical returns data used to compute this descriptor (we set this to 65 days), and \( N_{days} \) is the number of trading days in a month (set to 23).
Ratio of High Price to Low Price Over the Last Month

The ratio of high price to low price over the last month is calculated as follows:

$$\log\left(\frac{P_H}{P_L}\right)$$

where $P_H$ and $P_L$ are the maximum price and minimum price attained over the previous month.

Log of Stock Price

The log of the stock price at the end of the previous month.

Cumulative Range

Let $Z_t$ be defined as follows:

$$Z_t = \sum_{i=1}^{t} \log(1 + r_{i,s}) - \sum_{i=1}^{t} \log(1 + r_{f,s})$$

where $r_{i,s}$ is the return on stock $i$ in month $s$, and $r_{f,s}$ is the risk-free rate for month $s$. In other words, $Z_t$ is the cumulative return of the stock over the risk-free rate at the end of month $t$. 
If we define $Z_{\text{max}}$ and $Z_{\text{min}}$ as the maximum and minimum values of $Z_t$ over the last 12 months, cumulative range is computed as follows:

$$
\log \left( \frac{1 + Z_{\text{max}}}{1 + Z_{\text{min}}} \right)
$$

**Sensitivity of Changes in Trading Volume to Changes in Aggregate Trading Volume**

The sensitivity of changes in trading value to changes in aggregate trading value may be estimated by the following regression:

$$
\frac{\Delta V_{i,t}}{N_{i,t}} = a + b \frac{\Delta V_{M,t}}{N_{M,t}} + \xi_{i,t}
$$

where $V_{i,t}$ is the change in share volume of stock $i$ from week $t-1$ to week $t$; $N_{i,t}$ is the average number of shares outstanding for stock $i$ at the beginning of week $t-1$ and week $t$; $V_{M,t}$ is the change in volume on the aggregate market from week $t-1$ to week $t$; and $N_{M,t}$ is the average number of shares outstanding for the aggregate market at the beginning of week $t-1$ and week $t$. 
Serial Dependence

Captures serial dependence in residuals from the market model regressions. It is computed as follows:

\[
SERDP = \frac{1}{T-2} \sum_{t=3}^{T} (e_t + e_{t-1} + e_{t-2})^2
\]

\[
- \frac{1}{T-2} \sum_{t=3}^{T} (e^2_t + e^2_{t-1} + e^2_{t-2})
\]

where \( e_t \) is the residual from the market model regression in month \( t \); and \( T \) is the number of months over which this regression is run (typically, \( T = 60 \) months).

Option-Implied Standard Deviation

The implied standard deviation from the Black-Scholes option-pricing formula, using the price on the closest to at-the-money call option that trades on the underlying stock.
Momentum

Relative Strength

The cumulative excess return (using continuously compounded monthly returns) over the last 12 months:

\[ RSTR = \sum_{t=1}^{T} \log(1 + r_{i,t}) - \sum_{t=1}^{T} \log(1 + r_{f,t}) \]

where \( r_{i,t} \) is the arithmetic return of the stock in month \( i \), and \( r_{f,t} \) is the arithmetic risk-free rate for month \( i \). This measure is usually computed over the last one year (that is, \( T \) is set to 12 months).

Historical Alpha

The alpha term (that is, the intercept term) from a 60-month regression of the stock's excess returns on the S&P 500 excess returns.

Size

Log of Market Capitalization

The log of the market capitalization of equity (price times number of shares outstanding) for the company.
Size Nonlinearity

Cube of the Log of Market Capitalization

The cube of the normalized log of market capitalization.

Trading Activity

Share Turnover Over the Last Year

The annualized share turnover rate, using data from the last 12 months, expressed as follows:

\[
\frac{V_{ann}}{\bar{N}_{out}}
\]

where \( V_{ann} \) is the total trading volume (in number of shares) over the last 12 months, and \( \bar{N}_{out} \) is the average number of shares outstanding over the previous 12 months—that is, it is equal to the average value of the number of shares outstanding at the beginning of each month over the previous 12 months.
Share Turnover Over the Last Quarter

The annualized share turnover rate using data from the most recent quarter, and can be computed as follows:

\[
\frac{4V_q}{N_{out}}
\]

where:

\(V_q\) is the total trading volume (in number of shares) over the most recent quarter.

\(\overline{N}_{out}\) is the average number of shares outstanding over the period—that is, \(\overline{N}_{out}\) is equal to the average value of the number of shares outstanding at the beginning of each month over the previous three months.

Share Turnover Over the Last Month

The share turnover rate, using data from the most recent month. In other words, it is equal to the number of shares traded in the previous month, divided by the number of shares outstanding at the beginning of the month.
Share Turnover Over the Last Five Years

The annualized share turnover rate, using data from the last 60 months. It can be expressed as follows:

$$STO5 = \frac{12\left[\frac{1}{T}\sum_{s=1}^{T} V_s\right]}{N_{out}}$$

where $V_s$ is equal to the total trading volume in month $s$ and $N_{out}$ is the average number of shares outstanding over the last 60 months.

Indicator For Forward Split

A 0-1 indicator variable used to capture the occurrence of forward splits in the company's stock over the last two years.

Volume To Variance

Volume to variance is calculated as follows:

$$VLVR = \log\left(\frac{12\sum_{s=1}^{T} V_s P_s}{\sigma^2}\right)$$

where $V_s$ equals the number of shares traded in month $s$, $P_s$ is the closing price of the stock at the end of month $s$, and $\sigma^2$ is the estimated residual standard deviation. The sum in the numerator is computed over the last 12 months.
Growth

Payout Ratio Over Five Years

Payout ratio over five years is computed as follows:

\[
PAYO = \frac{1}{T} \sum_{t=1}^{T} D_t \frac{1}{1/T} \sum_{t=1}^{T} E_t
\]

where \(D_t\) is the aggregate dividend paid out in year \(t\), and \(E_t\) is the total earnings available for common shareholders in year \(t\). This descriptor is computed using the last five years of data on dividends and earnings.

Variability In Capital Structure

Variability in capital structure is measured as follows:

\[
V\text{CAP} = \frac{1}{T-1} \sum_{t=2}^{T} \left( |N_{t-1} - N_t|P_{t-1} + |LD_{t-1} - LD_t| + |PE_{t-1} - PE_t| \right) \frac{CE_t + LD_t + PE_t}{CE_t + LD_t + PE_t}
\]

where \(N_{t-1}\) is the number of shares outstanding at the end of time \(t-1\); \(P_{t-1}\) is the price per share at the end of time \(t-1\); \(LD_{t-1}\) is the book value of long-term debt at the end of time period \(t-1\); \(PE_{t-1}\) is the book value of preferred equity at the end of time period \(t-1\); and \(CE_t + LD_t + PE_t\) are the book values of common equity, long-term debt, and preferred equity as of the most recent fiscal year.
Growth Rate In Total Assets

Growth rate in total assets is computed first by running the following regression:

\[ TA_{it} = a + bt + \zeta_{it} \]

where \( TA_{it} \) is the total assets of the company as of the end of year \( t \), and the regression is run for the period \( t = 1, \ldots, 5 \). It is then computed as follows:

\[
AGRO = \frac{b}{1 \sum_{i=1}^{T} TA_{it}}
\]

where the denominator average is computed over all the data used in the regression.

Earnings Growth Rate Over Last Five Years

Earnings growth rate over the last five years is computed first by running the following regression:

\[ EPS_{i} = a + bt + \zeta_{t} \]
where $EPS_t$ is the earnings per share for year $t$. This regression is run for the period $t=1,...,5$. It is then computed as follows:

$$EGRO = \frac{b}{1 \sum_{t=1}^{T} EPS_t}$$

**Analyst-Predicted Earnings Growth**

Analyst-predicted earnings growth is computed as follows:

$$EGIBS = \frac{(EARN - EPS)}{(EARN + EPS)/2}$$

where $EARN$ is a weighted average of the median earnings predictions by analysts for the current year and next year, and $EPS$ is the sum of the four most recent quarterly earnings per share.

**Recent Earnings Change**

A measure of recent earnings growth, as follows:

$$DELE = \frac{(EPS_t - EPS_{t-1})}{(EPS_t + EPS_{t-1})/2}$$

where $EPS_t$ is the earnings per share for the most recent year, and $EPS_{t-1}$ is the earnings per share for the previous year. We set this to missing if the denominator is non-positive.
Earnings Yield

Analyst-Predicted Earnings-To-Price

The weighted average of analysts’ median predicted earnings for the current fiscal year and next fiscal year, divided by the most recent price.

Trailing Annual Earnings-To-Price

The sum of the four most recent quarterly earnings per share divided by the most recent price.

Historical Earnings-To-Price

Historical earnings-to-price is computed as follows:

\[
ETP5 = \frac{\frac{1}{T} \sum_{t=1}^{T} EPS_t}{\frac{1}{T} \sum_{t=1}^{T} P_t}
\]

where \(EPS_t\) is equal to the earnings per share over year \(t\), and \(P_t\) is equal to the closing price per share at the end of year \(t\).
Value

Book-To-Price Ratio

Book value of common equity as of the most recent fiscal year end divided by the most recent value of the market capitalization of the equity.

Earnings Variability

Variability In Earnings

Variability in earnings is computed as follows:

\[
VERN = \left( \frac{1}{T-1} \sum_{t=1}^{T} (E_t - \bar{E})^2 \right)^{1/2} \left( \frac{1}{T} \sum_{t=1}^{T} E_t \right)
\]

where \( E_t \) is the earnings at time \( t (t=1, \ldots, 5) \) and \( \bar{E} \) is the average earnings over the last five years. \( VERN \) is the coefficient of variation of earnings.
Standard Deviation of Analysts’ Prediction To Price

The weighted average of the standard deviation of IBES analysts’ forecasts of the firm’s earnings per share for the current fiscal year and next fiscal year, divided by the most recent price.

Variability In Cash Flows

The coefficient of variation of cash flow using data over the last five years—computed in an identical manner to VERN—with cash flow used in place of earnings. Cash flow is computed as earnings, plus depreciation, plus deferred taxes.

Extraordinary Items In Earnings

Extraordinary items in earnings is computed as follows:

\[ EXTE = \frac{1}{T} \sum_{t=1}^{T} \left| EX_t + NRI_t \right| - \frac{1}{T} \sum_{t=1}^{T} E_t \]

where \( EX_t \) is the value of extraordinary items and discontinued operations, \( NRI_t \) is the value of non-operating income, and \( E_t \) is the earnings available to common before extraordinary items. The descriptor uses data over the last five years.
Leverage

Market Leverage

Market leverage is computed as follows:

\[ MLEV = \frac{ME_t + PE_t + LD_t}{ME_t} \]

where \( ME_t \) is the market value of common equity, \( PE_t \) is the book value of preferred equity, and \( LD_t \) is the book value of long-term debt. The value of preferred equity and long-term debt are as of the end of the most recent fiscal year. The market value of equity is computed using the most recent month’s closing price of the stock.

Book Leverage

Book leverage is computed as follows:

\[ BLEV = \frac{CEQ_t + PE_t + LD_t}{CEQ_t} \]

where \( CEQ_t \) is the book value of common equity, \( PE_t \) is the book value of preferred equity, and \( LD_t \) is the book value of long-term debt. All values are as of the end of the most recent fiscal year.
**Debt-To-Assets Ratio**

Debt-to-assets ratio is computed as follows:

\[
DTOA = \frac{LD_t + DCL_t}{TA_t}
\]

where \( LD_t \) is the book value of long-term debt, \( DCL_t \) is the value of debt in current liabilities, and \( TA_t \) is the book value of total assets. All values are as of the end of the most recent fiscal year.

**Senior Debt Rating**

A multi-level indicator variable of the debt rating of a company.

**Currency Sensitivity**

**Exposure To Foreign Currencies**

To construct the exposure to foreign currencies, the following regression is run:

\[
\begin{align*}
r_{it} &= \alpha_i + \beta_i r_{mt} + \epsilon_{it}
\end{align*}
\]
where \( r_{it} \) is the excess return on the stock, \( r_{mkt} \) is the excess return on the S&P 500, \( \mu_t \) is the residual returns from this regression. These residual returns are in turn regressed against the contemporaneous and lagged returns on a basket of foreign currencies, as follows:

\[
epsilon_{it} = c_i + \gamma_{i1}(FX) + \gamma_{i2}(FX)_{t-1} + \gamma_{i3}(FX)_{t-2} + \epsilon_{it}
\]

where \( \epsilon_{it} \) is the residual return on stock \( i \), \( (FX) \) is the return on an index of foreign currencies over month \( t \), \( (FX)_{t-1} \) is the return on the same index of foreign currencies over month \( t-1 \), and \( (FX)_{t-2} \) is the return on the same index over month \( t-2 \).

The risk index is then computed as the sum of the slope coefficients, \( i1, i2, i3 \). Therefore, the exposure to foreign currencies = \( i1 + i2 + i3 \).

**Dividend Yield**

**Predicted Dividend Yield**

The last four quarterly dividends paid out by the company, along with the returns on the company’s stock and future dividend announcements made by the company are used to come up with a Barra-predicted dividend yield.
Non-Estimation Universe Indicator

Indicator For Firms Outside USE3L Estimation Universe

A 0-1 indicator variable equal to 0 if the company is in the USE3L estimation universe or equal to 1 if the company is outside the USE3L estimation universe.
Appendix B

USE3L Sector and Industry Definitions

This appendix lists the definitions and examples of companies for the USE3L industry definitions.
<table>
<thead>
<tr>
<th>Sector</th>
<th>USE3L Industry Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic Materials</td>
<td><strong>Chemicals</strong>&lt;br&gt;Chemicals, paints, plastics, plastic containers, coatings, gases, adhesives, inks, fibers, fertilizers; does not include household chemicals and plastics&lt;br&gt;&lt;br&gt;<em>Example:</em> Dow Chemical</td>
</tr>
<tr>
<td>Forestry &amp; Paper</td>
<td><strong>Lumber, wood, paper, newsprint, paper and cardboard containers, paper machine clothing (e.g., conveyor belts)</strong>&lt;br&gt;&lt;br&gt;<em>Example:</em> Boise Cascade Corp.</td>
</tr>
<tr>
<td>Gold</td>
<td><strong>Gold and silver</strong>&lt;br&gt;&lt;br&gt;<em>Example:</em> Homestake Mining</td>
</tr>
<tr>
<td>Mining &amp; Metals</td>
<td><strong>Aluminum, coal, iron and steel, stainless steel, steel castings, copper, beryllium, nickel, titanium, uranium; metal and glass containers; metal recycling; does not include aluminum foil for households</strong>&lt;br&gt;&lt;br&gt;<em>Examples:</em> ALCOA, Inc., Bethlehem Steel</td>
</tr>
<tr>
<td>Commercial Services</td>
<td><strong>Industrial Services</strong>&lt;br&gt;Janitorial and housekeeping services, plumbing, pest control, employment agencies, office temps, uniform rental, truck and auto fleet management, relocation services, employee and student education, training, and testing; security services, industrial plant management, car and truck leasing; auto repair&lt;br&gt;&lt;br&gt;<em>Example:</em> Manpower Inc.</td>
</tr>
<tr>
<td></td>
<td><strong>Information Services</strong>&lt;br&gt;Account keeping, payroll processing, news feeds, user-searchable databases, research-based financial information, translation services, computer centers, market research, management consulting, advertising, electronic publishing; travel agents&lt;br&gt;&lt;br&gt;<em>Examples:</em> Automatic Data Processing, Dun &amp; Bradstreet</td>
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<tr>
<td>Sector</td>
<td>USE3L Industry Definition</td>
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<td>-----------------------------</td>
<td>-------------------------------------------------------------------------------------------</td>
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<tr>
<td><strong>Consumer Cyclicals</strong></td>
<td>Apparel &amp; Textiles&lt;br&gt;Manufacturing of apparel, textiles, shoes, textile home&lt;br&gt;</td>
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<tr>
<td></td>
<td>furnishings (towels, sheets, etc.), processing of fibers for&lt;br&gt;textiles, wholesale, but&lt;br&gt;</td>
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<td></td>
<td>not retail, sale of apparel&lt;br&gt;&lt;br&gt;Example: Liz Claiborne, Nike&lt;br&gt;&lt;br&gt;Clothing Stores&lt;br&gt;</td>
</tr>
<tr>
<td></td>
<td>Specialty apparel retailers; does not include fabric stores&lt;br&gt;&lt;br&gt;Example: Gap&lt;br&gt;&lt;br&gt;</td>
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<tr>
<td></td>
<td>Construction &amp; Real Property&lt;br&gt;Building materials, residential lighting and fixtures, home&lt;br&gt;</td>
</tr>
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<td></td>
<td>builders, building managers, equity REITs; wholesale&lt;br&gt;construction materials; engineering&lt;br&gt;</td>
</tr>
<tr>
<td></td>
<td>and construction firms&lt;br&gt;&lt;br&gt;Example: Kaufman &amp; Broad&lt;br&gt;&lt;br&gt;Consumer Durables&lt;br&gt;Home&lt;br&gt;</td>
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<td>furniture, office furniture, appliances, lawn mowers, snow&lt;br&gt;blowers, televisions, floor&lt;br&gt;</td>
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<td>coverings, non-textile home furnishings, luggage, cutlery, china; consumer durable&lt;br&gt;</td>
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<td>wholesaling&lt;br&gt;&lt;br&gt;Examples: Maytag, Sunbeam, Black &amp; Decker&lt;br&gt;&lt;br&gt;Department Stores&lt;br&gt;Retailers&lt;br&gt;</td>
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<tr>
<td></td>
<td>who sell widely diverse products, department stores.&lt;br&gt;&lt;br&gt;Example: Dayton Hudson, Wal-Mart&lt;br&gt;</td>
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<td>Motor Vehicles &amp; Parts&lt;br&gt;Car and auto part manufacturing; car batteries; not auto&lt;br&gt;</td>
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<td>parts retailing; truck manufacturing (excluding heavy&lt;br&gt;equipment); motor homes&lt;br&gt;&lt;br&gt;Example:&lt;br&gt;</td>
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<td></td>
<td>Ford, Navistar International Corp.&lt;br&gt;&lt;br&gt;Specialty Retail&lt;br&gt;Sells one type of&lt;br&gt;item or&lt;br&gt;</td>
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<td></td>
<td>uses one concept (e.g., items under $10); not apparel retailers; includes retail auto supply&lt;br&gt;</td>
</tr>
<tr>
<td></td>
<td>stores, consumer electronics stores, fabric stores, catalog marketing, telemarketing, auctioneers&lt;br&gt;</td>
</tr>
</tbody>
</table>
|                             | Example: Home Depot, Circuit City
<table>
<thead>
<tr>
<th>Sector</th>
<th>USE3L Industry Definition</th>
</tr>
</thead>
</table>
| **Consumer Noncyclicals** | Alcohol  
Beer, wine, and spirits; alcohol wholesaling  
*Example: Anheuser-Busch*  
**Food & Beverages**  
Food companies (processed food), food distribution and wholesaling, raw agriculture products, beverage companies, soda bottling, flower-growing, veterinary services; does not include fertilizer  
*Example: Kellogg's, Tyson Foods, Coca-Cola*  
**Grocery Stores**  
*Example: Safeway*  
**Home Products**  
Soaps, housewares, cosmetics, personal care, skin care, beauty care, dental care, household chemicals, household plastics  
*Example: Procter & Gamble*  
**Tobacco**  
Cigars, cigarettes, pipe tobacco, leaf tobacco dealers, chewing tobacco, tobacco distribution  
*Example: Philip Morris* |
| **Consumer Services**   | **Entertainment**  
Movies, TV programming, theaters, theme parks, cruises  
*Example: Disney*  
**Hotels**  
Hotels and motels; gaming  
*Example: Hilton Hotels*  
**Leisure**  
Golf clubs, boats, toys, photography, entertainment systems, bicycles, novelties, camps, recreational vehicle parks, jewelry; gaming equipment; motorcycles  
*Example: Eastman Kodak*  
**Media**  
Radio and TV stations, cable TV stations; satellite communications; not TV programming  
*Example: CBS* |
<table>
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<tr>
<th>Sector</th>
<th>USE3L Industry Definition</th>
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</table>
| Consumer Services (cont.)  | Publishing  
Newspaper and magazine publishers, book publishers, greeting cards; commercial printing  
*Example:* Dow Jones, New York Times  
Restaurants  
*Example:* McDonald's  |
| Energy                     | Energy Reserves  
Oil and gas exploration, reserves, and production  
*Example:* Exxon Mobil  
Oil Refining  
Oil refining and marketing; gas pipelines and distribution  
*Example:* Quaker State  
Oil Services  
Oil drilling, oil platform operating, oil platform support services, drill bits, drilling tools  
*Examples:* Parker Drilling, Schlumberger  |
| Financial                  | Banks  
Local and regional banks; money center banks  
*Examples:* BayBank, J. P. Morgan  
Equity REITs  
Property-backed REITs  
Example: Liberry Property Trust  
Financial Services  
Mortgage brokers, consumer loans, car loans, student loans, home loans, credit cards, tax preparation, loan agencies of the U.S. government; also credit card processing, check guarantee, loan guarantee and collection, wire transfer, mortgage REITs; credit unions, pawnshops, patents; insurance services and brokers  
*Example:* FNMA  
Life and Health Insurance  
Includes life insurance, annuities, health insurance, disability insurance; not HMOs  
*Example:* Equitable, UNUM |
## Sector | USE3L Industry Definition
--- | ---
Financial (Cont.) | **Property and Casualty Insurance**<br>Property insurance, casualty insurance, municipal bond insurance, title insurance, liability insurance, workers' compensation insurance; not HMOs<br>*Example:* Allstate
**Securities & Asset Management**<br>Brokers, mutual fund companies, asset management companies<br>*Example:* Merrill Lynch
**Thrifts**<br>*Example:* Washington Mutual, Inc.
Health Care | **Biotechnology**<br>Drug production via DNA technology, monoclonal antibodies.<br>*Examples:* Genentech, Amgen
**Drugs**<br>Drug production via traditional chemical processes<br>*Examples:* Merck, Eli Lilly
**Medical Products & Supplies**<br>High-tech and low-tech medical products; medical technology<br>*Examples:* Advanced Technology, Kendall
**Medical Services**<br>Hospitals, nursing homes, surgical centers, HMOs, rehabilitation providers, laboratories, hospital pharmacy management, apartments with available nursing, cemetery and funeral homes, medical equipment rental<br>*Example:* Humana
Industrials | **Environmental Services**<br>Waste management, hazardous materials disposal, cogeneration and independent power; environmental consulting<br>*Example:* Browning-Ferris Industries (BFI)
<table>
<thead>
<tr>
<th>Sector</th>
<th>USE3L Industry Definition</th>
<th>Example</th>
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</table>
| Industrials (Cont.)   | **Heavy Electrical Equipment**  
Electrical equipment, electric power generation equipment, cables, wire, insulation, connectors; not electronics, batteries, elevators  
*Example*: GE, Westinghouse  

**Heavy Machinery**  
Tractors, cranes, fire trucks, sweeper machines  
*Example*: Caterpillar  

**Industrial Parts**  
Industrial manufacture of bearings, gears, pumps, equipment, batteries; elevators; machine tools; hand tools; manufacturing of machinery, engines  
*Example*: Applied Industrial Technologies, Inc. |
| Technology            | **Computer Hardware & Business Machines**  
Computers, disk and tape drives, network equipment, copiers, office machines, bar code scanners, credit card verification equipment, automatic toll collectors, and automatic teller machines  
*Example*: IBM  

**Computer Software**  
*Example*: Microsoft  

**Defense & Aerospace**  
Manufacturing for defense, including aircraft, tanks, submarines, and supplies. Civil aeronautics, space exploration; aircraft parts  
*Examples*: Lockheed, Boeing, General Dynamics  

**Electronic Equipment**  
Analytical instruments, instrumentation, test and measurement electronics, high precision motors, electronic power supplies, electronic controls, circuit board manufacturing, antennas, electrical switches, circuit protectors, sensors, thermostats, security systems, membrane separation technology; equipment for digital switching, telecommunications, broadcasting, voice and data networking, voice mail and electronic mail, and telephones; optical fiber manufacturing, paging equipment  
*Example*: Tektronix, Alcatel |
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<tr>
<th>Sector</th>
<th>USE3L Industry Definition</th>
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<tr>
<td>Technology</td>
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<td>(Cont.)</td>
<td>Internet</td>
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<tr>
<td></td>
<td>Internet Services, Internet Providers &amp; Content, Internet Software</td>
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<td></td>
<td>Examples: Amazon, Yahoo!</td>
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<td>Semiconductors</td>
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<td></td>
<td>Manufacture of semiconductors, chips, equipment for semiconductor manufacturing</td>
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<td>Example: Intel</td>
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<td>Telecommunications</td>
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<td>Telephones</td>
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<td></td>
<td>Long-distance and local phone companies</td>
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<td>Examples: AT&amp;T, SBC</td>
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<td>Wireless Telecommunications</td>
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<td></td>
<td>Cellular phones and pagers</td>
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<td>Examples: GTE Contel, Nextel Communications</td>
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<tr>
<td>Transport</td>
<td>Airlines</td>
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<tr>
<td></td>
<td>Airlines and airport ground services (excludes air freight)</td>
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<td></td>
<td>Example: Delta</td>
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<td></td>
<td>Railroads</td>
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<td></td>
<td>Railroads and rolling stock leasing</td>
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<td></td>
<td>Example: Burlington Northern</td>
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<td></td>
<td>Trucking, Sea, &amp; Air Freight</td>
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<td></td>
<td>Examples: CNF Transportation, Inc., Alexander &amp; Baldwin</td>
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<tr>
<td>Utilities</td>
<td>Electric Utilities</td>
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<td></td>
<td>Example: Pacific Gas &amp; Electric</td>
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<td></td>
<td>Gas &amp; Water Utilities</td>
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<td></td>
<td>Example: Brooklyn Union Gas</td>
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</tbody>
</table>
Appendix C

Equity Asset Selection
Risk Modeling

This appendix presents several formulae relating to asset selection risk in Barra’s equity multiple-factor models.

- Modeling the Average Level of Asset Selection Risk
- Modeling the Relative Level of Asset Selection Risk
- Estimating the Scaling Coefficients
- Final Asset Selection Risk Forecast
Modeling the Average Level of Asset Selection Risk

Barra models the average level of asset selection risk via a time series model, where the average asset selection risk is related to its own lagged values, as well as to lagged market excess returns:

$$S_t = \alpha + \sum_{i=1,k} \beta_i S_{t-i} + \beta_{k+1} r_{m,t-1} + \epsilon_t$$

where

$S_t$ = average asset selection risk at time $t$

$\beta_i$ = sensitivity of average asset selection risk to lagged values

$S_{t-i}$ = lagged average asset selection risk

$\beta_{k+1}$ = sensitivity of average asset selection risk to market returns

$r_{m,t-1}$ = market return at time $t-1$

$\epsilon_t$ = residual level of asset selection risk

This simple model captures mean reversion or trends and persistence in volatility, as well as lower average asset selection risk following market rises, and higher asset selection risk following market declines (provided that the sensitivity of average asset selection risk to market returns is negative).

We evaluate the performance of alternative weighting schemes by regressing realized average asset selection risk against predicted asset selection risk out of sample:
The results of these tests help us determine the appropriate coefficients in Equation 1 on page 124.

Modeling the Relative Level of Asset Selection Risk

To model the relative level of asset selection risk, we first identify factors that may account for the variation in asset selection risk among assets. These factors will vary depending on the country model and may include:

- Industry membership.
- Risk index exposures.

Having identified these factors, we then estimate the relationship by performing the “pooled” cross-sectional regression:

\[ V_{it} = \sum_{k=1,K} X_{ik} \gamma_k + \epsilon_{it} \]  

Eq. 3

where

- \( V_{it} \) = realized relative asset selection risk for asset \( i \) at time \( t \)
- \( t \) = 1 to \( T \) months
- \( i \) = 1 to \( N \) assets
- \( X_{ikt} \) = exposure of asset \( i \) to factor \( k \) at time \( t \)
In estimating this relationship, we mitigate the effects of outliers on the estimators by Winsorizing descriptors and risk indices.

### Estimating the Scaling Coefficients

Average asset selection risk can vary widely over the full capitalization range of an equity market. To correct for this effect, scaling coefficients are used to adjust for any asset selection risk bias. To estimate the scaling coefficients, we divide the set of securities into capitalization deciles, and for each decile, we compute the bias in predicted asset selection risk from the previous steps. The scaling coefficients are then computed as a piece-wise linear function of an asset’s relative capitalization in a manner that makes the average bias in predicted asset selection risk zero for each capitalization decile.

\[
V_{it} = \sum_{k=1, K} X_{ikt} \gamma_k + \varepsilon_{it} \quad \text{Eq. 3}
\]

where

\[
\gamma_k = \text{factor } k's \text{ contribution to relative asset selection risk}
\]
Final Asset Selection Risk Forecast

The above three components—average level of asset selection risk, relative level of asset selection risk, and decile scaling coefficients—are combined to produce the final asset selection risk forecast:

\[ Std(\varepsilon_{it}) = scale_{it} \cdot \hat{S}_t \cdot \left(1 + \hat{V}_t\right) \]  

Eq. 4

where

\[ \varepsilon_{it} = \text{asset selection risk of asset } i \text{ at time } t. \]

\[ scale_{it} = \text{scale coefficient for the decile that asset } i \text{ falls into at time } t. \]

\[ \hat{S}_t = \text{average level of asset selection risk across all assets.} \]

\[ \hat{V}_t = \text{relative level of asset selection risk for asset } i \text{ at time } t. \]
Bibliography

Investment Theory and Statistics


Multiple Factor Models


Risk Analysis


Portfolio Construction


Performance Attribution


U.S. Equity Model


Global Equity Model


Europe Equity Model


Fixed-Income Securities


Returns Forecasting and Information Analysis


**The Value of Long-Short Investing**


This glossary provides definitions for some of the common terms used in this handbook. For more terms, refer to the glossaries located in the online help for the individual Barra applications.

A

Active management
The pursuit of investment returns in excess of a specified benchmark.

Active return
Return relative to a benchmark. If a portfolio’s return is 5%, and the benchmark’s return is 3%, then the portfolio’s active return is 2%.

Active risk
The risk (annualized standard deviation) of the active return. Also called tracking error.

Alpha
The expected residual return. Beyond the pages of this book, alpha is sometimes defined as the expected exceptional return and sometimes as the realized residual or exceptional return.

Asset Selection risk
The risk (annualized standard deviation) of the specific return. Also known as Specific Risk.

B

Benchmark
A reference portfolio for active management. The goal of the active manager is to exceed the benchmark return.

Beta
The sensitivity of a portfolio (or asset) to a benchmark. For every 1% return to the benchmark, we expect a -1% return to the portfolio. Historical beta measures the response of a company’s return to the market return, ordinarily computed as the slope coefficient in a 60-month historical regression. Predicted beta (also called fundamental beta) is the predicted common-factor risk coefficient (predictive of subsequent response to market return) that is derived, in whole or in part, from the fundamental operating characteristics of a company.

C

Coefficient of determination (R^2)
See R-squared.

Common factor
An element of return that influences many assets. According to multiple-factor risk models, the common factors determine correlations between asset returns. Equity
model common factors include industries and risk indices (or styles). Fixed-income model common factors include currencies and emerging market spreads (global factors) and term structure shifts, twists, and butterflies, swap spreads, and credit spreads (local market factors).

**Constraint**
In portfolio optimization, a limitation imposed upon the portfolio so that it will have desired characteristics.

**Correlation**
A statistical term giving the strength of linear relationship between two random variables. It is a pure number, ranging from -1 to +1: +1 indicates a perfect positive linear relationship; -1 indicates a perfect negative linear relationship; 0 indicates no linear relationship. For jointly distributed random variables, correlation is often used as a measure of strength of relationship, but it fails when a nonlinear relationship is present.

**Covariance**
The tendency of different random investment returns to have similar outcomes, or to “co-vary.” When two uncertain outcomes are positively related, covariance is positive, and conversely, negatively related outcomes have negative covariances. The magnitude of covariance measures the strength of the common movement. For the special case of a return’s covariance with itself, the simplified name of variance is used. Covariance can be scaled to obtain the pure number, correlation, that measures the closeness of the relationship without its magnitude.

**D**

**Descriptor**
A variable describing assets, used as an element of a risk index. For example, a volatility risk index, distinguishing high volatility assets from low volatility assets, could consist of several descriptors based on short-term volatility, long-term volatility, and residual volatility, etc.

**Distribution**
The function which describes the frequency with which a random variable takes on any given value. A normal distribution is the familiar bell-shaped curve which that occurs when large numbers of independent factors are added together. It is a symmetrical distribution, with approximately two-thirds of all outcomes falling within ± 1 standard deviation and approximately 95 percent of all outcomes falling within ± 2 standard deviations.

**Diversification**
A reduction of risk obtained by investing positive wealth in assets that are not perfectly positively correlated. Since the assets are not perfectly positively correlated, losses of any one asset tend to be offset by gains on other assets.

**Dividend yield**
The dividend per share divided by the price per share. Also known as the yield.

**E**

**Earnings yield**
The earnings per share divided by the price per share.

**Efficient frontier**
A set of portfolios, one for each level of expected return, with minimum risk. We
sometimes distinguish different efficient frontiers based on additional constraints, e.g., the fully invested efficient frontier.

**Exceptional return**
Residual return plus benchmark timing return. For a given asset with beta equal to 1, if its residual return is 2%, and the benchmark portfolio exceeds its consensus expected returns by 1%, then the asset’s exceptional return is 3%.

**Excess return**
Return relative to the risk-free return. If an asset’s return is 3% and the risk-free return is 0.5%, then the asset’s excess return is 2.5%.

**Factor portfolio**
The minimum risk portfolio with unit exposure to the factor and zero exposures to all other factors. The excess return to the factor portfolio is the factor return.

**Factor return**
The return attributable to a particular common factor. We decompose asset returns into a common factor component, based on the asset’s exposures to common factors times the factor returns, and a specific return.

**Information coefficient**
The correlation of forecast return with their subsequent realizations. A measure of skill.

**Information ratio**
The ratio of annualized expected residual return to residual risk. A central measurement for active management, value added is proportional to the square of the information ratio.

**Market**
The portfolio of all assets. We typically replace this abstract construct with a more concrete benchmark portfolio.

**Multiple-factor model (MFM)**
A specification for the return process for securities. This model states that the rate of return on any security is equal to the weighted sum of the rates of return on a set of common factors, plus the specific return on the security, where the weights measure the exposures (or sensitivity) of the security to the factor. These exposures are identified with microeconomic characteristics, or descriptors of the firms (see descriptor).

Several simplifications of this model have been used historically. If there is only one factor, it becomes a single-factor model; if this one factor is identified with an index, it is called a single-index model; if the single-factor is identified with the market factor, it becomes the market model. Depending on the statistical specification, some of these could become a diagonal model, which simply indicates that the covariance matrix between security returns is (or can easily be transformed into) a diagonal matrix.

**Normal portfolio**
A benchmark portfolio.

**Normalization**
The process of transforming a random variable into another form with more desirable properties. One example is standardization in which a constant (usually the mean) is subtracted from each number to shift all numbers uniformly. Then each number is
divided by another constant (usually the standard deviation) to shift the variance.

**Optimization**
The best solution among all the solutions available for consideration. Constraints on the investment problem limit the region of solutions that are considered, and the objective function for the problem, by capturing the investor’s goals correctly, providing a criterion for comparing solutions to find the better ones. The optimal solution is that solution among those admissible for consideration which has the highest value of the objective function. The first-order conditions for optimality express the tradeoffs between alternative portfolio characteristics to provide the optimum solution.

**Outlier**
A data observation that is very different from other observations. It is often the result of an extremely rare event or a data error.

**Passive management**
Managing a portfolio to match (not exceed) the return of a benchmark.

**Payout ratio**
The ratio of dividends to earnings. The fraction of earnings paid out as dividends.

**Performance analysis**
Evaluation of performance in relation to a standard or benchmark with the purpose of assessing manager skill.

**Performance attribution**
The process of attributing portfolio returns to causes. Among the causes are the normal position for the portfolio, as established by the owner of funds or the manager, as well as various active strategies, including market timing, common factor exposure, and asset selection. Performance attribution serves an ancillary function to the prediction of future performance, in as much as it decomposes past performance into separate components that can be analyzed and compared with the claims of the manager.

**R-squared**
Coefficient of determination. A statistic usually associated with regression analysis, where it describes the fraction of observed variation in data captured by the model. It varies between 0 and 1.

**Regression**
A data analysis technique that optimally fits a model based on the squared differences between data points and model fitted points. Typically, regression chooses model coefficients to minimize the (possibly weighted) sum of these squared differences.

**Residual return**
A data analysis technique that optimally fits a model based on the squared differences between data points and model fitted points. Typically, regression chooses model coefficients to minimize the (possibly weighted) sum of these squared differences.

**Residual risk**
The risk (annualized standard deviation) of the residual return.

**Risk**
The uncertainty of investment outcomes. Technically, risk defines all uncertainty about the mean outcome, including both upside and downside possibilities. The more intuitive concept for risk measurement is the standard deviation of the distribution, a natural
measure of spread. Variance, the square of the standard deviation, is used to compare independent elements of risk.

**Risk-free return**  
The return achievable with absolute certainty. In the U.S. market, short maturity Treasury bills exhibit effectively risk-free returns. The risk-free return is sometimes called the time premium, as distinct from the risk premium.

**Risk index**  
A common factor typically defined by some continuous measure, as opposed to a common industry membership factor defined as 0 or 1. Risk index factors include Volatility, Momentum, Size, and Value.

**Risk premium**  
The expected excess return to the benchmark.

**S**  
**Sharpe ratio**  
The ratio of annualized excess returns to total risk.

**Significance, statistical**  
A statistical term which measures the spread or variability of a probability distribution. The standard deviation is the square root of variance. Its intuitive meaning is best seen in a simple, symmetrical distribution, such as the normal distribution, where approximately 2/3 of all outcomes fall within ± 1 standard deviation of the mean, 95% of all outcomes fall within ± 2 standard deviations, and 99% of all outcomes fall within ± 2.5 standard deviations. The standard deviation of return—or, more properly, of the logarithm of return, which is approximately symmetrically distributed—is very widely used as a measure of risk for portfolio investments.

**Specific return**  
The part of the excess return not explained by common factors. The specific return is independent of (uncorrelated with) the common factors and the specific returns to other assets. It is also called the idiosyncratic return.

**Specific risk**  
The risk (annualized standard deviation) of the specific return. Also known as *Asset Selection Risk*.

**Standardization**  
Standardization involves setting the zero point and scale of measurement for a variable. An example might be taken from temperature, where the centigrade scale is standardized by setting zero at the freezing point of water and establishing the scale (the centigrade degree) so that there are 100 units between the freezing point of water and the boiling point of water. Standardization for risk indices and descriptors in Barra equity models sets the zero value at the capitalization-weighted mean of the companies in the universe and sets the unit scale equal to one cross-sectional standard deviation of that variable among the estimation universe.

**Systematic risk**  
The standard deviation of systematic return, which is the portfolio return due to the same forces that influence the return on the benchmark. Systematic risk can be thought of as the portfolio risk that can’t be diversified away.

**T**  
**Tracking error**  
*See Active risk.*
Transaction costs
Costs incurred for a portfolio when securities are changed for other securities. Transaction costs are deducted from the value of the portfolio directly, rather than paid as fees to the money manager. These costs arise from three sources: (1) commissions and taxes paid directly in cash; (2) the typical “dealer's spread” (or one-half of this amount) earned by a dealer, if any, who acts as an intermediary between buyer and seller; and (3) the net advantage or disadvantage earned by giving or receiving accommodation to the person on the other side of the trade. The third component averages out to zero across all trades, but it may be positive or negative, depending on the extent to which a trader, acting urgently, moves the market against the selected strategy.

U
Universe
The list of all assets eligible for consideration for inclusion in a portfolio. At any time, some assets in the universe may be temporarily ruled out because they are currently viewed as overvalued. However, the universe should contain all securities that might be considered for inclusion in the near term if their prices move to such an extent that they become undervalued. Universe also defines the normal position of a money manager, equating the normal holding with the capitalization-weighted average of the securities in the universe or followed list.

Utility
A measure of the overall desirability or goodness of a person’s situation. In the theory of finance, utility is the desirability of a risky series of outcomes. The utility (or expected utility) of a set of risky outcomes is assumed to measure its goodness, so that a package with higher utility is always preferred to one with lower utility. In portfolio theory, utility is almost always defined by a function of the mean and variance of portfolio outcomes, which is then called a mean/variance utility function. The further assumption that the utility function is linear in its two arguments (mean and variance) results in a linear mean/variance utility function (LMVU).

V
Value added
The utility, or risk-adjusted return, generated by an investment strategy: the return minus a risk aversion constant times the variance. The value added depends on the performance of the manager and the preferences of the owner of the funds.

Variance
A statistical term for the variability of a random variable about its mean. The variance is defined as the expected squared deviation of the random variable from its mean—that is, the average squared distance between the mean value and the actually observed value of the random variable. When a portfolio includes several independent elements of risk, the variance of the total arises as a summation of the variances of the separate components.

Volatility
A loosely-defined term for risk. Here we define volatility as the annualized standard deviation of return.

Y
Yield
See Dividend yield.
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   - ___ Strongly agree
   - ___ Agree
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