Measuring Yield

Joel R. Barber
Notation

- CR is Coupon Rate
- \( c \) is annual coupon
- \( c/2 \) is semiannual coupon
- \( y \) is nominal interest rate (annual rate with same compounding frequency as coupons)
- \( i \) is periodic rate (rate between payment dates)
- \( N \) is number of payments
- \( M \) is maturity value, par value, face value, ...
- \( P \) is price
- \( CY \) is current yield = \( c/P \)
- \( FV \) is the future value
- \( r \) is reinvestment rate
Sources of return

- coupon payments
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- recovery of principal
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  - coupon bond: par value
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Chapter 3
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- capital gains – if sold prior to maturity
- reinvestment of cash flows
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2. \( Y = 8\% \)
3. Current Yield = \( \frac{8}{100} = 8\% \)
4. one year later yield jumps to 9%
5. no built in gain or loss
6. Future price:

\[
P_1 = 4 \cdot \frac{1}{.045} \left( 1 - (1.045)^{-58} \right) + \frac{100}{(1.045)^{58}}
\]

\[
= 89.754
\]
neglect interest on first coupon

\[
\text{Return} = \frac{89.754 + 8}{100} - 1
\]
\[
= -2.25
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capital loss

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\frac{89.754}{100} - 1 \\
= -10.25
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return:

\[
\frac{89.754 + 4(1.02) + 4}{100} - 1
\]

\[
= -2.16\%
\]
Yield measures

1. **coupon rate**
2. **current yield**: annual coupon divided by price
3. **yield to maturity**
   1. IRR on bond. Annual:

\[ P = \frac{c}{1+y} + \frac{c}{(1+y)^2} + \cdots + \frac{M + c}{(1+y)^N} \]

2. Semiannual: \( P = \frac{c/2}{1+y/2} + \frac{c/2}{(1+y/2)^2} + \cdots + \frac{M+c/2}{(1+y/2)^N} \)
Perpetuity - the bond that keeps on paying.

\[ y = \frac{c}{P} \]
Easy special cases

1. Perpetuity - the bond that keeps on paying.

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2. Zero-coupon - the bond that pays only once.

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Easy special cases

1. **Perpetuity** - the bond that keeps on paying.
   
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2. **Zero-coupon** - the bond that pays only once.
   
   \[ y = \left( \frac{M}{P} \right)^{1/N} - 1 \]

3. **Par-bond** - redeemed for selling price. All yield comes from coupon.
   
   \[ y = CR \]
Relationship between bond price and yield

1. par
2. discount
3. premium
4. example: three bonds: CR = 4%, 6%, 8%, priced at 6% yield

![Graph showing relationship between bond price and yield]

- The graph illustrates the relationship between bond price and yield over time (half years).
- At par bonds, the price is exactly the face value (100 in the graph).
- Discount bonds have a price below par (below the 100 line).
- Premium bonds have a price above par (above the 100 line).

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Chapter 3
Yield to maturity and true return

two are equal only if cash flows are reinvested at yield to maturity.
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3. BEY (Bond Equivalent Yield) - annual rate semiannually compounded
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   - semiannual
     
     \[
     (1 + y/2)^2 - 1
     \]
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2. return > (<<) yield if coupons reinvested at rate > (<<) yield
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4. EAR or annual-pay basis
   1. semiannual
      \[(1 + y/2)^2 - 1\]
   2. monthly
      \[(1 + y/12)^2 - 1\]
Yield to first call

1. Assume bond is called at first call date

2. Reduce maturity to first call date

3. Substitute call price for par value

4. Compute yield

As a practical matter you should use $\min(y, YTC)$ if the call price is greater than the present value of the remaining cash flows, then $YTC > y$. 

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   \[\min\{y, YTC\}\]
6. If the call price is greater than the present value of the remaining cash flows, then
   \[YTC > y\]
Yield to first call

Example

- 8% callable bond with maturity 20 years
- price = $110
- first call date 5 years
- call price 100

1. yield to maturity = 7.06%
Yield to first call

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1. yield to maturity = 7.06%
2. yield to call = 5.67%

What if call price = 110

PV of remaining cash flows = 108.61

YTC = Yield to maturity
Yield to first call

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5. should use yield to maturity of 7.06%
6. PV of remaining cash flows = 108.61
7. at call price equal to 108.61, YTC = Yield to maturity
1. project cash flow based upon assumed prepayment speed
2. prepayment speed is fraction of total par value prepaid in a year
3. yield = IRR of projected cash flow stream
Yield spread (or discount margin) for floating rate security

1. project cash flow under assumption future reference rate equals current rate

\[ \text{Yield spread} = \text{IRR} - \text{reference rate} \]
Yield spread (or discount margin) for floating rate security

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Yield spread (or discount margin) for floating rate security

1. Project cash flow under assumption future reference rate equals current rate
2. Compute IRR
3. Yield spread = IRR - reference rate
Yield spread (or discount margin) for floating rate security

Example

- floating rate security
- price 99.3098 per 100 par value
- \( CR = \text{LIBOR} + 80 \text{ bps} \)
- reset every six months
- current reference rate = 10%

\[ CR_0 = 10 + .8 = 10.8\% \]
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3. \( C_{12} = 105.4 \)
4. IRR:

\[
99.3098 = 5.4A(y/2, 12) + \frac{100}{(1 + y/2)^{12}}
\]

solution is: \( y = 10.960 \)
Yield spread (or discount margin) for floating rate security

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solution is: \( \{y = 10.960\} \)

5. discount margin = 10.960 – 10 = .96\% = 96 bps
Sources of return

1. coupon payments

2. capital gain or (loss)

3. reinvestment (of coupons over holding period) income

4. return of principal or amortization
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calculate future value to maturity of coupon stream
Determining Interest on Interest

1. calculate future value to maturity of coupon stream
2. subtract dollar value of all coupons
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2. subtract dollar value of all coupons
3. remember future value of one dollar annuity with $N$ payments at the rate $r$ is given by

$$S(i, N) = \frac{1}{i}((1 + i)^N - 1)$$
Determining Interest on Interest

Example

- 10-year, 6% (paid semiannually) bond selling for 100
- coupons reinvested at $r = 4\%$ compounded semiannually

$1$ yield: $y = 6\%$
Determining Interest on Interest

Example

- 10-year, 6% (paid semiannually) bond selling for 100
- Coupons reinvested at $r = 4\%$ compounded semiannually

1. Yield: $y = 6\%$
2. $FV @ 4\%$ csa

\[
FV = 3S(.02, 20) \\
= \$72.892
\]
Determining Interest on Interest

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$$72.892 - 3 \times 20 = 12.892$$
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   - total coupon = 60
Determining Interest on Interest

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4. sources of return:
   - total coupon = 60
   - interest on interest = 12.892
   - built-in-capital gain (or loss) =

\[
100 - P = 100 - 100 = 0
\]
Promise return equals the yield to maturity if

1. the bond is held to maturity
2. all coupon interest payments are reinvested at the yield to maturity

Total return is the return on the bond that equates the price to the future value of the reinvested coupons.

Suppose you pay $100 for a ten-year bond and reinvest the coupons for the life of the bond. The reinvested coupons grow to $80 at the end of ten years.

You started with $100 dollars; now you have $180.

What is your total return?

\[
\left( \frac{180}{100} \right)^{1/10} - 1 = 6.054\%
\]
Total return

Example

- 10-year 6% csa bond selling for 100
- cash flows reinvested at $r = 4\%$ csa

1. **yield:**

\[
y = 6\%
\]
10-year 6% csa bond selling for 100
- cash flows reinvested at $r = 4\%$ csa

1 yield:
$$y = 6\%$$

2 $FV$ @ $4\%$ csa

$$FV = 100 + 3S(.02, 20)$$
$$= \$172.892$$
Total return

Example

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1. yield:
   
   \[
   y = 6\%
   \]

2. FV @ 4\% csa
   
   \[
   FV = 100 + 3S(0.02, 20) = \$172.892
   \]

3. Total Return as BEY:
   
   \[
   \frac{TR}{2} = \left( \frac{172.892}{100} \right)^{1/20} - 1
   \]
Total return

Example

1. 10-year 6% csa bond selling for 100
2. Cash flows reinvested at $r = 4\%$ csa

1. **yield:**

   \[ y = 6\% \]

2. **FV @ 4\% csa**

   \[ FV = 100 + 3S(.02, 20) = \$172.892 \]

3. **Total Return as BEY:**

   \[ \frac{TR}{2} = \left( \frac{172.892}{100} \right)^{1/20} - 1 \]

4. So

   \[ TR = 2 \left[ \left( \frac{172.892}{100} \right)^{1/20} - 1 \right] \]
Total return versus reinvestment rate

HPY versus reinvestment rate graph.