1 Creating an Inverse Floater

Say, we have collateral of $M$ dollars in a portfolio of bonds that pays $cM$ annually. We want to create a floating rate bond that pays $a + r$ percent annually and an inverse floater that pays $K - Lr$ annually for five years. Create two tranches with par value of $M_1$ and $M_2$ such that total cash flows from the two equals the income generated by the collateral and the sum of the par values equals the par value of the collateral:

\[
Mc = M_1 (a + r) + M_2 (K - Lr) \\
M = M_1 + M_2
\]

Divide both sides of both equations by $M$:

\[
c = m_1 (a + r) + m_2 (K - Lr) \\
1 = m_1 + m_2
\]

where $m_1, m_2$ are the fractions in each tranche. For this to work, the floating rate components must offset each other:

\[m_1 r - m_2 Lr = 0\]

so that

\[L = \frac{m_1}{m_2} = \frac{m_1}{1 - m_1}\]  

(1)

If the floating portions offset, then the fixed components must sum to $c$:

\[c = m_1 a + m_2 K\]
\[= m_1 a + (1 - m_1)K\]
\[= m_1 (a - K) + K\]

Then

\[m_1 = \frac{c - K}{a - K} = \frac{K - c}{K - a}\]  

(2)

Substitute for $m_1$ from (2) into (1):

\[L = \frac{K - c}{c - a}\]

or

\[K = c + L(c - a)\]

Solve (1) for $m_1$:

\[m_1 = \frac{L}{L + 1}\]

The quoted margin $a$ must be the same as other floating rate bonds with the same risk. Given $a$, we are free to choose $K$ or $L$ – but not both.
2 Example

A. Suppose the collateral pays \( c = 4\% \), the market determines \( a = 0 \), and we choose \( L = 1 \). Then \( m_1 = \frac{1}{1+1} = \frac{1}{2} \), \( m_2 = \frac{1}{2} \), and \( K = 4 + (4 - 0) = 8\% \). So that

\[
CR_F = r \\
CR_{IF} = 8 - r
\]

Check results:

\[
4 = \frac{1}{2}r + \frac{1}{2}(8 - r)
\]

B. Suppose \( L = 2 \). Then \( m_1 = \frac{2}{2+1} = \frac{2}{3} \), \( m_2 = \frac{1}{3} \), and \( K = 4 + 2(4 - 0) = 12 \). So that

\[
CR_F = r \\
CR_{IF} = 12 - 2r
\]

Check results:

\[
4 = \frac{2}{3}r + \frac{1}{3}(12 - 2r)
\]

3 Floor and Ceiling

Since the coupon rate on the inverse floater can never be negative (why?), we need to attach a floor to the inverse floater. In Example A, this means that \( r_{\text{max}} \) must be less than or equal to 8\%. Suppose \( r_{\text{max}} = 6\% \). Then the inverse floater has a floor of 2\% and the floater a ceiling of 6\%. The investors in the floater may not be happy that the maximum rate is capped but there is no floor rate. We resolve this problem by setting a floor on the floater of 2\%. Now the inverse floater will have a ceiling at \( 8 - 2 = 6\% \). The rate from the floater is determined by

\[
CR = \begin{cases} 
2 & \text{if } r \leq 2 \\
r & \text{if } 2 < r < 6 \\
6 & \text{if } r \geq 6 
\end{cases}
\]
The rate from the inverse floater is determined by

\[
CR = \begin{cases} 
6 & \text{if } r \leq 2 \\
8 - r & \text{if } 2 < r < 6 \\
2 & \text{if } r \geq 6
\end{cases}
\]

Notice that \( \frac{1}{2} CR_F + \frac{1}{2} CR_{IF} = 4\% \) for all \( r \).

The payoff rate from the floater looks like a collar. We bought a pure floater. Sold a call option on the underlying rate \( r \) with a strike of 6\% and bought a put
with a strike of 2%. For the quoted margin to stay a 0, the collar would have to be zero cost. In other words, the cost of call and put would have to be equal.

4 Case

Create a floating and inverse floating tranche secured by $10 million in par bonds with a coupon rate of 5.0% percent paid semiannually for five years. Suppose the market rate on a floater with the same default rate is six-month T-bill rate (reference rate) plus 40 basis points and a floor of 2% and a ceiling of 8%. The floating rate tranche has a maturity of five years and the rate is reset every six months.

1. If the leverage factor \((L)\) equals 1, design both the floating rate and inverse floating rate tranches. For both the tranches, specify the coupon rate formula, the par values, maturity, and ceiling rate, and floor rate.

2. Demonstrate that the total cash flow from both tranches equals the income generated by the underlying collateral, regardless of the reference rate. Must demonstrate result for three regions: floor, ceiling, and in between.

3. Graph the coupon rate versus the reference rate for both the floating and inverse floating bonds.

4. Suppose in one year (immediately after the coupon is paid) the value of the collateral drops to $9 million due to rising interest rates. Suppose the floating rate bond sells for par. Estimate the value of the inverse floating bond (per 100 par value).

5. If the leverage factor \((L)\) equals 1.5, design both the floating rate and inverse floating rate tranches.

6. Graph the coupon rate versus the reference rate for both the floating and inverse floating bonds.

7. Suppose in one year (immediately after the coupon is paid) the value of the collateral drops to $9 million due to rising interest rates. Suppose the floating rate bond sells for par. Estimate the value of the inverse floating bond (per 100 par value).