HW4 Solutions

1. The effect of a government tax increase of $100 billion on (a) government saving, (b) private saving, and (c) national saving can be analyzed by using the following relationships:

\[
\text{National Saving} = \text{[Private Saving]} + \text{[Government Saving]}
\]
\[
= [Y - T - C(Y - T)] + [T - G]
\]
\[
= Y - C(Y - T) - G.
\]

a. Government Saving—The tax increase causes a 1-for-1 increase in public saving. \(T\) increases by $100 billion and, therefore, government saving increases by $100 billion.

b. Private Saving—The increase in taxes decreases disposable income, \(Y - T\), by $100 billion. Since the marginal propensity to consume (\(MPC\)) is 0.6, consumption falls by 0.6 \(\times\) $100 billion, or $60 billion. Hence,
\[
\Delta \text{Private Saving} = -100b - 0.6 (-100b) = -40b.
\]

c. National Saving—Because national saving is the sum of private and public saving, we can conclude that the $100 billion tax increase leads to a $60 billion increase in national saving. Another way to see this is by using the third equation for national saving expressed above, that national saving equals \(Y - C(Y - T) - G\). The $100 billion tax increase reduces disposable income and causes consumption to fall by $60 billion. Since neither \(G\) nor \(Y\) changes, national saving thus rises by $60 billion.

d. Investment—To determine the effect of the tax increase on investment, recall the national accounts identity:
\[
Y = C(Y - T) + I(r) + G.
\]
Rearranging, we find
\[
Y - C(Y - T) - G = I(r)
\]
\[
\text{National Saving} = I(r)
\]

The left-hand side of this equation is national saving, so the equation just says that national saving equals investment. Since national saving increases by $60 billion, investment must also increase by $60 billion.

2. a. Private saving is the amount of disposable income, \(Y - T\), that is not consumed:

\[
\text{Private Saving} = Y - T - C(Y - T) = Y - T - MPC(Y - T)
\]
\[
= 5,000 - 1,000 - (250 + 0.75(5,000 - 1,000))
\]
\[
= 750.
\]

Public saving is the amount of taxes the government has left over after it makes its purchases:

\[
\text{Public (Government) Saving} = T - G
\]
\[
= 1,000 - 1,000
\]
\[
= 0.
\]

Total saving is the sum of private saving and public saving, which will be 750.

b. The equilibrium interest rate is the value of \(r\) that clears the market for loanable funds. We already know that national saving is 750, so we just need to set it equal to investment:
\[
S = I
\]
\[
750 = 1,000 - 50r
\]
Solving this equation for \(r\) we find \(r = 5\%\).

c. When the government increases its spending, private saving remains the same as before (notice that \(G\) does not appear in the private saving above) while government saving decreases. Putting the new \(G\) into the equations above:

\[
\text{Private Saving} = 750
\]
\[
\text{Government Saving} = T - G
\]
Thus, $S = 750 + (-250) = 500$.

d. Once again the equilibrium interest rate clears the market for loanable funds:

$$S = I$$

$$500 = 1,000 - 50r$$

Solving this equation for $r$ we find $r = 10\%$.

3. i. Saving: National saving $= Y - T - C(Y - T) + T - G = Y - MPC(Y - T) - G$

Thus, the change in saving will be:

$$\Delta S = (1 - MPC)\Delta Y + MPC * \Delta T - \Delta G$$

Since the production function is $Y = AF(K,L)$, then an increase in $A$ will push output up which give us a positive change in $Y$. Since $MPC$ less than one, then $1 - MPC$ is positive which makes $(1 - MPC)\Delta Y$ positive. $\Delta T = \Delta G = 0$, so $\Delta S = (1 - MPC)\Delta Y > 0$. Thus, national saving will increase.

ii. Investment: $I = \Delta K_t + dK_t = K_{t+1} - K_t + dK_t$

$K_t$ is decided by the benefit and the cost of investment, which are $MPK$ and $(r+d)\rho_k$. When $A$ increases, $MPK$ increases too. Then, the equilibrium level of $K$ will also increase. Thus $\Delta K_t > 0$ and investment increases.

iii. Interest rate: If $MPK$ increases, then in a competitive market the rental price of capital will also increase, since $MPK = r/P$. Thus, the interest rate will also increase.

In the graph, both investment and saving move to the right. The new equilibrium level of investment, saving, and interest rate are higher.