HW 8: Due on Oct. 17th before class

1. (Textbook, 7th e, chapter 7, question 1)

Country A and country B both have the production function

\[ Y = F(K, L) = K^{1/2}L^{1/2} \]

a. Does this production function have constant returns to scale? Explain.
b. What is the per-worker production function, \( y = f(k) \)?
c. Assume that neither country experiences population growth or technological progress and that 5 percent of capital depreciates each year. Assume further that country A saves 10 percent of output each year and country B saves 20 percent of output each year. Using your answer from part (b) and the steady-state condition that investment equals depreciation, find the steady-state level of capital per worker for each country. Then find the steady-state levels of income per worker and consumption per worker.
d. Suppose that both countries start off with a capital stock per worker of 2. What are the levels of income per worker and consumption per worker? Remembering that the change in the capital stock is investment less depreciation, use a stable to show how the capital stock per worker will evolve over time in both countries. For each year, calculate income per worker and consumption per worker. How many years will it be before the consumption in country B exceed the consumption in country A?

2. (Textbook, 7th e, chapter 7, question 2)

In the discussion of German and Japanese postwar growth, the text describes what happens when part of the capital stock is destroyed in a war. By contrast, suppose that a war does not directly affect the capital stock, but that casualties reduce the labor force. Assume the economy was in a steady state before the war, the saving rate is unchanged, and the rate of population growth after the war returns to normal.

a. What is the immediate impact of the war on total output and on output per person?
b. What happens subsequently to output per worker in the postwar economy? Is the growth rate of output per worker after the war smaller or greater than normal?

3. (Textbook, 7th e, chapter 7, question 4)

“Devoting a larger share of national output to investment would help restore rapid productivity growth and rising living standard.” Do you agree with this claim? Explain.

4. (Textbook, 7th e, chapter 7, question 6)

Many demographers predict that the United States will have zero population growth in the twenty-first century, in contrast to average population growth of about 1 percent per year in the twentieth century. Use the Solow model to forecast the effect of this slowdown in population growth on the growth of total output and the growth of output per person. Consider the effects both in the steady state and in the transition between steady states.

5. (Textbook, 7th e, chapter 7, question 8)

Consider how unemployment would affect the Solow model. Suppose that output is produced according to the production function \( Y = K^{\alpha}(1-u)L^{1-\alpha} \), where \( K \) is capital, \( L \) is the labor force, and \( u \) is the natural rate of unemployment. The national saving rate is \( s \), the labor force growth at rate \( n \), and capital depreciates at rate \( \delta \).

a. Express output per worker \( (y=Y/L) \) as a function of capital per worker \( (k=K/L) \) and the natural rate of unemployment. Describe the steady state of this economy.

b. Suppose that some change in government policy reduces the natural rate of unemployment. Describe how this change affects output both immediately and over time. Is the steady-state effect on output larger or smaller than the immediate effect? Explain.